

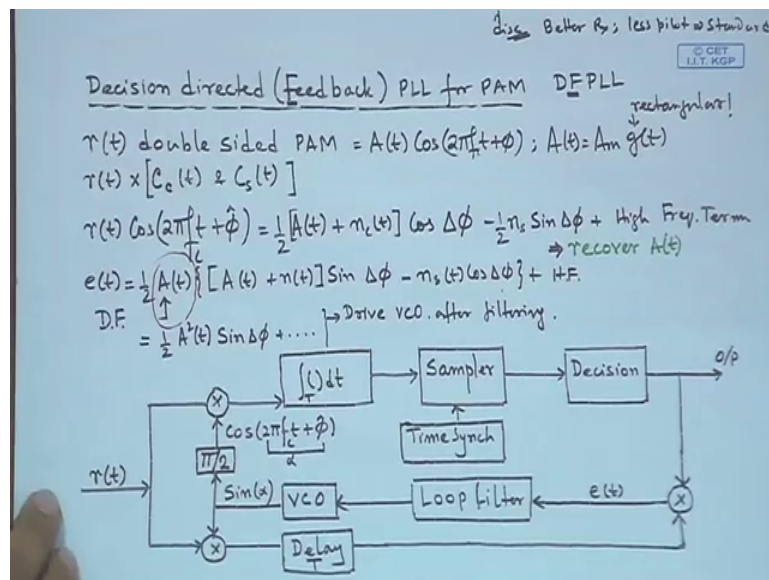
**Modern Digital Communication Techniques**  
**Prof. Suvra Sekhar Das**  
**G. S. Sanyal School of Telecommunication**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 60**  
**Synchronization Techniques (Contd.)**

Welcome to the lectures on modern digital communication techniques. So, far we have looked at transmitter, the channel, the receiver in ideal conditions in AWGN channel, under channel impairments and even under carrier phase impairments. Whatever we have discussed on the carrier in phase impairment would help us in recovering instantaneous phase of the signal.

We could also extend whatever we have done in the similar manner for estimating the carrier frequency as well, So that I give you to as an exercise for you to try out. So, we will go ahead a little bit further in this particular lecture, and present to you couple of more very popular or other techniques to recover the carrier phase and then we will move onto discuss symbol timing synchronization.

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So, we will take a look at the decision feedback or decision directed PLL for pulse amplitude modulated signals. So, it is called d f PLL and in this case the received signal  $r(t)$  is a double side band signal. So, the PAM is  $A(t) \cos 2\pi f_c t$  and  $A(t) = A_m g(t)$ . So, that is how we have this particular signal.

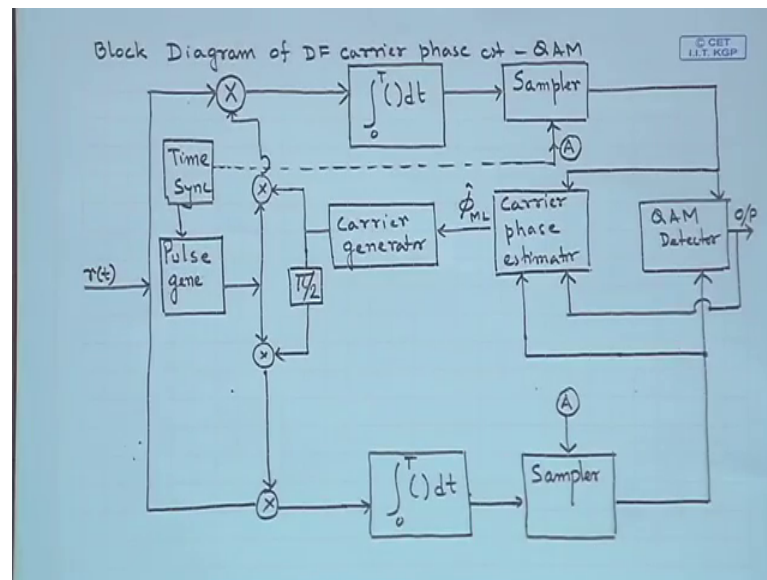
The received signal is multiplied by a cosine carrier and a sin carrier; that means,  $C$  indicates carrier reference carrier at the receiver. Sub  $c$  indicates the cosine and sub  $s$  indicate sin; that means, we generally as we have always drawn that you get a signal multiply by a cos and you multiplied by sin.

And then the  $r$  of  $t \cos 2\pi f_c t + \phi_{cap}$  that particular signal would look like half of  $A t$  plus noise and  $\cos \Delta \phi$ , because of  $\phi - \phi_{cap}$  and half  $n_s \sin \Delta \phi$  again. Because of the sin phase difference right. The high frequency components get removed. The So, this particular one that we have here as the product could be looked as the error term after we have removed the high frequency components. So, this error term if you look at it here, here we have  $A^2$  term along with the  $\sin \phi$  right. So, this is another alternative which is used as a phase recovery and this could be used to drive the VCO because it has a  $\sin \Delta \phi$  term available with it right.

So, the typical block diagram as it would turn out what the situation is the received signal, you have passed on to both directions you have VCO, which generates a  $\sin 2\pi f_c t + \phi_{cap}$ . So, I have written  $\alpha$  just to indicate the it is phase shifted by  $\pi$  by 2 feed on the other side. You integrated the loop filter you can think of So, which also removes the have high frequency high frequency signals. Sample it So, you need this sample symbol timing. And then make a decision this decision is  $A$  of  $t$  which is fed over here and multiplied. So, the decision is  $A t$  what we have decoded is multiplied and since we have  $A^2$  the  $\sin$  would go away. So, it will only have the magnitude and hence when pass through the loop filter removes the high frequency components and then it drives the VCO.

So, we will be left with only the  $\sin \Delta \phi$  term even the fluctuations would go away and therefore, we could build this kind of a circuit. So, this kind of a structure is also possible to use of course, this is an there is an approximation involved in this kind of implementation. For QAM we have a block structure as depicted here you get the received signal, split it into 2 directions, on one of the sides you multiply, let say by cosine the other side we multiply by the sinusoids.

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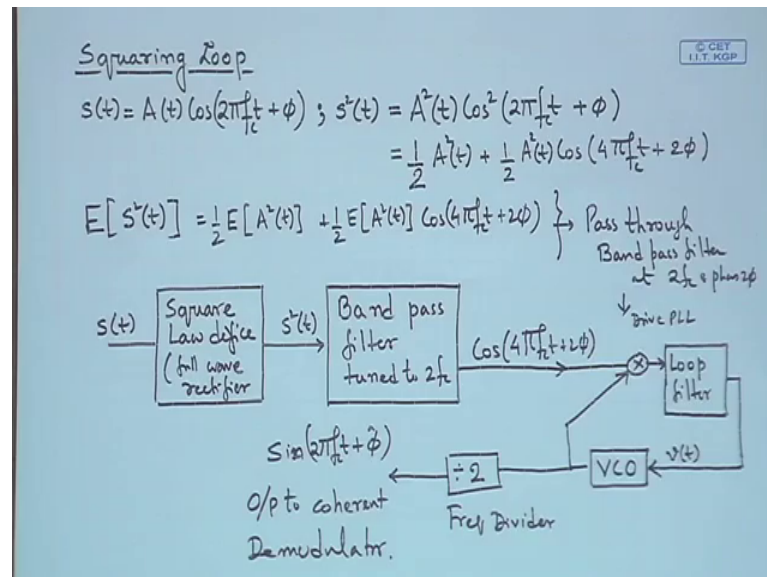
So, there is  $\pi$  by 2 shift from the carrier, integrate them. So, this part is generally common in most of the receivers. Then you sample them you sample them to sample you require the symbol timing. So, there you have the symbol timing, and of course, you are need to have the pulse which is multiplied with the received generator and the carrier received signal and the carrier.

So, once you have the pulse then you send it to the QAM detector. So, till this point you have the matched filter output of the in phase component. On this side you have the match filter output of the quadrature phase component. So when both of them go into the QAM detector you detect the I side PAM and you detect the q access pam. So, once you detected the I axis and q axis PAM you can decode the QAM. Once you decoded the QAM you would feed it to the carrier phase generator as well as you would feed these outputs of the samplers.

So, you can what you can recall over here is in the decision feedback for PAM, we had used  $I_N$  that is the data sequence and the output of the matched filter which is the  $Y_N$ . So, you have the output  $I_N$ , and you have  $Y_N$ . So, using them you can find the  $\phi_{ML}$ . So, this diagram is extending the result only to the QAM case. So, we are using the same expression as we have got and simply using that to derive the different decoding structures or the carrier phase estimators for different modulation techniques.

So, we will not discuss the non decision needed, we will skip that as there are certain philosophies involved which could be bit time consuming we will not keep it is part of our course.

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So, next we will go to the other important thing and which is very, very popular is the squaring loop, and many a times when you study digital communications or even analogue communications, you would implement this in a typical lab exercise, even we using discrete components.

So, let us say we have the signal  $s(t) = A(t) \cos(2\pi f_c t + \phi)$ . The first step is squaring this. So, when you square this you are going to get  $A^2(t) \cos^2(2\pi f_c t + \phi)$ . If you expand you going to get half  $A^2(t)$  plus half  $A^2(t) \cos(4\pi f_c t + 2\phi)$ . So now, you have twice the frequency components here.

So, if we filter this components and use this components; that means, we pass this output through a band pass filter, what we are going to get is a signal with twice the frequency component. So, that is what we have over here. So, we take the average of it because  $A(t)$  is a random sequence. So, you take the expected value of it.

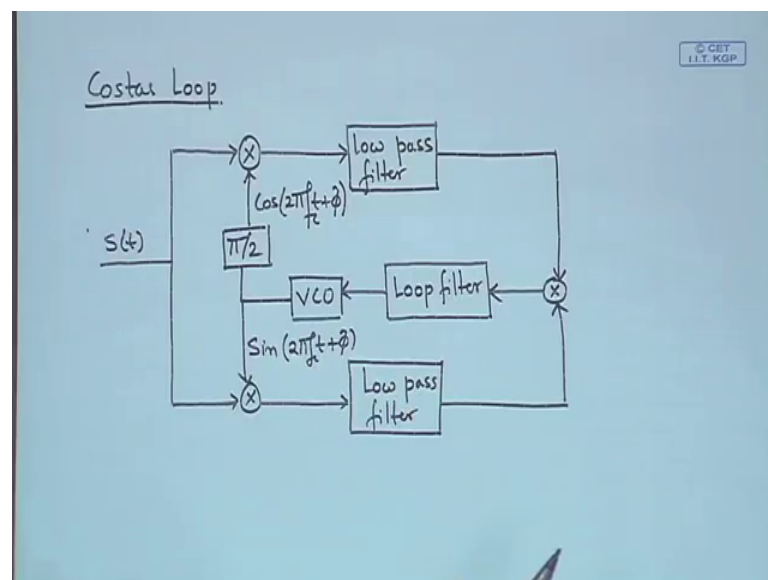
So, if you pass this signal so, this is the motivation. So, if you pass the average of this through a band pass filter, the end result will be a signal at 2 times  $f_c$  and of course, 2 times the phase. This could be used to drive a PLL, PLL we have seen before. So, if it

drives a PLL the outcome will be at twice the frequency then we can pass it through a frequency divider and we can get a outcome. So, this is one of the simplest carrier recoveries, but of course, it requires a PLL and what we had seen before is the realization of the PLL.

So, we have  $s(t)$  pass it through A square law device it produces  $A^2 \cos^2(2\pi f_c t + \phi)$ , pass it through a band pass filter tuned to  $f_c$ . So, you get  $\cos(4\pi f_c t + 2\phi)$  and then pass it through a loop filter right. It produces the signal which is at  $2f_c$  and send it through a VCO, it will produce  $2f_c$  divide it or send it through a frequency divider you are going to get  $\sin(2\pi f_c t + \phi)$ .

So, by this way you can generate the reference signal at the receiver and you can recover the carrier phase. So, this is again another very popular way, by using the squaring loop and in this case you the advantage of this scheme is you are not using the PDF or the likelihood function of the signal. You are directly using the signal model and using algebraic expressions by expanding them and seeing their behavior you can build the receiver structure to estimate the carrier phase, this is one of the very popular techniques.

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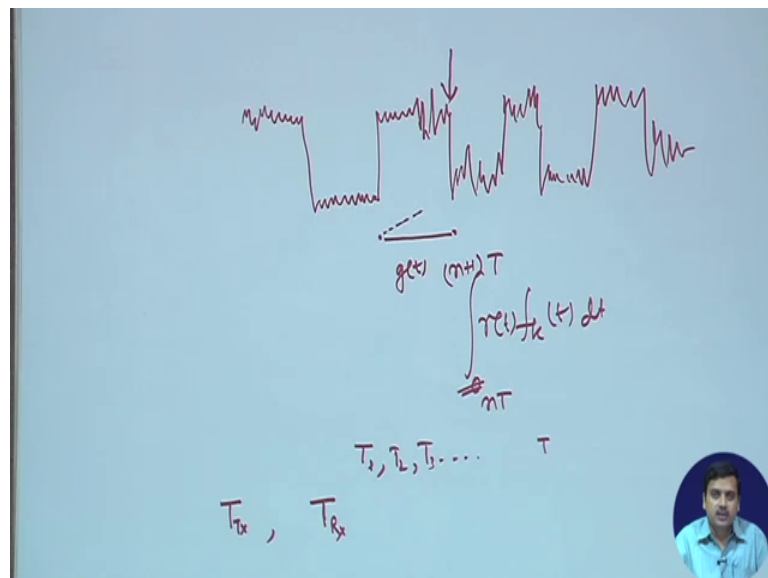
The next one is again a very famous, one known as the cost as loop. So, you have the signal arriving you multiply them by cosine and sins. So, this part is spate pretty, pretty common, pass them through low pass filter is almost like the integrator, and you multiply them feed it through a low pass filter. So, loop filter and drive the VCO. So, is almost a

similar structure, but there are subtle differences which you compare once you compare the figures, you will be able to get a better view of things.

With this we now move to the discussion of symbol timing estimation. So, if we summarize and try to see how we did this particular subject in the initial part we assumed all ideal conditions; that means, the channel is ideal there is no distortion, there is no ISI. We assumed that the phases are synchronous, we also assumed that the matched filter is working perfectly, In fact we did make any assumptions about the matched filter.

So, inherently we made a lot of assumptions where we dint say anything we made a lot of assumptions about it we did not write them specifically.

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So, at the receiver if you try to see signal arrives in some form. So, let us take a binary PAM and this is the noise signal which arrives right. At the receiver we said we are going to multiply in this period with  $g$  of  $t$ . So,  $g$  of  $t$  gives us this period and integrated; that means, you multiply and integrate that is the match filter from 0 to  $t$ . Or we sometime said  $nT$  to  $n$  plus 1  $t$   $d$   $t$  right. And we said sample it at this instant.

So that means, at the receiver we must have a clock with generates at a frequency one upon  $t$ ; that means, it produces the clock at every  $t$  interval of time without fail. If you take real systems and real clocks there are jitters in the clock right. As well as since it is an output of the oscillator, it is not necessary to produce the same frequency

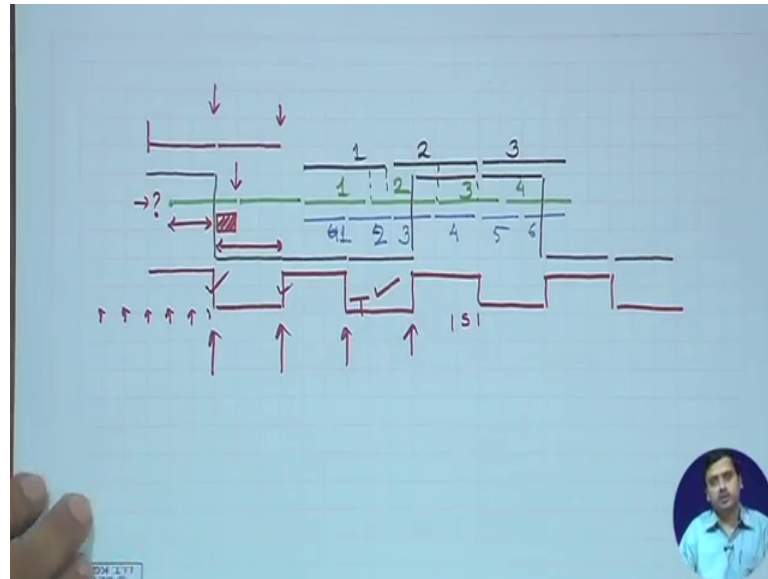
continuously it will oscillate around some center frequency. And you describe such behaviors through a power spectral density of the oscillator.

What it means is that it does not continuously generate the time  $t$ , and there are 2 issues in this. One is it could be generating  $T_1 T_2 T_3$  and so on and so forth. On an average it produces  $T$ , and the second could be that the receive the transmitter uses some clock which is phased at let me explicit write  $T T_x$  whereas, the receiver generates a clock which is  $T R_x$ . And they are different in terms of the mean value because there 2 different clocks made up of physical materials.

So, if let us say the transmitter clock runs at a few gigahertz let us say 1 gigahertz the receiver clock might be running at a slightly higher or at a slightly lower frequency. Mean is slightly different right. And they could be jitter around the mean as well. So that means, whatever we assumed as ideal in our earlier study is actually not ideal. So, let see what happens. So, just a quick reminder we have seen what happens if the phase of the receiver is different than that of the oscillator at the transmitter. In case of clock we will see it qualitatively and not quantitatively and we will rather directly going to the estimator of timing.

So, let us assume that in an ideal case we have the transmitted sequence, let us let us take an other one. So, this I have by mistake I made a clock. So, let us assume that the transmitted sequence is this. The receiver starts, first question when does the receiver start? Does it start here?

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Does it start here? Does it start here? Here? Or here? Is the question, right? So if the receiver starts here, then it could measure a time  $t$  and it will sample at this instant of time again it will measure a time  $t$  it will sample at this instant of time and everything is perfect and fine there is no problem.

But the receiver could by mistake start here and measure  $t$ , start here measure  $t$ , start here measure  $t$  and start here and measure  $t$  and so on and so forth. So, similarly So, these are the intervals which the receiver is measuring. So, note all of them a  $t$ . So, when you multiply by  $g t$  you are taking a small part of the desired signal and you are taking additionally the interference signal. So, this is this is the second symbol, and you are taking this portion of the second symbol at the output of the matched filter. Because you are sampling at this instant of time because you do not know the starting point. So, although your  $t$  is perfect, but the delay at which to start is not known the exact sampling instants are not known.

So, what we have is inter symbol interference right 2 symbols get interfered. The second problem if  $t$  is not perfect in that case suppose you start at the right time. So, what happens? So, there are 1 2 3 symbols which have been sent from the transmitter, but the receiver records 1 2 3 and 4 in this time interval. So, if you record all of them, there are 4 symbols which are send and the receiver records there are 5 symbols.



So obviously, there is going to be huge error. The rivers could also happen that means the receiver has a clock which done slower; that means, the time interval is larger. So that means, in that same period the receiver has recovered only 3 symbols. And in this case there is inter symbol interference you can clearly see, there is inter symbol interference right. So, the performance is terrible. So at least through this we can suggest or we can definitely understand that we need to recover the clock and the receiver we need to use or find the clock which has an appropriate phase, otherwise things are not going to be as could as we have described in the earlier part of a course.

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Rx should not only know  $T$  { ref freq  $1/T$  }  
 but also where to take samples (phase)  
 to take correlator o/p  
 → else ISI of diapa.

Symbol synch can be achieved by

- (1) Master clock shared between Tx & Rx
- (2) Tx sends clock info
- (3) Rx extracts clock from Tx data

Decision Directed  
 non Decision Directed

So, the receiver can know the clock by using a master clock shared between the transmitter and receiver. So, there could be some clock generator which sends clock to the transmitter as well as to the receiver. So, this is the master, or the transmitter can send clock info. So that means, what the transmitter can do is while sending signal. So, while it sends the signal, the transmitter can along with it send a clock. So, look at this it can send a clock signal. So, this is a clock signal. What the receiver will do? It will look at this edge the moment there is an edge it will decode the data, the moment there is an edge it will decode the data. So, by; that means, it will be able to get the appropriate clock, this is also well explained.

So, if you do either of these 2 things you require extra communication extra channel. In this case you have to send this signal through additional bandwidth. So that means, it is a

very inefficient way of communication. Because bandwidth is very, very costly. So, receiver should extract clock from the receive data. Of course, these are used in critical cases, but we would general like to receive extract the clock from the received data So that we are using this spectrum as efficiently as possible.

So, this receive extraction of clock is basically estimation of clock or phase synchronization of the clock. It could be decision directed on non decision directed, just like we have done carrier phase estimation. So, since we have setup the general model for phase estimation or general model for signal parameter estimation, we will use that same framework to get the receiver clock or extract the receive clock from the received signal.

So, we will again resort to the ML criteria or the ML method.

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ML - Estimation  
 Consider PAM:  $r(t) = s(t, \tau) + n(t)$ ,  $s(t, \tau) = \sum_n I_n g(t - nT - \tau)$

$$\Lambda_L(\tau) = C_L \int_{T_0} r(t) s^*(t, \tau) dt = C_L \sum_n \int_{T_0} r(t) g(t - nT - \tau) dt$$

$$= C_L \sum_n y_n(\tau)$$

where  $y_n(\tau) = \int_{T_0} r(t) g(t - nT - \tau) dt$ .

A necessary condition for  $\hat{\tau}$  to be the ML estimate of  $\tau$

$$\frac{\partial \Lambda_L(\tau)}{\partial \tau} = \sum_n \frac{\partial y_n(\tau)}{\partial \tau} = 0$$

$\Rightarrow$  suggest implementation of tracking loop!

So, by this let us consider a pulse amplitude modulation, where the received signal is the transmitted signal parameterized by some delay, tau plus noise. And this signal  $s(t, \tau)$  can be the pulses of  $I_n$  taking different values and  $g(t - nT - \tau)$ . So, this simply means that you do not know the timing instant. It is still of duration  $t$ , but the starting point is offset by a delay  $\tau$ .

The likelihood function. We have the correlation, if it is complex, you take the complex. And then you replace  $s$  with  $I_n$  and  $g(t)$  over here as described here. So, you left with

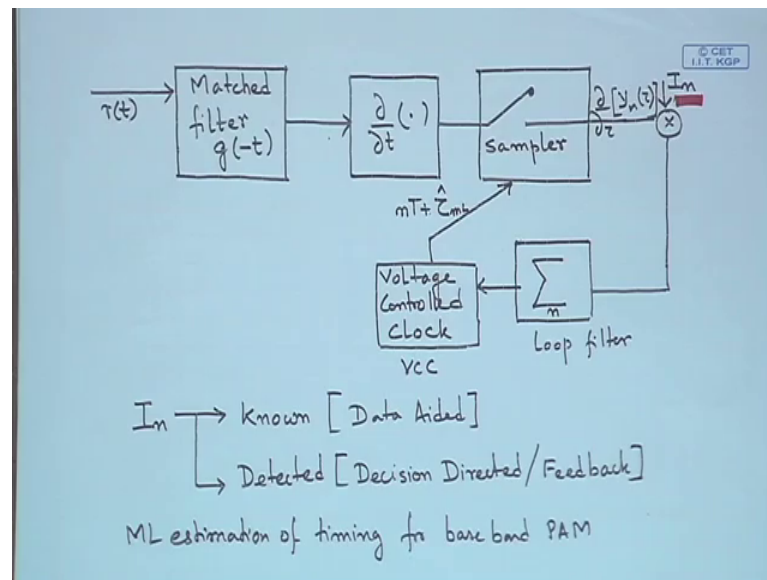
some constant value received signal integrated over the period  $t_0$  multiplied by  $g(t)$ . So, this clearly comes from the above step.

So, if we look at  $r(t)$  integrated on  $g(t)$ , it is basically the output of the matched filter; however, it is parameterized by  $\tau$ . Earlier it was at  $t$ , but now it is at a  $\tau$ . So, if  $\tau$  is equal to  $t$  you are at the appropriate time. If it is not equal to  $t$  you are not at the appropriate time. So now, you can guess the next step, we should take the derivative of log likelihood function, with respect to this parameter  $\tau$ , set it to 0 and get our solution the same procedure.

So, again we use the same philosophy, as we used for estimation of carrier phase; that means, we take the derivative of log likelihood function set it to 0, and we assume that the parameter assume gets the perfect value, feed it in to the expression and the whole expression should be equal to 0, when the parameter takes the optimum value. So, that is what we have, we have taken the first derivative.

So, from this we have So, were moved  $c$  because when we set it equal to 0 it is no longer required assuming it to be non 0. IN is in the first derivative of this and if  $\tau$  is the optimal solution then  $\tau_{cap}$  is the optimal solution. So, we can feed  $\tau_{cap}$  in to this and that should be equal to 0 right. This should be solving it. So that means, again we are getting to the point of using tracking loop solution. So, this is the this is the similar procedure. So, that is why we started off with the maximum likelihood estimator described it once we have the template we can almost use the same template continuously for most of the things. For our case the likelihood function is very, very similar. So, our job is easier in this situation.

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So, when we look at the expression as we have there, and we try to translate this expression into a block diagrammatic representation, what we get is the picture below. We have  $r(t)$  the matched filter  $g(-t)$  because it is the flipped version. So, this is the matched filter to  $g(t)$  derivative. So, output here is  $y_n(t)$ . So, you have this  $d(t)$  right. This is the derivative you take the sampler; that means, you are reading the value at a particular time instant. So, what you have at this point is first derivative it is already there of  $y_n(t)$  right. And here you multiply  $I_n$ . So, this  $I_n$  could be from prior known symbol sequence or it could also be from decision detected.

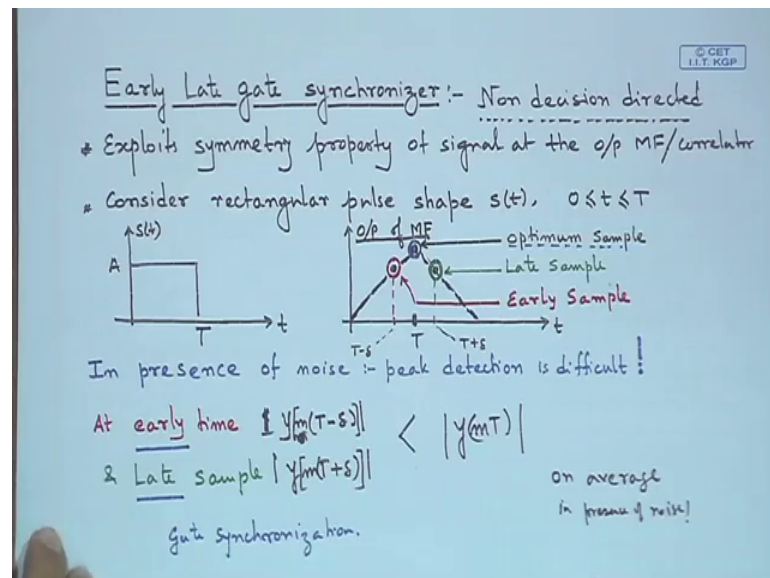
So, this product; that means, this product is said to the summation which is the loop filter and the outcome is fed to the VCO right. So, if the error is zero; that means, once you have found the optimal solution this would produce a 0 error and hence it is already released  $nT$  that means  $n$  is an integer value. So, it is going to produce clock at multiples of capital  $T$  if it is not the exact phase; that means, there is some error it will try to adjust the error by a providing the appropriate phase So that this loop converges to a point where this goes to 0 right. That is that is the basic idea.

So, in this way we can use the similar ML technique or them l technique is the same, and the algorithm turns out to be a similar algorithm and we have the block diagram the receiver.  $I_n$  could be known in that case it is data detected and if it is based on the certain decision feedback you can use decision directed.

So, one question could be that when I do not know the clock, then how about deciding on I nit could be erroneous the answer is very true, but we can also think of when the system starts they could be some initial amount of pilot a very tiny amount of pilot which can be used to get the initial acquisition which is almost correct. Once you get that then you start driving your loop and once you start driving your loop then it is already synchronized. So, the amount of errors that it can produce are expected to be small So that errors in IN would be small. In that case you could continued to use this loop. And you could use lesser amount of pilot in such a fashion.

If you using completely based on decision directed well there is criticality in terms of performance there is starting how much of noise. So, generally it may be preferred that you have some kind of pilot initially, and then you would run the loop So that you can keep on using estimated data of the sequence and feed it back to the loop. So, we will have a mixture of data added and non data added in practical systems. The next very interesting way of recovering clock is a very popular one and we will take a look at that and this is the early late get recovery.

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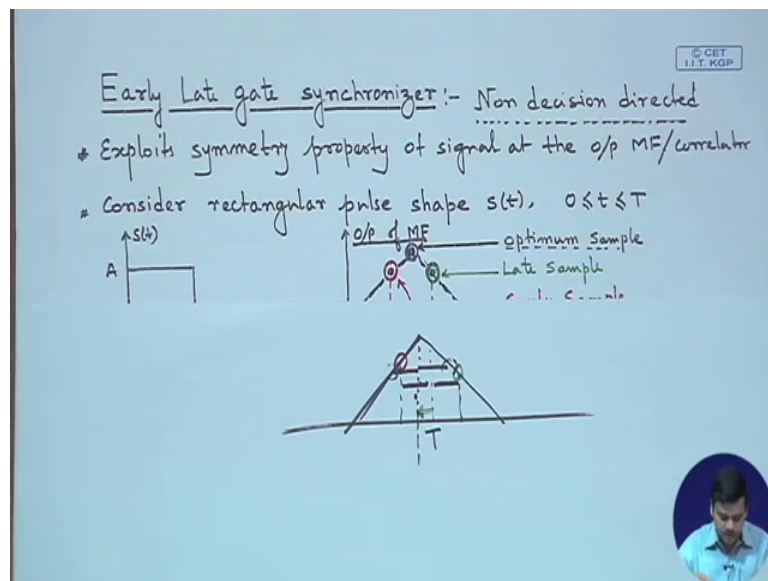


So, let us take a brief view on that, first and for most it is non decision directed; that means, it does not use the detected sequence IN back in to the loop right. It exploits symmetry property of the signal at the output of matched filter. So, let us look at what is the symmetry property of the matched filter. So, if you consider a rectangular pulse as in

this case, the output of the matched filter will be triangular form because you have convolved this with the flipped version of this, or you have correlated this with this right. So, which is the peak value at capital T. So, which follows this shape. At exact capital T it reaches the maximum values. So, y axis is output of the matched filter right. This output of matched filter. It reaches the optimum value at capital T. If the clock is operating bit earlier, you will be having a lower value. If it is operating a bit slower, you will also have some lower value.

So, this point is t minus delta this point is t plus delta right. So, what we will do is we will take 2 readings one at an early point one at a late point. So, if I have a certain time generated t suppose. So, we are going to take one point which is a bit ahead of t another point which is after t, and we can compare these 2 values. So, if these 2 values are exactly the same; that means, we can hope that the middle point is where the matched filter gives a peak output.

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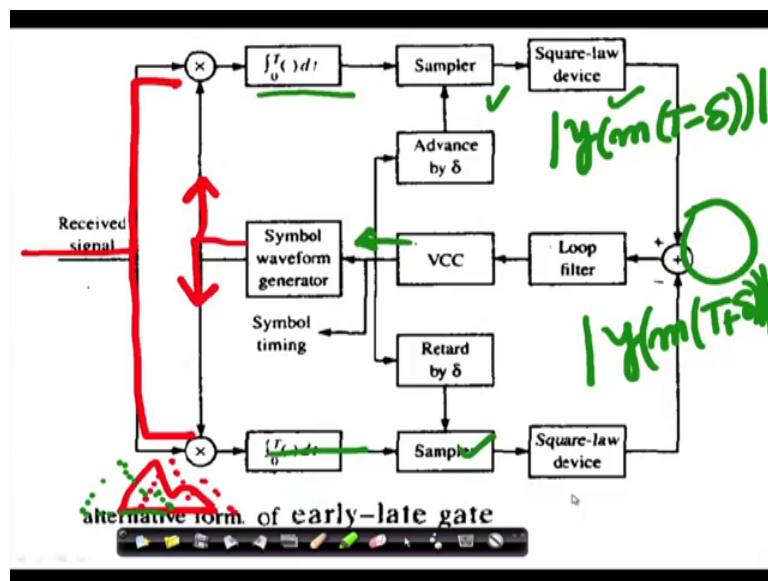


So, if we have a situation where we have this is the ideal matched filter output, and suppose the operation or the clock is producing sampling time there. All it says you record one value at a certain offset, you had a similar offset you record another value on the other direction. So, you are going to record one value here and you are going to record another value here.

So, this is the early this is the late. So, if I compare the early value with the late value what I can see is early is a bit higher and late is a bit lower. So that means, I need to shift my clock little bit of on this side. In that case if I am perfectly synchronized. Suppose I am there in that case and I shift and I take equal readings on both the sides. Then I will find that we are balanced in the early value and the late value and we can do the synchronization; that means, we are comparing these 2 values continuously in order to do the synchronization.

So, what we have with us is at early time the output of matched filter is  $y_m T h$ , that is  $m$  is an index multiplied by  $t$  minus  $\delta$  early, and  $y_m T$  plus  $\delta$ . So, if  $\delta$  is 0 it is  $y_m T$ ; that means, it both are at the same point. So,  $m$  is the time index. So, if  $\delta$  is 0 you have  $y$  into  $m$  multiple of time right.

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So, on an average these values would be less than the central value. So, that is the philosophy that we use. So, we use this philosophy in recovering the early late get receiver and what we have is the block diagrammatic representation of the early late get receiver. So, you have the received signal. So, you have the received signal here. This split it in 2 directions as we always do, multiply by the wave form generator that is the  $g$  of  $t$  minus  $nT$  right. And in one case the pulse is delayed by  $\delta$  in other case the pulses advanced by  $\delta$ . So, if you have let say for sake of argument are pulses like this, in one

case you are going to have pulse going like this, the other case you are going to have the pulse going like. So, these are the 2 pulses with which you are going to correlate.

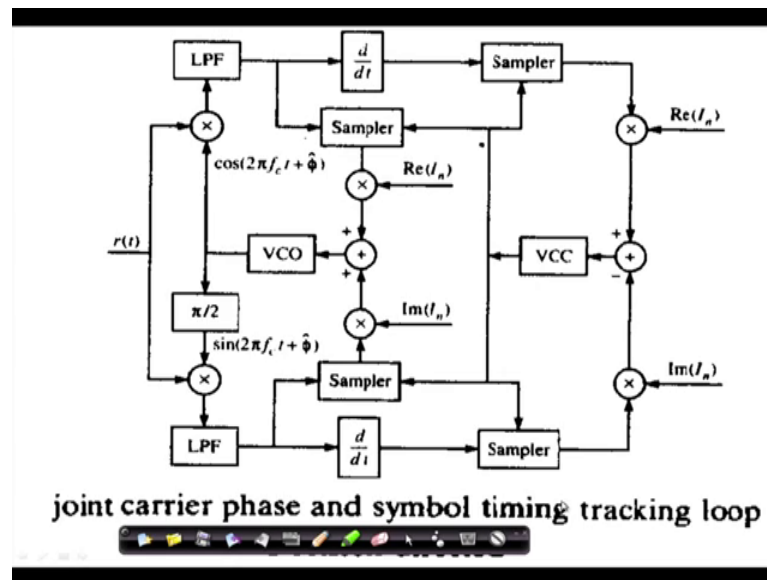
So, we are using the correlation philosophy; that means, we have created 2 functions with which we are correlating the received signal right. And then we are trying to see the output of this correlation right. We are trying to see that the correlation value is maintained the same, and again we are using the correlation philosophy in a in a in a grow sense. Pass it to the integrator stander procedure, we have seen sampler as we have already Seen.

Take the magnitude of it right; that means, mod of  $y_m T$  minus delta. And here what you have is a mod and here what you have mod of  $y_m$  is an integer number plus delta. So, we are delaying that is why we have plus we are advancing that is why we have a minus, and then we compare these 2 and feed it a loop filter to the voltage control clock, which will generate this symbol right. So, that way you can use the early late get recovery and find the exact value of timing.

It could be recovered in another way. And this particular picture, we leave it for you to go through at a comfortable time where the carrier and symbol tracking are jointly club together right. So, we will not going to the discussion of this here, but briefly you can see that there are some components I am not explaining the details of here leaving for you to merge the earlier to discussion. So, you have the derivative which in which indicates you are talking about the clock right. And the rest of it is about the carrier because it is generating this phase. So, in this picture you have clubbed both the things that is the timing and carrier recovery together.



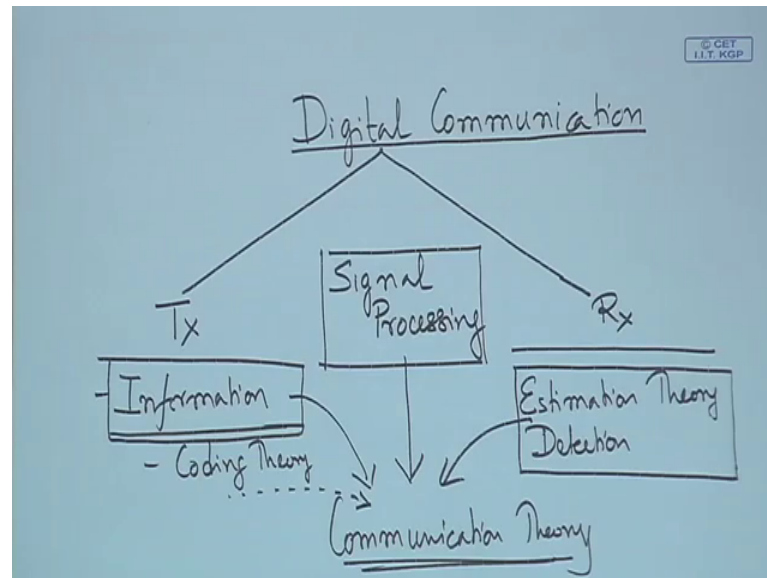
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So, we come to a discussion to an end of the topics that we had to discuss in this particular subject. And I would like to end the discussions by briefly telling you some of the fundamentals that you may take up in future in order to master the subject even better. So, I will take just couple of minutes on that. There are of course, many, many things to do which we could not cover in this particular course. But we have covered the fundamentals which we will equip you with the necessary tools by which you can take up advanced algorithms, whenever you are reading a research paper you would find similar signal models. If you use the theories that we have explained over here appropriately you should be able to build a fully operative receiver, because we have seen the non ideal channel, we have seen non ideal oscillator as well as non ideal clocks.

So, we have more or less covered most of the important things that are required to build a receiver. We have also seen how to build or how to choose the signal at the transmitter. So, together you should be able to build a communication link using whatever we have discussed in this particular course.

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Now, coming back to the issue which I was trying to bring out to you. So, if we look at digital communications, we can broadly divide it into 2 sides one is the transmitter side, the other is the receiver side. In every where you will find we are essentially doing signal processing right. All the things that we did at the receiver channel estimation equalization carrier frequency estimation clock estimation are signal processing.

So, at the receiver we can say we are using estimation theory, estimation and detection theory. Because we are detecting the signals. And the receiver we are using we are the closest that we can think of is information theory, because it gives a bound on the amount of information that could be sent. You can also use coding theory which we have not referred to in this particular subject, there are some other important subjects as well. But fundamentally what I want to convey to you the receiver is almost entirely developed using estimation and detection theory.

So, if you have to master the domain of digital communications you can pick up a subject on estimation detection theory in future. And at the transmitter side knowledge of information theory is going to greatly help you in further developing this particular field or taking a research activity in this particular field. Coding theory is also very active area which deals with error correction codes, and in all cases you are handling signal processing.

So, this domain of digital communications is broadly categorized as communication theory, which is an applied version of information theory, estimation detection theory, signal processing coding theory we have not done, but it of course, comes over here. So, along with this when you move further and advance in this particular direction the next immediate engineering subject which you can take up is the subject on wireless communications. So, if you take up the subject on wireless communications you will be using a similar signal model, but will extend the study on the channel, which usually takes up a huge portion of the study.

So, once you study the channel very carefully all the things that you have studied in this particular subject can be directly applied in designing a wireless communication link. Of course, we are talking about wireless digital communication links. So, I would like to congratulate you on being able to complete all the lectures. And I would also like to thank you for patiently covering up to this point. And I hope taking this lecture would add some value to the way you do your engineering problems, especially in the field of communications all would help you enhance your knowledge in this particular area.

Thank you.