Modern Digital Communication Techniques

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Lecture - 59
Synchronization Techniques (Contd.)

Welcome to the lectures on modern digital communication techniques. So, in the previous lecture, we have seen an application of using the maximum likelihood estimator in finding the carrier phase. So, we proceed further to the general class of estimating carrier frequency for the receiver synchronisation purpose. So, we have we have to begin with the signal parameter model estimation model. So, we are writing the expression of the received signal as were used to as S of t minus tau.

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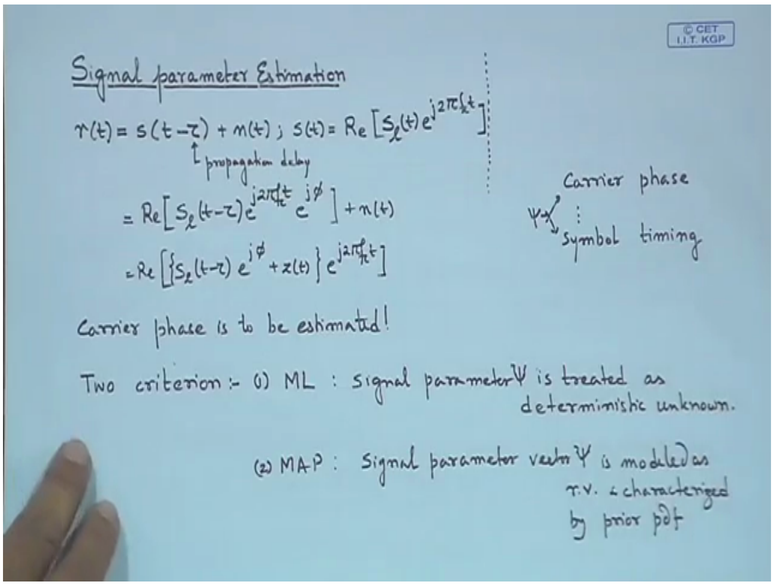

So, here we have included the propagation delay and plus of course, there is noise which is present with us and where S t is the real part of S L t to the power of j 2 pi f c t.

So, we are used to this kind of model. So, I am not explaining further on these models and one could expand the S t using whatever we have here S t minus tau, we have S t minus tau and there is a phase component associated with it now and this phase is mainly due to the propagation delay that is one and since we are talking about the low pass equivalent. So, what we have with us is at the received at the transmitter you would generally have e to the power of j 2 pi f c t this is what you use of the transmitter S L right and in the in the low pass from sorry, this is this is what you have at the receiver you would multiply this by e to the power of minus j 2 pi f c prime t; that means, there is a phase there is a difference in the frequency.

So, this phase is mainly due to this delay along with some additional phase that may be due to the frequency offset plus there may be some phase phenomena relative phase between the transmitter and receiver oscillators. So, this should actually capture all the axis phase. So, we need to estimate the carrier phase right so; that means, the phase has to be estimated and we can use the M L criteria or we can also use the m f t criteria.

In the M L criteria the signal parameter which can be denoted as psi sometimes it is a signal vector if there are more than one parameter. So, psi can have the carrier phase as well as symbol timing which we will see shortly. So, it can be treated as deterministic unknown if it is maximum likelihood if it is MAP, remember we have the prior probabilities.

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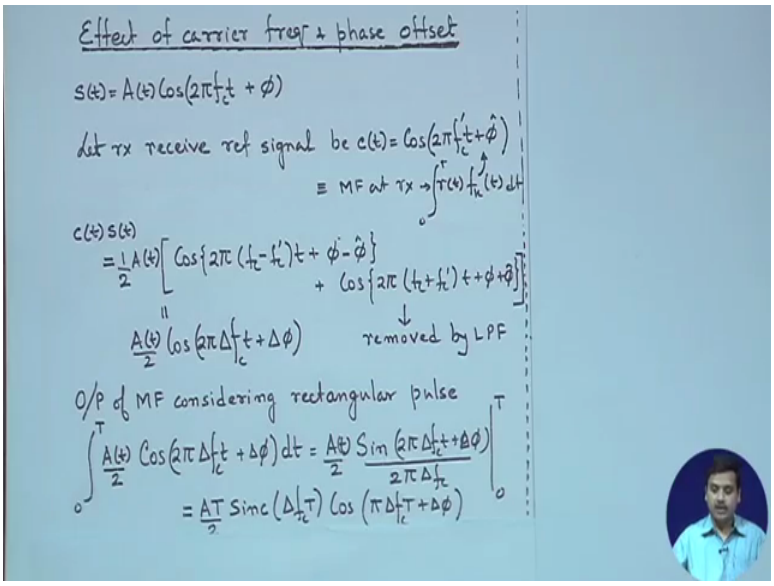

So, there will be the prior PDF; that means, these will be modelled as random with prior pdf, right. So, that is the difference; that means, if you have a prior PDF, you are going to use them L P criteria, if you do not have that then you are going to use the M L criteria. So, let us begin with the discussion of the effect of carrier phase and carrier frequency offset on the received signal.

So, this is the transmitted signal a t cos 2 pi f c t plus phi. So, note that there is a certain unknown phase in the oscillator and the received signal and the received reference signal so; that means, this is what we wrote the received after signal why. So, because again reminding you about the matched filter what do we do we have r t multiplied by f k t d t this is the component of r t.

So, when you do this generally r t is cos 2 pi f c t that is what we have taken along with the gate pulse right. So, what we say is that this we used as 2 pi f c t as if it was ideal situation that is what we did before, but now we are deviating from ideal and we are saying that no longer this has 2 pi f c t it is 2 pi f c t prime and there is a relative phase at the receiver.

So, you could take a single phase which is relative phase has been transferred receiver or you could say transmitter has a phase phi and receiver has a phase phi cap all right. So, with this justification we proceed. So, that is briefly explained over here because we have the matched filter at the receiver and hence if this could be a bit bigger I think then it will be advantages all right. So, this is the matched filter effect that have just explained.

So, you have to take the product of the received signal along with the reference carrier. So, when you do it. So, you have the S t and you have c t. So, you have cos multiplied by cos. So, you are going to get the terms cos of 2 pi f c minus f c prime and the phase difference of the phases and the next term would be the plus; that means, costwo pi f c plus f c prime and additional phases as well.

So, now if you pass this signal, this particular product through a low pass filter, this having twice the frequency can be filtered off, right. So, what will be have available with you is this particular signal part; that means, we could write it as a t upon 2 the coefficient cos 2 pi delta f c plus delta phi. So, we can filter this off because it is a twice the frequency the output of the matched filter considering rectangular pulse, right.

So, what we remain is integration of C t S t that is what we have been explaining right here that is what we have been explaining in this. So, the output of the matched filter will be this is the product already now the integration part is integration of 80 by 2 cos of this term that that is what we have over here. So, integrator integration of cos leads to a sin and the inside parameter with t comes in the denominator and you integrate this value from 0 to t; that means, you calculate the value between t and 0 and evaluated what do you get is 80 by 2 sinc function of delta f c t and there is a cosine term right. So, if you look at these expressions as if delta f c is 0.

What do you have you have 80 by 2 and this is a 1 a cos, let us assume delta phi 0 and delta f c 0. So, cos of 0 is one. So, there is no problem. So, if there is no phase mismatch or no carrier mismatch you are going to get the maximum value as delta f c increases sinc function drops like this and goes like that and cosine function also drops in a similar fashion, right.

So, what we get is d is drop very very fast; that means, the peak of the matched filter comes down this curve at a very fast rate as the delta f increases. So, what it means is that the signal to noise ratio is reduced the signal power gets reduced and hence signal to noise ratio is reduced therefore, what we can conclude is the effect of career and phase offset is to reduce the effective signal to noise ratio at the receiver.

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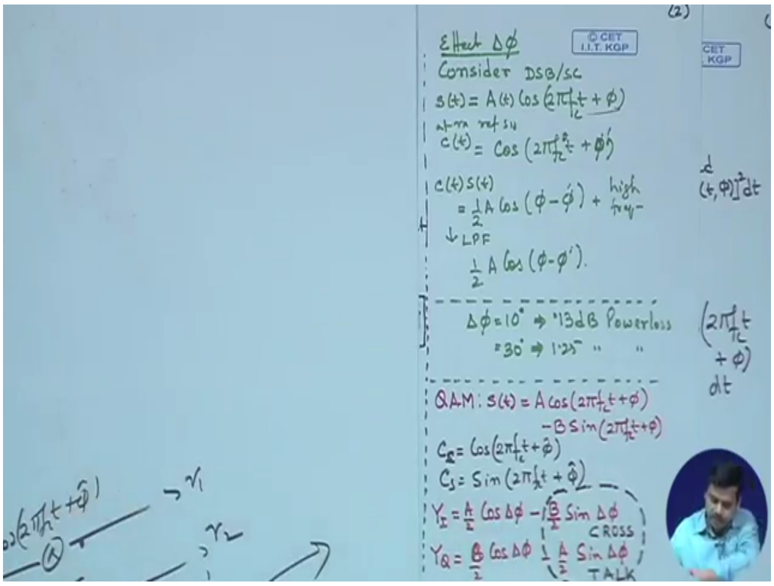

Further, we will consider a double sideband suppressed carrier situation where of course, the signal S t, we have modelled, I do not know if we can zoom it a little bit things will be better. So, we consider DSB/SC and this is the signal model at the receiver we have. So, we are now studying the effect of only delta phi because there are 2 effects delta frequency as well as delta phi. So, let us first study the effect of only delta phi.

Now, since this is also a phase term, this is a phase absolute phase term, this is the instantaneous phase term, but of course, at time t this is the total phase. So, we can study the overall effect of a phase, right. So, to study that we are saying that let the carrier frequency be perfect only phase effect the effect of delta f c would be similar. So, that is why we restrict our study to phase at the receiver the carry the reference signal has S phi prime.

So, the product would now have half cos phi minus phi prime because these 2 are going to go away and the additive terms would be filtered by the low pass filter. So, high frequency components are filtered away. So, when you pass this through a low pass filter what we have is a upon 2 cos of phi minus phi prime this if we say the delta phi this difference is 10 degrees, it will result in a loss of void 13 dB; 1 can calculate if it is 30 degrees it would result in a loss of 1.2 phi d.

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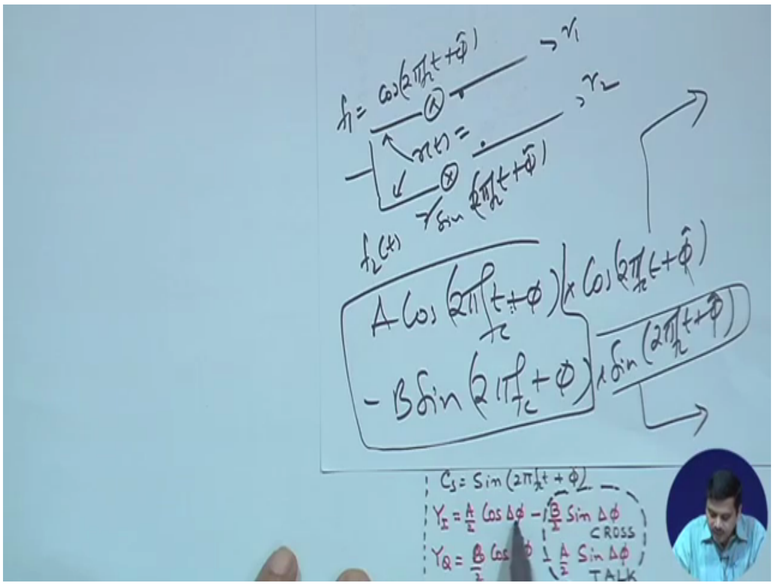

So, now instead if we consider a Q m constellation if you considered a Q m signal we are going to have S t equals to a cos 2 p t f c t plus phi minus b sin 2 pi f c t plus phi right. So, this is the typical expression that we have a cos 2 pi f c t plus phi and this is the carrier minus b sin 2 pi f c t plus phi right this is what we are having you term read the reference carriers are cos 2 pi f c t plus phi prime phi cap and sin carrier is sin 2 pi f c t plus phi prime because you multiply this with the cos 2 pi f c t plus phi cap and you multiply this the sin 2 pi f c t plus phi cap, right.

So, there is difference in phase in the carrier in the cosine carrier as well as a sin carrier. So, this is at the end of the multiplication what do you get from here; that means, end of multiplication and filtering through a low pass filter you are left with a cos delta phi that is what is here and you are also having b sin delta phi so; that means, you are going to multiply this with cos going to one channel and this with sin going to the other channel right. So, you have a by 2 that is how the receiver works you have the received signal getting multiplied by cos and getting multiplied by sin because this is cos 2 pi f c t plus phi cap because this is your f 1 of t and this is your f 2 of t that is the basis function right and what is coming in going into both the things is r of t and what is r of t the signal component is a t cos 2 pi f c t minus b sin 2 pi f c t of course, there is noise.

So, this is the same signal going into both the paths. So, you decode you will take the component r one take the component r 2 and then pass through detector. So, we are looking at this point we are looking at these points. So, the I component; that means, the I channel you are going to get a upon 2 cos delta phi and b upon 2 sin delta phi. So, if delta phi is 0 you have upon 2 because cos 0 is one and sin 0 is 0 right and the Y component the Q component would be b by 2 cos delta phi.

So, whichever goes along this direction would be b by 2 cos delta phi minus a by 2 sin delta phi. So, by similar reason when delta phi is 0 you are going to get b by 2 sin delta phi is 0 as delta phi increases your cosine term how does the cos function look like it decreases right and how does the sin function look like it grows.

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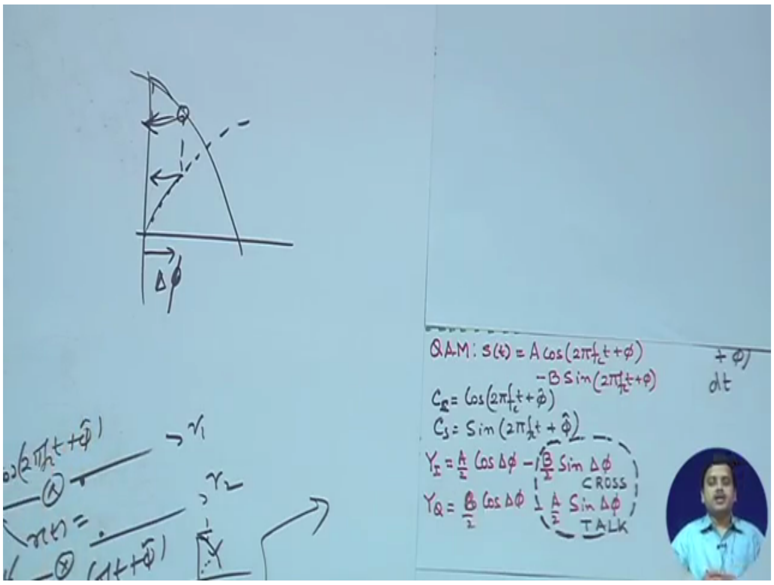

So, what does it tell as your phase increases your desired signal decreases it falls down, right. So, we can draw bigger picture if you want. So, if you look at these expressions what we have is the cos decreases and the sin slowly increases, right; that means, as we increase the delta phase the desired signal falls. So, it has reduced the value while the interference has also grown. So, what happens there is a huge amount of interference and what we have is a cross talk appearing in the I channel and the Q channel. So, what happens the in phase produces a component on the quadrature phase and the quadrature phase produces a component on the in phase.

So, as there is relative phase offset between these 2 this is also sometimes known as no this is not the IQM balance. So, so because of the phase offset between the transmitter and the receiver some components of the I channel gets projected onto the Q and some components of the Q channel gets projected on to the I. So, there is cross talk and this not only causes interference the desired signal fall is reduced and interference fall increases as there is gap in phase.

So, one could also easily say that as the carrier frequency offset increases you get a similar effect right because ultimately is the phase. So, this at least tells you why this phase offset between the transmitter and this receiver has to be estimated and compensated at the receiver if we do not compensate then the performance will be unacceptably poor, alright.

So, next let us look at the Emily estimation M L estimation of the carrier phase.

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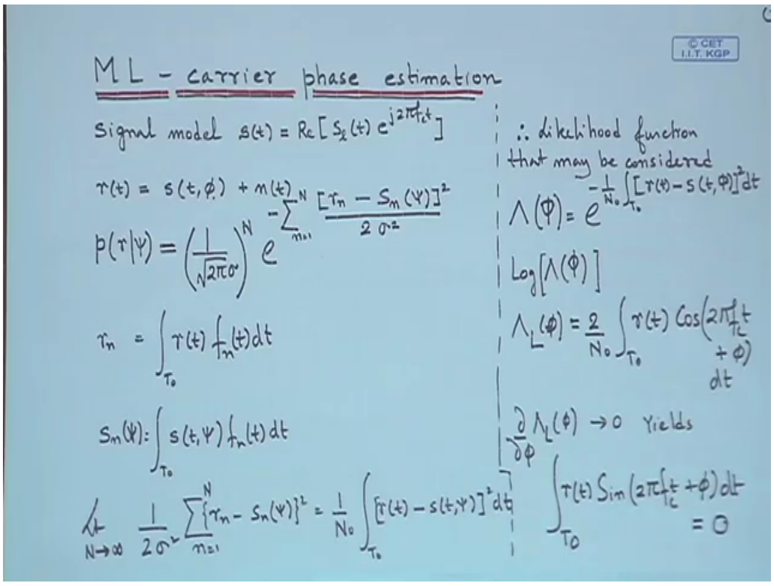

So, we have already done such a thing in the earlier discussion when we just finished the discussion on Emily estimator. So, we have inadvertently already addressed this problem. So, we will now see this problem purely from the communication prospective. So, things will be easier for us since we are already discussed partially this.

So, the received the S t is this model which I do not need to explain which your well aware of and received signal is the transmitted signal which is parameterized by the phase and there is a noise term and the likelihood function is also we have explained since it is the vector r and we said the components of r r independent being affected by Gaussian noise this we have discussed several times the joint PDF of r conditioned on psi. Psi is the parameter vector to be estimated in this case it contains only the phase is given by the expression here and of course, the nth component is found by taking the r t on the basis function.

So, it contains the signal which is the component of the signal on the basis function. So, we have discussed all these things right. So, we could also say that you could expand this expression, if we have large dimension ASN tends to infinity to a continuous function, right. So, you could actually take instead of these components you could write it terms of this expansion, right, you could write in terms of the signal directly instead of taking the summation over n if you have taken all the components then you could write in terms of the integration.

So, the likelihood function that may be considered now is this right. So, we have instead of the summation we have gone to the integration the log likelihood function, if we have stated you can use the log likelihood function if you take the log likelihood then the exponent goes away this becomes the direct cos function. So, that is what we are interested in. So, we have r t. So, there is a square term you are going to have an r square term you are going to have an S square term and you we having an r multiplied by S term with the 2 in front of it.

Now, r square term; we have already discussed several times is not necessary S squared if it is a constant modulus signal is also irrelevant rather the most important term is the cross correlation, right, further because when you square you are removing the phase component from it. So, when you take this expression the log likelihood function the way to find the phase is to take the derivative of the log likelihood function with respect to 0 and get the solution. So, you write the derivative of the log likelihood function with respect to the phase set to 0 yields the solution; that means, we could say that the derivative which is integrate over the period t 0 r t when you take the derivative of this you get cos becomes the sin you get the expression which looks like this.

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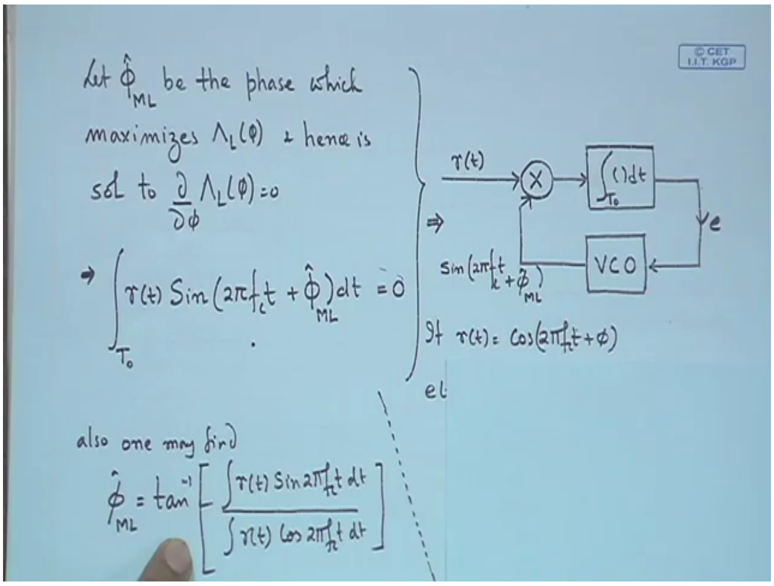

So, now there is a very interesting turn of things how we look at this expression as we have got there what we say is suppose phi is the M L solution maximum likelihood solution if phi is the maximum likelihood solution then what would happen put into this expression the maximum likelihood solution disintegrator should turn out to give a value of 0 is a bit settle, but we use that. So, let phi M L with the phase which maximizes the log likelihood function so; that means, the first derivative of log likelihood function at phi ml should be equal to 0 and hence it is the solution to this expression that is equal to 0.

So, what we do is we write the expression as we had seen before, but except instead of phi we put phi M L and then we say that this is equal to 0; that means, if I put phi M L in this; this is equal to 0. Now this motivates the solution that instead of actually solving this you could find a PLL like structure a loop like structure. So, what you do is you have r t as a received signal you multiply by some sinusoid with certain phase you integrated right once you integrated you feed it to the v c o right and this would settle in such a way that it produces the phi for which this produces a 0 error term this integrator motivates this PLL solution.

So, what you can think of is r of t is cos 2 pi f c t plus phi. So, the error term is cos 2 pi f c t which is r t coming in multiplied by sin 2 pi f c t plus phi m l. So, that is produced by the v c o right. So, this product can be factored as half sin of phi minus phi M L because they have the same frequency that is what we have assumed plus half sin four pi f c t plus phi plus phi M L.

Now, since you are passing through the integrator right this acts like a loop filter and it filters out the high frequency component. So, what is left is just the sin of phi minus phi M L and if this angle is very small this can be approximated as phi minus phi M L and this is the error term which drives the v c o right if the error signal is 0 the v t the v c o of produces 2 pi f c t and if it produces a certain voltage it is going to produce sin 2 pi f c t plus a phase. So, the input signal the error term is going to drive the V c o.

So, in this way you could realise a carrier frequency synchronisation just to remind one could also get the phi M L as stated in this particular expression which we had used in the previous discussion when we did sinusoidal phase estimation the only difference here is we have used the integration instead of a summation. So, in the earlier picture that we had referred to here; so, the summation is replaced by an integration. So, that is what you have over here. So, you could get a single shot estimation of the carrier phase whereas, here through a loop you are able to continuously track the filter phase right the received signal phase. So, if this phase changes if they receive phase changes this will be able to track the phase again.

So, now we move further to discuss the carrier phase recovery of a modulated carrier. So, till now we have seen that you have a signal which is unmodulated we just have cos 2 pi f c t plus phi, but now we say let there be a modulated carrier. So, for the modulated carrier what will refer to is in this case where a is not constant, but it is fluctuating with time.

So, in all the previous cases, you would have seen that we have taken the received signal as cos 2 pi f c t now there will be a in front of it. So, if there is in front of it in that case the integration output that you will get is going to go away.

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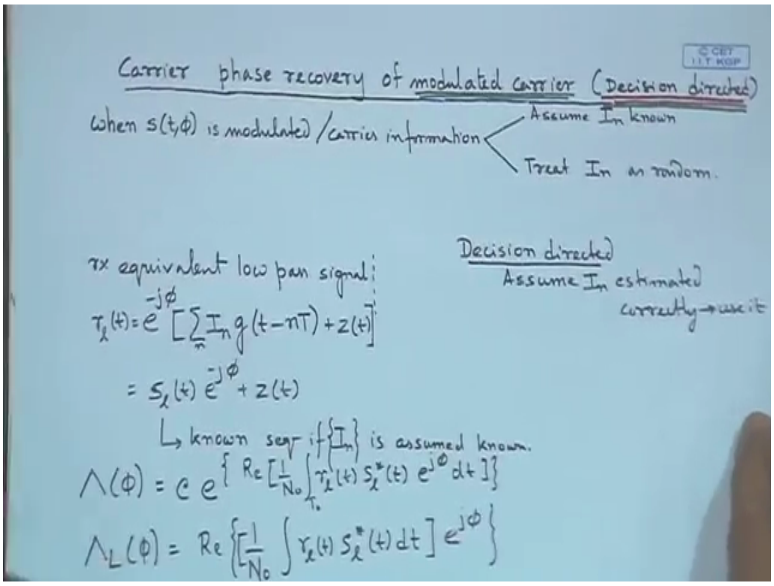

Because the modulation which will be in front of it could take a plus and minus values and the integration could result in a 0. So, when we have a modulated carrier. So, S of t contains modulated information; that means, it carries information through let say the amplitude. So, in recovering the phase of a modulated carrier one can assume that the sequence I n is known at the receiver.

So, in this case it is pilot sequence for data added as we have discussed for channel estimation. So, now, you see that how the pilot is also important for carrier recovery if it is available the other option is you treat I n as unknown and the and the other option and there are 2 options in case of being known one is pilot the other one is decision directed right decision directed means that we decode I n the transmitted sequence and we assume whatever we have decoded is correct and we use it back into the estimation procedure. So, without delaying further lets go into it. So, in the decision directed mode the low pass equivalent received signal. So, we are directly jumping to low pass equivalent we are not going through the past bend as usual we have done before there is a phase term which is available, right.

So, we are still talking about the carrier phase recovery now please note that carrier phase is very important because if we can match the instantaneous phase, we have actually match the total phase of the carrier. So, that is efficient. So, this could be summation I n which is the transmitted sequence and the pulse and there is of course, noise and this is the residual phase right. So, we could write this as the transmitted sequence with a residual phase e to the power of j phi and there is noise.

So, now this S L of T, we could assume this to be unknown sequence right either I means of pilot or by means of decision directed which we will see shortly the likelihood function would be some constant term e to the power of real part of this which we have seen before in the previous expression we have been looking at the likelihood function. So, it is not new.

So, we did explain here that you have this square term r squared is not relevant for S having a constant modulus again, this is not important. So, we are left with the product term. So, we have this expression that is that is importance and the square remove the phase component of s. So, that is not going to help we are related only with the correlated term. So, we have r l t, S l t e to the power of j phi, right, if S L is complex you can take complex conjugate and the log likelihood function this is the likelihood function the log likelihood function is going to take the inner part of it which is left in this fashion, right.

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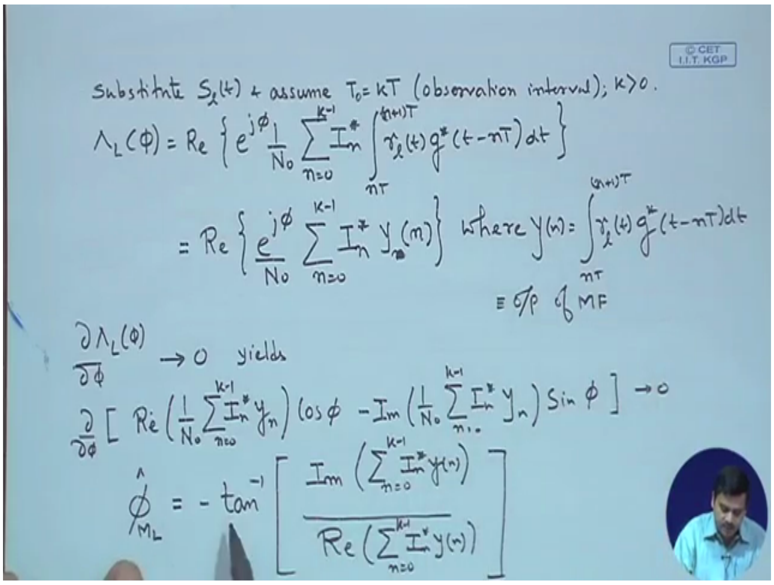

Now, is the important part? So, we will substitute S L t because we know it we assume we know it and assume that t 0 is equal to k t; that means the observation interval.

So, here we first time introduce that this t maybe unknown which will address when we talk about timing synchronisation. So, when we replace S L t in the previous expression. So, we had S L t write. So, S L t we replace over here is I n because we can clearly see I n right. So, S L t is known we have r L times this, right. So, basically this is the gate function; so, now, if you carefully look at received signal right projected onto the basis function in the baseband. So, what you could write this component as the output of the matched filter which is the Y n we had a similar situation earlier. So, we just using our earlier knowledge appropriately by studying the expression what it means. So, Y n is the output of the matched filter.

So, now if I take the derivative of the log likelihood function set to 0 we can get our receiver. So, you do the same procedure you take the derivative of log likelihood function with respect to phi and set it to 0 by doing. So, we can get an expression which is negative of tan inverse imaginary part. So, because when you do this it has real part over here. So, when you take the derivative of this you are going to get an expression which looks like this we are due to time shortage we are free writing the equations. So, if you do a few steps you can arrive at this expression right and when this is set to 0 naturally you get an expression which looks very familiar to you so, but now we have the real part and imaginary part and we have I n star and Y n.

So, earlier you may remember the expression had an x n and a sin and here it was x n and a cosine; so, which we are again reminiscent of this kind of expression. So, they are all very similar because we are using the similar signal model and we are using the log likelihood function and finally, the M L estimator.

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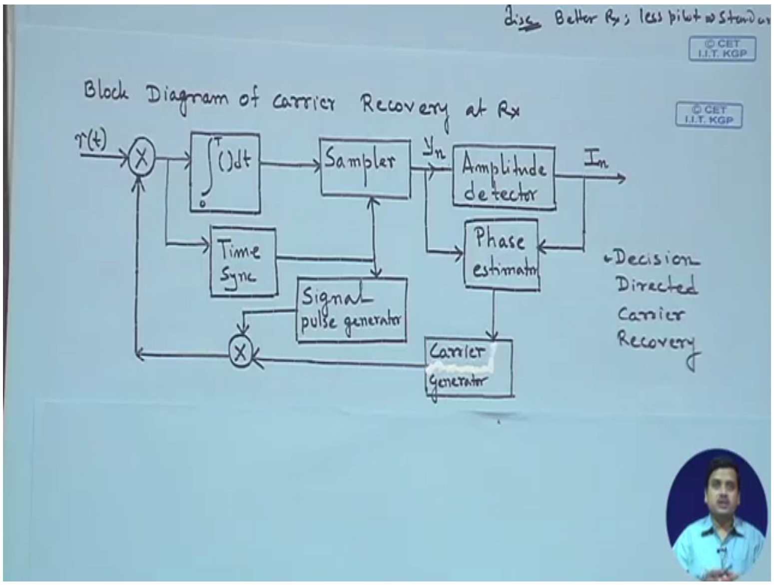

So, in this way we could translate the expression that we have seen in the previous page; that means, that we have here expression that we have above to a block diagrammatic representation received the signal right integrated sample it get Y n. So, that is what we had Y n Y 1 was here and how do you get Y n received signal integrated with the basis function. So, you integrate in case of rectangular pulse if it is rectangular you just need to integrate get sample it and get Y n as we have here in this part.

So, once you have Y n, you do an amplified detection and you would find I n star. So, that is has been detected. So, this decision directs the phase later on. So, this is that I n that we have detected if it is known a priory it could be pilot otherwise it could be decision directed.

So, moving further; so, using Y n and I n is what you need to estimate the phase. So, if you look at this you just need the I n and the output of the matched filter. So, if you do this you can estimate the phase once you have estimated the phase you pass it to the carrier generator the carrier generator multiplied by the pulse generator is what is multiplied by the received signal. So, that is what you have the pulse generator is g t right and once it passes through this filter or this integrator your left with the residual phase and the expressions of this finally, yielding to this has been translated to a block diagramatic representation which is realizable.

So, we stop the discussion on carrier recovery at this point and just trying to summarise that we have started with the philosophy of estimating signal parameter we have used the log likelihood function and we have made a parameter estimation using the maximum likelihood estimator. So, when you finally, arrived at the last phase of the estimation the very important task remains over there is to read the expression carefully and try to see how the signal at each stage gets processed processing in the sense multiplication integration summation trigonometric like tan inverse and so on and so forth.

So, we translate each part of the equation to the corresponding block diagram to realise the carrier phase recovery loop and since there is a loop over here we generally call it the carrier phase recovery loop and you can see that matched filter is already part of it detection is already part of it. So, using detection feeding it back to the whole cycle; so, we have the clock the carrier recovery at the receiver.

So, once again, I would like to remind you about the most important thing to take from this is the use of maximum likelihood estimator and before that the effect of phase. So, that is what is very important in terms of evaluation and the rest of what we have discussed is very important in terms of designing appropriate receiver which can do career estimation in summary you have now been a exposed to channel estimation and you can now also estimate the instantaneous career phase by virtue of which you can almost build a receiver. The last thing which we will look at in the next lecture is symbol timing recovery which will complete the essential components required to build a complete communication system.

Thank you.