Digital Speech Processing
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Lecture - 10
Uniform Tube Modeling Of Speech Processing Part – II

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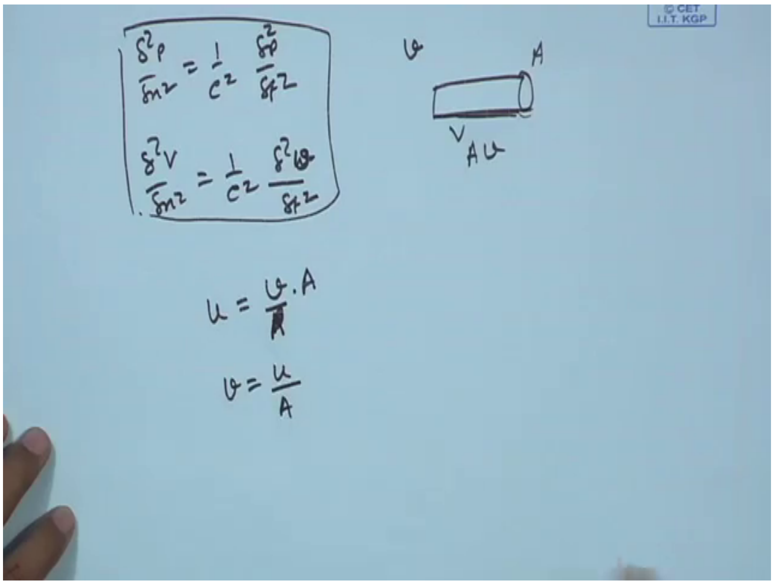

So we have derived that 2 wave equation. This 2 wave equation we have derived. Now here one things is there v is the particle velocity. Now if I say volume velocity. So, if you say volume velocity if the velocity is v of let us tube the cross sectional area is A. Then what is the volume velocity A into v the whole total volume v is the length of the tube and A is the cross sectional area. So, v is the a into v is the volume. So, if I say volume velocity is u then u is equal to v by a or sorry, u equal to v into A. Or I can say v is equal to u by A.

Now, if I put this thing on put this thing on this 2 equation.

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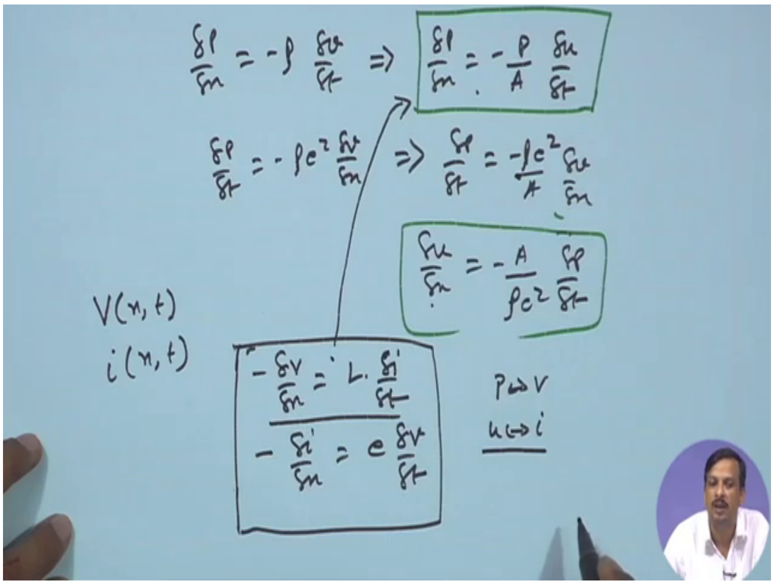

Where I can say that del square or del p by del x is equal to minus rho del v by del t. So, this will become v is equal to u by A. So, del p by del x is equal to minus rho p u by A, rho by A del u by del t. Similarly del p by del t is equal to minus rho c square, del v by del x this will become del p by del t is equal to minus rho c square, rho c square minus rho c square, or I can say del u sorry.

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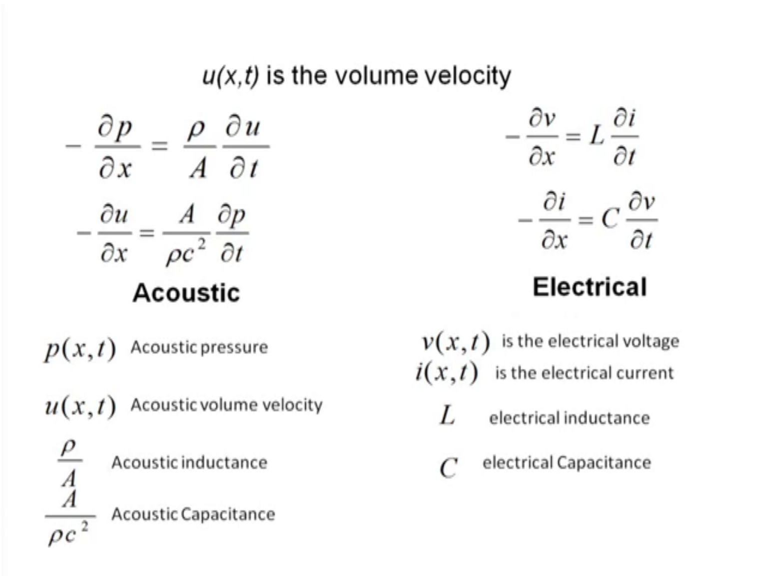

This is del u by del t, not del p, del u by del t is or rho c square by a del v by del x. Or I can say del u by del x is equal to minus A by rho c square del p by del t.

Now if you see this equation and this equation. These 2 equation if you see. This is analogous to electrical transmission line equation. So, electrical transmission line voltage v is a function of x and t and current I is also a function of x and t transition line. So, in that case in electrical transmission line minus del v by del x is equal to l into del I by del t. Similarly minus del I by del x is equal to c into del v by del t. Now this is analogous to So these 2 equation analogous to this is the electrical transmission line equation. Lossless electrical transmission line equation. Then I can say if this equation is analogous to this equation, then I can say that p is analogous to v.

So, pressure in acoustic domain is analogous to voltage in electrical domain. Similarly u is analogous to i. Volume velocity in acoustic domain is equivalent to electrical current in electrical domain. And if I say l is equal to rho by A. So, this rho by a is called acoustical inductance, acoustical inductance. Similarly I can say that a by rho c square is analogous to c. So, it is called acoustical capacitance A by rho c square is acoustical capacitance and rho by A is the acoustical inductance, p is analogous to voltage, current is analogous to volume velocity.

So, if you see this slide I have written this slide same things I have written in this slide. Now So, this I know So, I can say that if I know the equation I can analogous electric transition line I can also draw. This same as electrical transmission line lossless transmission line now. So, what I want to know what I want to do is that I want to find out the transfer function of a uniform tube.

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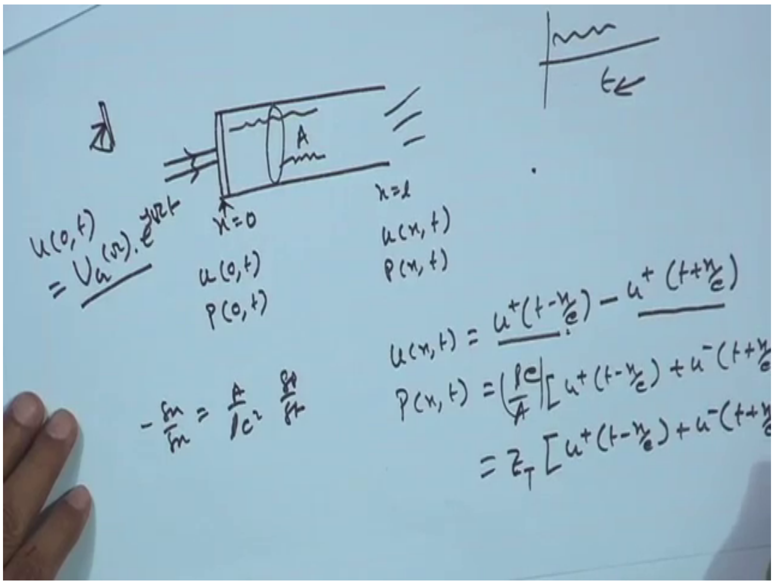

Which is I can say this side is the vocal cords which acts as a piston. And so, x is equal to 0 where the force is applied in the tube and x is equal to l where the tube radiate the sound energy in the air.

Now if I see that that this one; this tube whole tube is nothing but a this kind of switches of pressure or you can say the pressure is applied in here. The acoustical pressure is applied in here. So, I can say the particle velocity in this point I can say or volume velocity in this point is u 0 t. And volume velocity at this point is u x t. Pressure at this point is nothing but p 0 t, pressure at here is p x t and cross section of the tube is A the area cross sectional area of the tube is A. I will consider throughout the tube a remain constant. So now, A remain constant. Now try to find out the traveling wave solution in the tube what is travelling wave solution? Now if I see that minus del u by del x is equal to a by rho c square del p by del t, or I can write down these 2 equations where I said this is the this is the pressure wave equation or not this not this one where it is this will be here.

So, this is the pressure wave equation, is the pressure wave equation and this is the velocity wave equation. Now I have to find out the solution of this second order differential equation, and again solution of this second order differential equation will give me the velocity. So, if I draw the solution of the second order differential equation, you know that second order differential is total solution is nothing but A. So, I can say that u x t is equal to u plus t minus x by c minus u plus t plus x by c. What is that? T minus x by c. So, I can say this is called forward wave this is called backward wave. So, in the tube there is a forward wave and there is a backward wave.

In the forward wave I can say the if the time axis this is the time axis in the forward wave, then t is minus x by c, if it is the backward wave. So, negative direction t; t plus x by c; c is the velocity of the sound x is the position. Now similarly what is p x t, p x t is nothing but a rho c by A u plus t minus x by c plus u minus t plus x by c. So, I this I can say is nothing but A z z t which is called impedance of the transmission line or the impedance of the acoustic line is nothing but a u plus t minus x by c plus u minus t plus x by c. So, what z d is equal to rho c by A. So, u plus is the forward wave u minus is the backward traveling wave.

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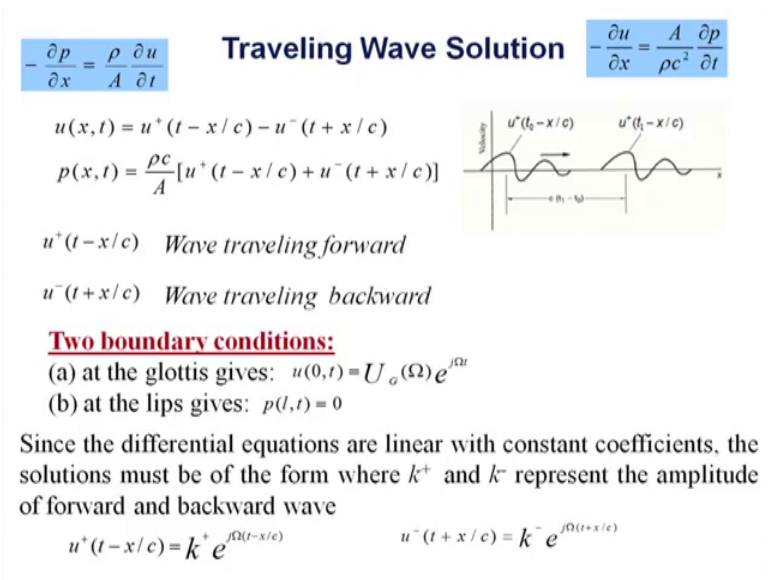

Now, if it is that then there is there is a 2 boundary condition in the tube if you see what is the boundary condition at the glottis at the here the glottis point where the vocal cords are vibrating. Here u 0 t is nothing but a glottal excitation, which is nothing but a let us glottal excitation is U G omega and equal to it into e to the power j omega t.

Where this omega is the continuous frequency. So, all frequency the frequency of u. So, I can say I excited the tube with unit impulse or I guess every frequency and I have to find out the every frequency response at the output. That is why I said omega e j omega t and this excitation is if it is impulse then that or you can say the excitation contain all the frequencies equal amplitude, and that amplitude is modified as for the glottal response glottal frequency response that is these U G omega. So, I can say the glottal frequency response which is U G omega has to be passed through the tube, and the tube output of the tube I have to find out and then output by input is the tube transfer function.

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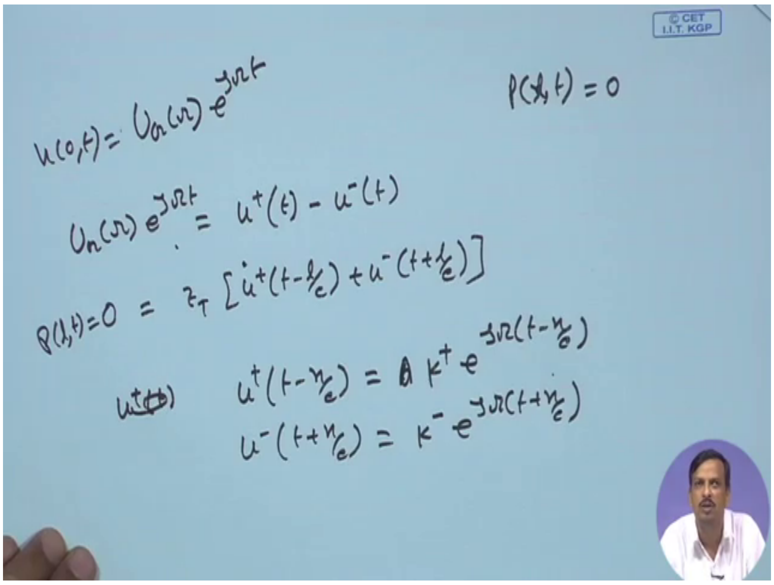

So, u 0 t u 0 t is nothing but a U G omega e to the power j omega t. And at the lips let us there is a radiation load is for the low frequency radiation load is completely not there. So, what will happen at the lips? It again it is atmospheric pressure. So, I can say p x p l t at x equal to l is equal to 0. So, if I put p l t. So, what is u 0 t? So, u 0 t is U G omega e to the power j omega t is equal to u plus. So, x is equal to 0. So, I can say u plus t minus u minus sorry, this will be minus u minus t x equal to 0. So, x equal to 0 will be not there. So, I can say u u plus t u minus t.

And if it is p l t then p l t is equal to 0. So, 0 is p l t is equal to 0. So, I can say z t into u plus t minus l y c plus u minus t plus l y c. So, there is a differential equation. So, since the differential equation are linear with constant coefficient the solution must be in the form of k plus k minus represent the amplitude and the forward and backward wave. So, I can say if it is linear wave equation, forward wave u plus t or you can say that u plus t minus x by c can be represented in the form of amplitude, let us this is k plus e to the power j omega t minus x by c. Differential equation second order differential equation, and u minus I can say t plus x by c is nothing but a k minus e to the power j omega t plus x by c, exponential solution. So, this is the exponential solution. Now if I put that value in 2 equation; equation number one in here and equation number 2 in here.

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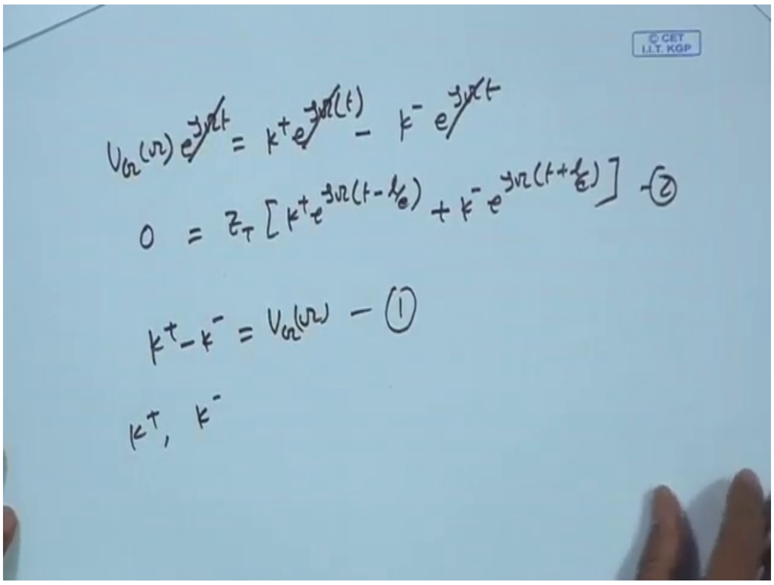

So, in one I can say U G omega e to the power j omega t is nothing but a k plus e to the power j omega t. Because x is equal to 0, minus k minus e to the power j omega t x equal to 0. And similarly 0 is equal to z t into u plus k plus e to the power j omega t minus l y c plus k minus e to the power j omega t plus l y c. From here I can say e to the power j omega t e to the j omega t cancel. So, I can say k plus minus k minus is equal to U G omega, and here I can say z t is equal to k to this is equal to this. So, I can this is the equation number one and this is equation number 2 again I can solve for k plus. And k minus I am not deducing the solution because you can do it easily.

So, you can just deduce the solution and find out the k plus and k minus.

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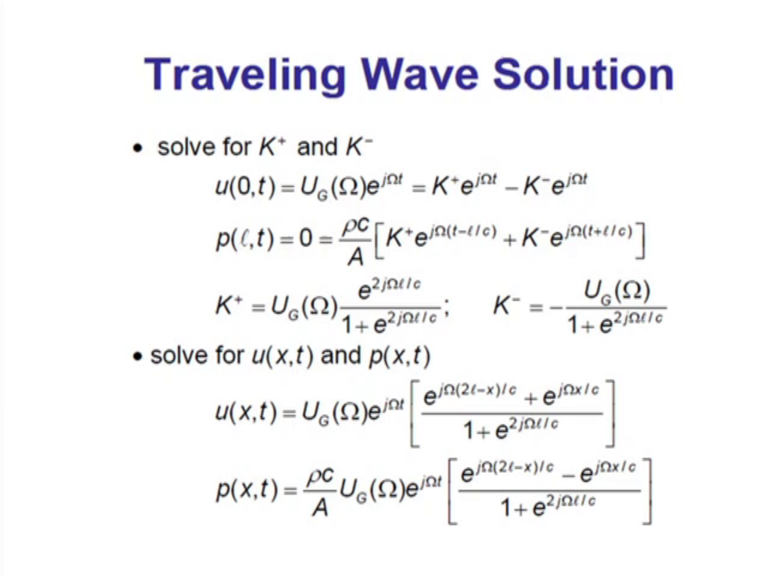

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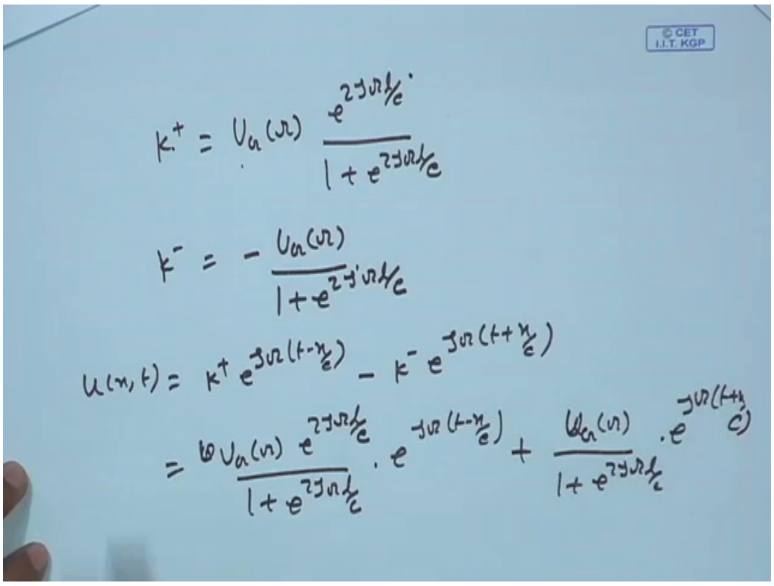

So, k plus and k minus will be in the form of k plus will be equal to U G omega e to the power 2 j omega l by c divided by 1 plus e to the power 2 j omega l y c, this is the k plus. And k minus will be minus U G omega divided by 1 plus e to the power 2 j omega l by c. Once I know the k plus and k minus, then I can write down the u x t, u x t is nothing but a k plus e to the power j omega t minus x by c minus e to the k minus e to the power j omega t plus x by c. Now I put the value of k k plus is nothing but a U G omega this value in here and k minus I can put the value in here. Then if I put the value then I can say U G u g omega e to the power 2 j omega l by c divided by 1 plus e to the power 2 j omega l y c, into e to the power j omega t minus x by c minus u g. So, it is minus, minus plus U G omega U G omega divided by 1 plus e to the power 2 j omega l by c into e to the power j omega t plus x by c.

Now, I can say that U G omega is common. So, U G omega e to the power j omega t will be same. E to the power j omega t then I can say it is nothing but a e to the power j omega 2 l 2 l minus x divided by c minus plus e to sorry; plus e to the power j omega x by c divided by 1 plus e to the power 2 j omega l by c. Now if it is that then I can solve for that this is the U G u omega.

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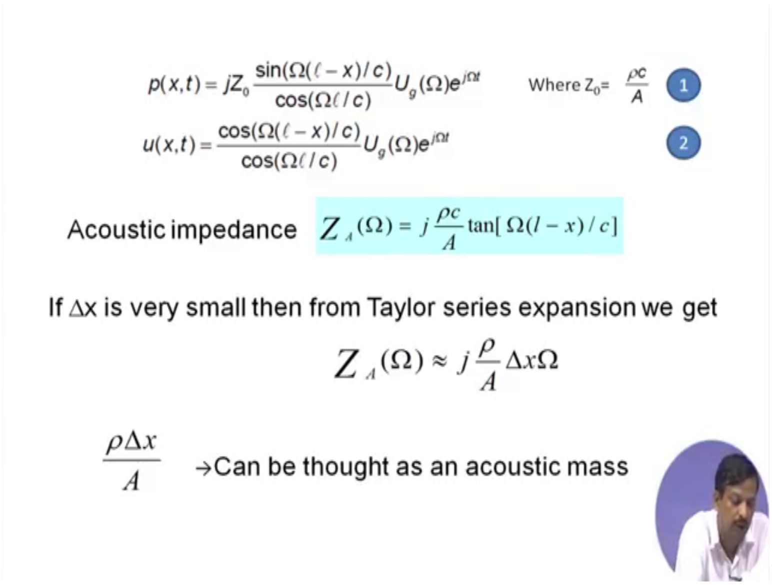

So, U G omega if I simplify it then u j omega sorry this is a u x t.

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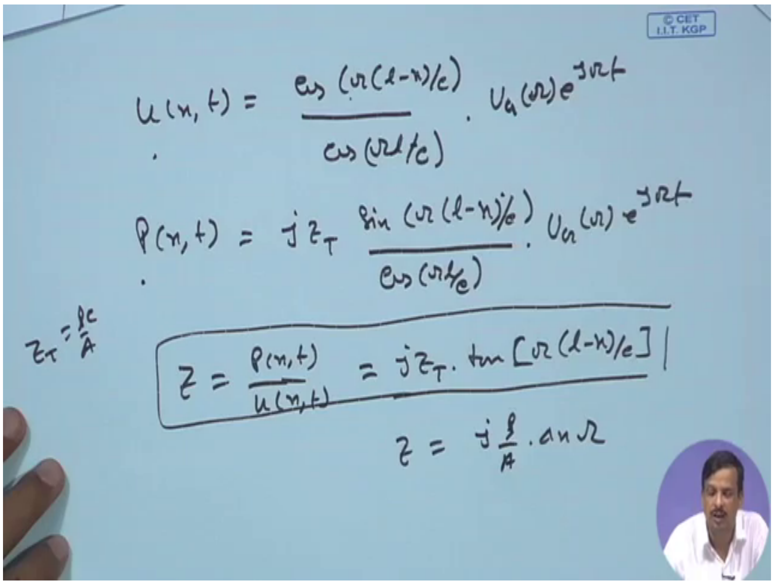

So, ultimately If I again simplify it u x t will be in the form of cos omega l minus x by c divided by cos omega l by c cos omega l by c into U G omega e to the power j omega t.

Here if I say that e to the power j theta, look that way if I do it l minus l minus x if I do that then it will cos omega l minus x by c into cos omega l if I simplify it. Similarly I will get the p x t p x t will be j z t sin omega l minus x divided by c divided by cos omega l by c into U G omega e to the power j omega t. Where z t is equal to rho c by A. Now I get this to u and p. Now what is if you see p is analogous to voltage, u is analogous to current. Now if I say what is the; you can say the impedance, z is nothing but a p x t divided by u x t if I do that p x t divided by u x t.

So, it will come j z t into tan omega l minus x by c. Tan omega l minus x by c this divided by this. So, that this divided by this. So, cos by sin sin by cos tan this will come. So, this is nothing but a acoustics impedance inside the tube at any point of x. So, if x is equal to something I can find out the acoustic impedance. So now, if the del x if the x is variation of x is very small then I can say that z is equal to nothing but a z z is nothing but a j omega a rho by c rho by a into del x omega. So, simplification of that for a small x del x is very small then I can do that now this is a z.

Now, if I interpreted this thing the part this is particle velocity; this is pressure wave, this is volume velocity and this is pressure wave. Now if I interpreted that thing. What is the envelope here envelop is z t pressure wave z t sin omega l minus x by c, this is part is the envelop part this part will be same variation e to the j omega t. That is that is like a envelop part, and this is the envelop part in here. Now if this portion is constant if the l is constant this portion is constant. So, one is sin another is cos. So, if I say how the pressure amplitude, pressure wave amplitude and volume velocity amplitude is very inside the tube.

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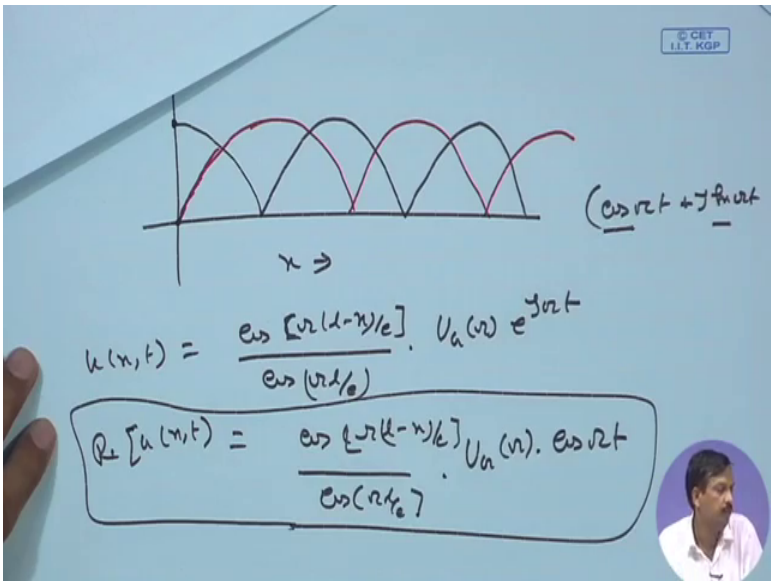

Then I can say only x axis here x only I am drawing the envelop portion.

So, one is sin another is cos. So, if this is cos is the volume velocity see the particle velocity is maximum then pressure wave is minimum 0. So, particle velocity will vary like this, then pressure wave for a volume velocity sorry, u is the volume velocity pressure wave will be look like this. One is cos another is sin. Now if I say real part of this part the real part of this. So, I can say if I write u x t is equal to cos of omega l minus x by c divided by cos omega l by c into U G omega e to the power j omega t. So, e to the power j omega t is nothing but a cos omega t plus j sin omega t.

So, cos part is the real part sin part is the imaginary part. So, I can say the real part of u x t is nothing but a cos omega l minus x divided by c, divided by cos omega l by c into U G omega cos omega t. Similarly I can get the real part of the pressure wave or not. So, real part of this u U G omega t is this one only. Now I come to the transfer function of the tube. So, in the tube I can say this is x equal to 0 this is x equal to l.

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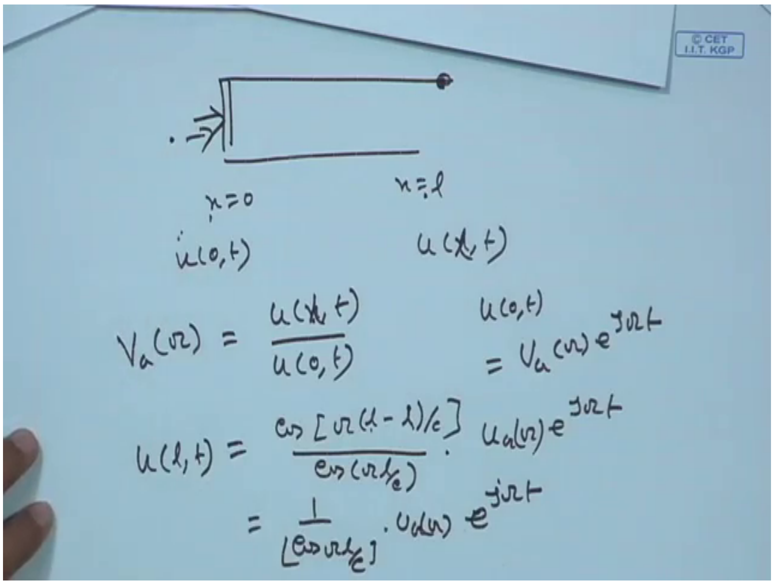

So, if I say input is u 0 t and output is u x t volume velocity. So, I can say the v a transfer function v a omega is nothing but a u x sorry u l t u l t divided by u 0 t. X is l and x is 0 here. So, what is that?

Now, if I say that u is equal to cos omega l minus x by c. So, at u at l x is equal to l cos omega l minus at x equal to l x equal to l divided by c divided by cos omega l by c into U G omega e to the power j omega t, which is nothing but u l t. So, u l t it is l minus l is 0. So, it is nothing but a 1 by cos omega l by c. U G omega e to the power j omega t. Now what is u 0 t, u 0 t is nothing but a U G omega it is what g omega t which is bottle impulse at the glottis point here. So, if I say u l t by u u 0 t.

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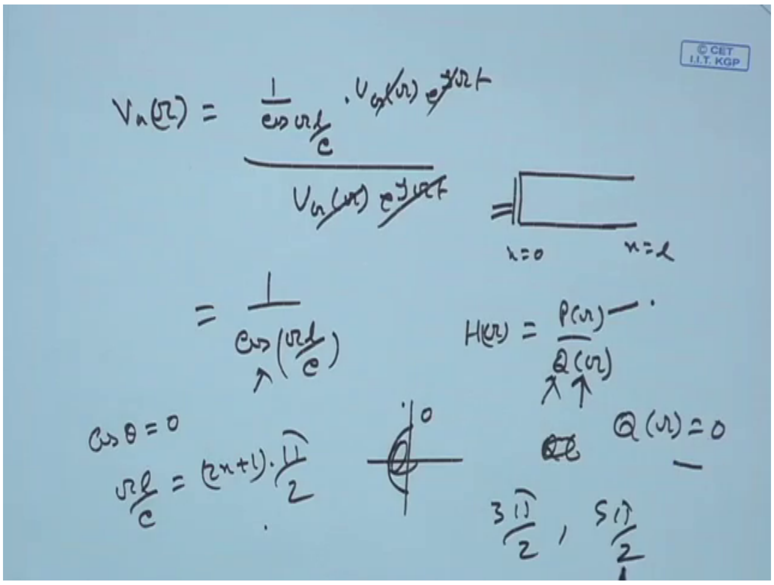

So, b a omega is nothing but a u l t 1 by cos omega l by c into U G omega e to the power j omega t divided by U G omega e to the power j omega t. So, this, this, this will cancel. So, it is nothing but a 1 by cos omega l by c.

So, transfer function of the uniform tube where this side is closed and mouth is open x equal to 0 and x equal to l is 1 by cos omega l by c. Now if it is 1 by cos omega l by c, then what is the pole position? What do you mean by pole position? Pole you know that pole. So, any transfer function h omega is nothing but consistence of p omega divided by Q omega. So, there is some pole and some solution of p will give me that 0 solution of Q will give me the pole. So, if you see solution of Q when Q omega Q omega is equal to 0 then what is the amplitude value value is infinite; that means, at the pole the resonance of the system is occur.

So, I get the highest energy maximum energy resonance will be occur at pole every pole. So, if I have a 5 pole I get 5 resonance frequency because. So, if the fifth order Q is the order is the 5th; that means, I can get the fifth solution 5 solution 5 solution give me the 5 resonant frequency how many solution I can get here. Cos theta is equal to 0, when? When omega l by c is equal to 2 n plus 1 into pi by 2 cos pi by 2 cos pi by 2 0. Again it will be 0 and wave pi. So, cos pi by 2 is equal to 0. So, it cos pi by 2 is equal to this way again it will be 0, but it is 3 pi by 2 when it is 5 pi by 2? That way if it is happened then it will be 0 ok.

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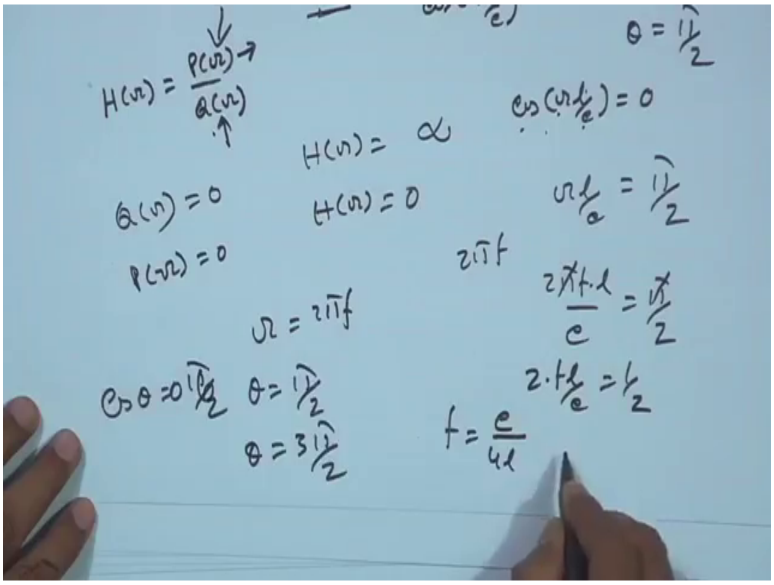

So, 1 by the b a is nothing but a 1 by cos omega l by c. So, what I described that any transfer function can be written any system transfer function h omega, can be written as p omega divided by Q omega, p omega divided by Q omega. Now if you know that solution of p omega this polynomial will give me the what? 0 and solution of this omega will give me the pole that is why p h mega is called the pole 0 function. So, 0 and pole now what is the physical significance of pole, suppose I said there is a pole, what do mean my pole? Physically what will happen? Now if you see what do you mean by solution of Q omega at solution point Q omega will be 0. If I put it 0, then h omega value will be infinite. At solution point of p omega p omega will be 0 the solution point of p omega then h omega will be 0.

So, I can see that if each solution point of Q omega; that means, in every pole the system response will be infinite; that means, system will resonant, and every solution for p omega which is nothing but a 0. The system response will be 0. That is why it is called pole 0. So, every pole corresponding to a resonance in speech those resonance are called formant. In speech those resonance are called formant. So, I know if I want to know the formant response or frequency response of b omega b a omega, then I have to find out where it is resonance is occur when the resonance will be occur when cos omega l by c is equal to 0.

So, if I want to know at which frequency this cos omega l by c is equal to 0, at which frequency? So, what is omega? Is continuous frequency is nothing but a 2 pi f. So, I put So, omega is equal to 2 pi f when cos theta will be 0? Cos theta will be 0 if theta is equal to pi by 2 theta is equal to pi by 2 cos theta is equal to 0. So, I know what is omega l by c should be pi by 2. Now when pi by 2 let us I put that 2 pi f l by c is equal to pi by 2. So, I know at theta is equal to pi by 2 cos theta is equal to 0. So, omega l by c is equal to pi by 2 pi if pi by 2 then cos omega l by c is equal to 0. So, resonance will be occur.

So, first resonance because cos theta can be 0 not only pi by 2, but it cos theta is equal to 0 if theta is equal to pi by 2 not only pi by 2 if theta is equal to 3 pi by 2 then also, it is 0 if it is 5 pi by 2 then also it is 0. So, let us minimum frequency first that is why we call first resonance frequency is theta is equal to pi by 2. So, pi cancel it is nothing but a 2 into f l by c is equal to half, or I can say f is equal to c by 4 l. Now what is c? C is the velocity of sound what is l? L is the length of the tube. Average human vocal cord length is equal to l is equal to call 17.5 centimeter.

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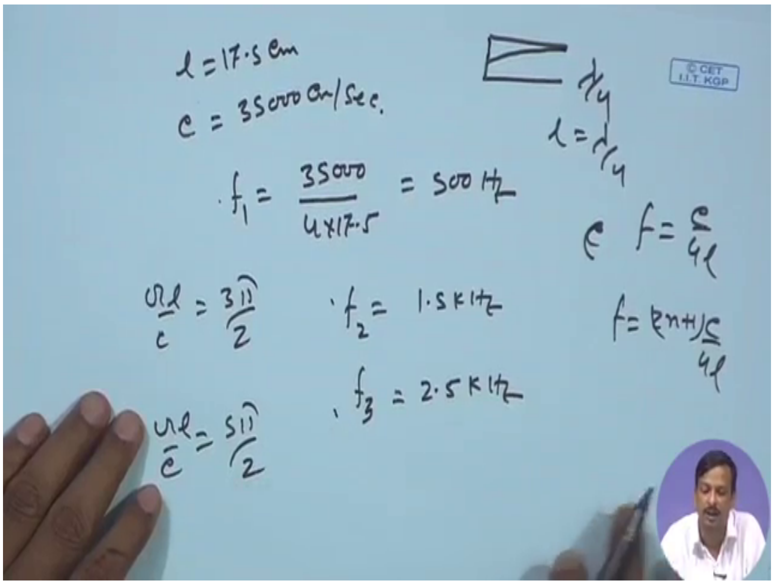

So, l is equal to 17.5 centimeters what is c? Velocity of sound velocity of sound is 3 5 centimeter per second.

So, I can say f is equal to 35000 divided by 4 into 17.5 which is equal to 500 hertz. If omega l by c is equal to 3 pi by 2 then f will becomes 1.5 kilo hertz. So, it is f 1 first formant second formant if it is third formant then omega l by c is equal to 5 pi by 2 then it will become 2.5 kilo hertz. If you see distance between the first formants and second formant is one kilo hertz second formant and third formant is one kilo hertz it is fixed because tube length is fixed. This is similar with your earlier physics theory that if this side is closed, then the first harmonics will occur at lambda by 4.

So, l is equal to lambda by 4. So, f is equal to c by lambda which becomes c by 4, l since the velocity and l is the length of the tube which is same in here also. Then it will be odd multiples of So, it is nothing f is equal to nothing but a 2 n plus 1 c by full length lambda. So, here also same. So, f 1 f 2 f 3 I can get now can I draw the frequency response of the uniform tube like there is no loss no nothing any loss. So, I can say if this is my frequency axis this is 500 hertz this is 1.5 kilo hertz, this is 2.5 kilo hertz. So, at 500 hertz power is maxima so on; it will going.

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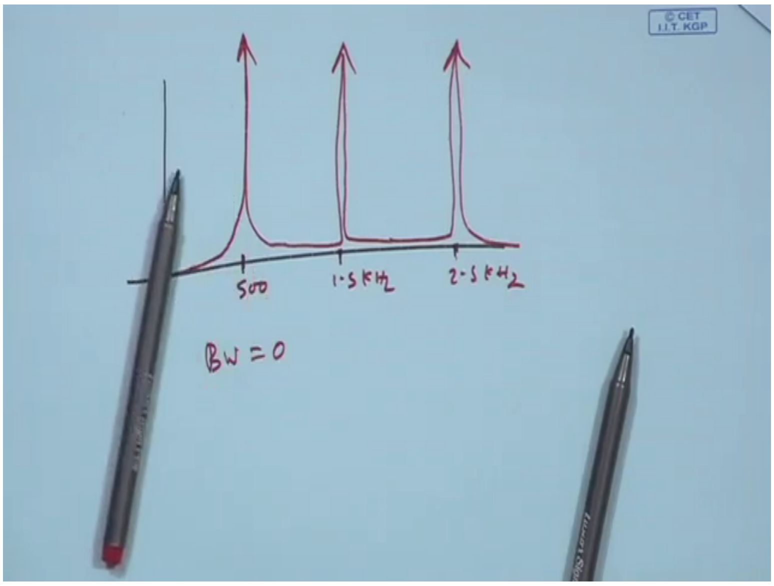

So, it is I can put an arrow in here. So, it is infinite power; infinite impulse with the impulse power at 500 hertz, and bandwidth is equal to 0. This is a real pole. So, bandwidth is equal to 0. Bandwidth is equal to 0 with 0 bandwidth, infinite power resonance will occur at 500 hertz, 1.5 kilo hertz, 2.5 kilo hertz, if the length of the vocal tube is 17.5 centimeter and velocity of the sound is 35000 centimeter per second.

So, that I know this is the frequency response for the uniform tube model with without considering any kinds of loss, or any walls is rigid wall not variable wall means walls is rigid. So, next lecture I will discuss how this response will be affected if I consider one loss at a time if I consider. One loss how it will be modified second loss how it will be modified. Then ultimately we can get what should be the frequency response or what should be the response of this vocal tract tube if I get it and what will be the transfer function.

Thank you.