Modern Digital Communication Techniques
Prof. Suvra Sekhar Das
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Lecture – 57
Synchronization Techniques

Welcome to the lectures on modern digital communication techniques. So, in the previous lectures, we have been looking at equalization techniques, the earlier side of equalization that we have seen are single shot equaliser namely the 0 forcing and MSC equaliser in both of them. We directly get the equaliser coefficients in one calculation in one step calculation then we put forward the situation where would like to do it in a fashion based on the error that we compute when we get the output of the equaliser and compare it with the desired response.

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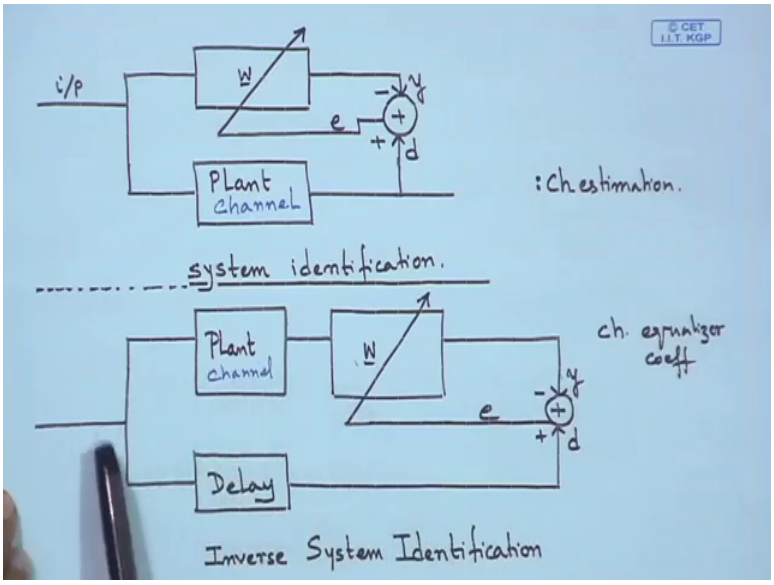

So, we have taken system identification modern by which we can estimate the channel coefficients, we are taken an inverse system identification model by which we could find the weight coefficients, if you find the channel coefficients then we can follow it up with equalization by procedures as mentioned earlier such as 0 forcing or the MMSC.

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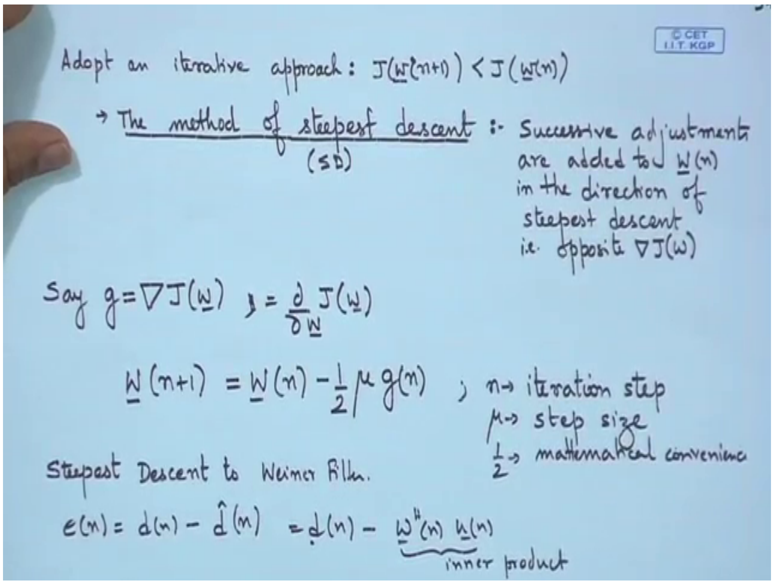

When we say; we want to adapt adaptively; calculate the channel coefficient, we want to make an iterative approach in which at every step; you would like the cost function to be lower than the cost function that is the means squires error less than the previous iteration. So, first we describe the method of steepest descent, it is excessively adjust the weights W; that means, whatever we have found to the next iteration in a manner or in the direction of the steepest descent that is opposite to the slope of this.

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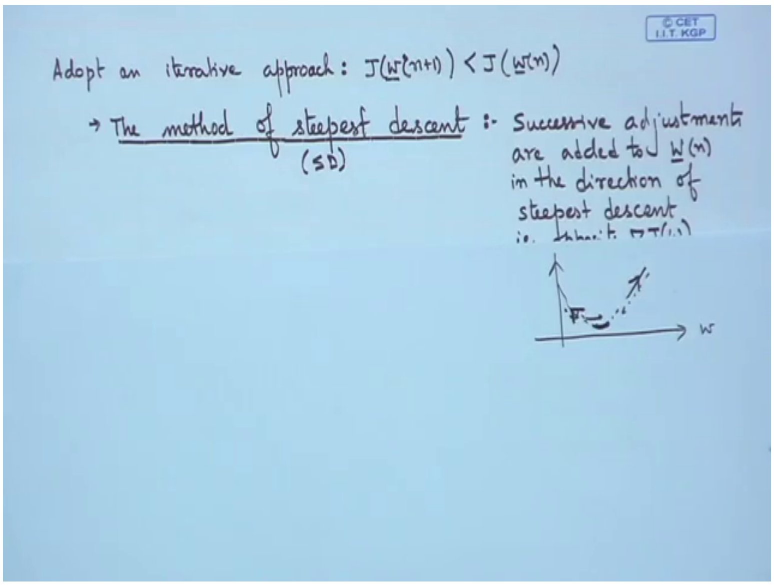

So, if the slope is in the direction; that means, if there is some W, let us say and this is the cost function. So, if the slope is in this direction; would like to go in this direction and if it is a bowl like this. So, if we here and the slope and the slope is in this direction would like to come down. So, that we attain the minimum of the cost function. So, just a note at this point, I would like to mention for a subject like this and the particular course that you are attending in this particular mode it may not be feasible to go through all the details of such advance mechanism.

However I am still exposing you to the known or most popular techniques which are used and can be easily accommodated in the receiver regarding examination we will try to ensure that the questions that is usually put forward in evaluating these kind of algorithms as we are discussing which is usually very very cumbersome is made in a way that you could address them within the limited time that is possible and I would highly encourage you to follow the assignments that would be given along with this lectures. So, that you practice what the kind of way, we can evaluate the understanding of knowledge in this particular area; however, the particular things that we have discussed till the point where we have come up to the maximum likelihood detector at the receiver seems to be pretty minimum basic requirement to understand this course.

So, moving ahead, so, let say we define g as the gradient of J which is derivative of J with respect to W, we can define the weights in the next iterations as the weights in the previous iteration and just it with some step which is a function of G. So, one has to calculate the slope once, one calculate the slope then with the certain steps, size, one would like to continuously improve the weights from the one iteration to the next iteration and in every iteration, one could get outputs y n which would add to the computation of the errors which would finally, drive this particular weight calculations.

So, when we apply this method of steepest descent, as you just described by to the winner filters which we have described earlier; that means, we have described in air filters which requires the value in single shot, select, apply this method to the method of winner filters. We define e that is the error as the desired signal minus the estimated value of the desired signal that the filter produces. So, which is the error term the d is the desired single d cap which is the estimate is the outcome of the filter; that means the inner product of the filter coefficients and the input to the filter. So, the cost function J in the n are iteration would work out to a expression which appears like this again and mention.

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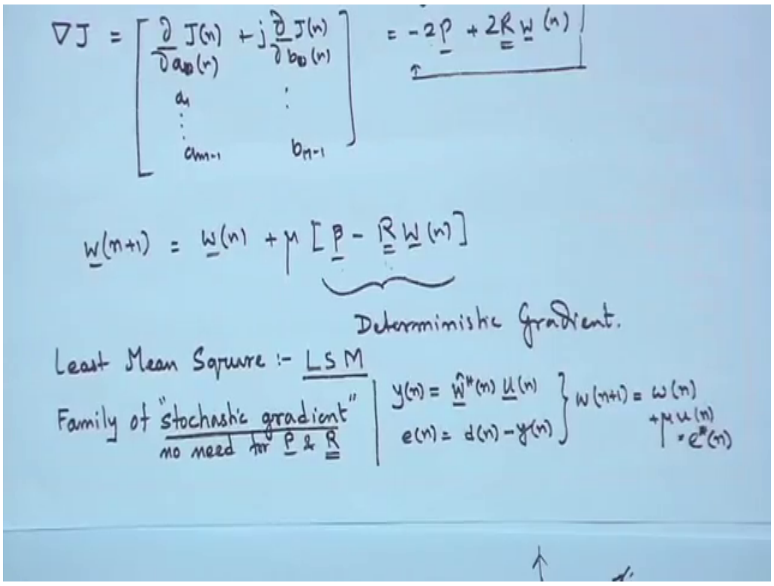

We need not worry much about how this expression comes, it is a matter of algebraic computations and we will not require you to remember this particular expression in the examinations and then if you produce; calculate the gradient, one would get a simpler expression which is given that again I mentioned one need not worry about remembering this expression either. So, as we proceed, we would define the weight in the next step, to be the weight in the previous step. So, we put an underline indicating the vector as we had done for signal expansion, we follow the similar notation plus a few that is the step factor and gradient of J. So, there is a half. So, half and to cancel south and there is a minus sign, earlier you may have noted that there was a minus an half this minus and half would cancel out this too and this minus becomes a plus and this plus becomes a minus where is the covariance matrix; where.

We find this earlier, similarly define P and these are the weights in the previous iteration. So, what we find is that this calculation is a deterministic gradient because we are as if calculate in this priory and it follows on improving in this manner which requires you to calculate the expectation of the output with respect itself. So, there is a covariance matrix to be calculated. In order to calculate this particular order to use the particular method; that means, you need to have the prior PDF of the output of the signal so; that means, of the channel response the other method which is followed comes from the family of stood caustic gradient we call it to pester gradient.

Because it does not use her deterministic gradient and it does not need one to calculate the cross correlation and autocorrelation mattresses we have the output y n which is the inner product of the input an filter coefficients and the errors as the desired signal minus the output. So, we have been shifting between y n and d cap of n is the same thing, the filter taps in the next iteration is the filter tap in the prier iteration plus the mew, there is the step size times the input and the error which we have computer here. So, in this kind of functions, we do not need to calculate the covariance matrices, we just need to calculate the error and what we can see is that the rate at which this changes is raring randomly because we have a random input sequence and the error which is a function of the previously learnt with values this relaxes some of the conditions.

So, there are convergence issues, but if we can design the input sequence appropriately, we could find the weights to convert pretty fast this kind of algorithm are also very commonly used for estimational channel coefficients as well as for estimation of equaliser coefficients. So, next; once we have found methods to find the channel coefficients as well as covered the underlying philosophy of channel equalization while maintaining that the received signal the overall trans of function of the received signal after getting processed for lost a raised cosine function we are now interested in studying some more non ideal characteristics which is the estimation of carrier phase and clock. So, till this point, we had assumed perfect synchronisation of the carriers between the transmitter and receiver as well as perfect synchronisation of the clock from the transmitter and receiver; that means, we assumed that there is no phase distortion there is no error in the sampling clock in clock instance and so on and so forth. So, when we look at this whole set of operation; what we essentially do is wear estimating; some of the important parameters of the communication system. So, when we write the received signal, it is R of t is equal to S of t plus noise and S of t the is S L of t into the par of J 2 pi of c t and the real part of that.

When demodulate at the; we do the correlation receiver or the matched filter receiver in both the cases we generally take the projection of the signal on the basis functions. So, projection on the basis function means to integrate from 0 to t times F K of t with the basis function and R of t; that means, multiply E t with F K t integrate from 0 to t, if your basis function, if it is a pan your basis function is cost 2 pi F C t times G t F t minus n t if it is QAM, you have a cosine and you have the sin if it is a emery FSK, you have E to the power of J 2 pi m delta F t.

So, most of the cases you have a sinusoid or a sinusoid getting multiplied by the received signal now this we have assumed to be 2 pi F C t there is a F C is the career and the transmit signal also had 2 pi F C t it is not necessary that the received oscillator has the same frequency F C and their could also be a phase difference between the transmit signal and the received signal further the clock which dance at the receiver which is used to sample at time instance capital t may not be perfect and it may not sample at exact t instants nor does the receiver know which is that particular instant of time where it should sample although the receiver can keep on sampling at regular capital T intervals, but when to start is not known at the receiver this is because of propagation delay and unknown clock start and end the oscillator start at the receiver.

So, when we try to estimate these parameters, it is very helpful that we look at the general class of estimation theory using which; we should be able to formulate a few expressions. The expressions would be used directly where we apply the signal model; we should develop. So, far and once we apply the signal model into the expression of the estimator that will be see shortly will be able to estimate the parameters of our interest.

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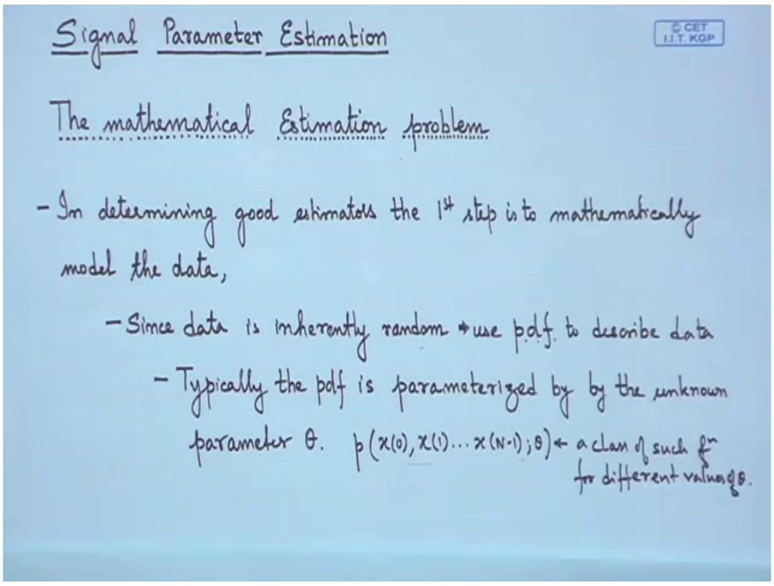

So, we have the general problem of estimating the signal and we would like to state the mathematical estimation problem and as we have stated since the beginning, we want to write down the expression of the signal at every stage of operation. So, that we get an unambiguous output which could be used for the designer of the implementation of the particular communication system.

So, we have we can state it as the mathematical problems and let us see what it gives us. So, in determining good estimators; that means, want to get an estimate just a small note; we already have discussed the channel estimation procedure and signal estimation where we have done channel equalization, channel equalization is as good as estimating the signal. So, we did not mention their estimation detection philosophy is there, but here would like to describe them with the little more time. So, that you get the basics of the philosophy which is used in deriving the estimators. Once we get those expressions, they will be helping us in getting estimation of carrier phase and clock. So, in determining good estimators, the first step is to mathematical model the data once again if you are not writing the appropriate expression of the data will not be able to write the algorithm, we want to operate on the data.

So, since data is inherently random we use the PDF to describe the data because random variable is well described by its probability density function typically the probability density function is parameterized by the unknown parameter theta; that means, we have P indicating the PDF of the observed data.

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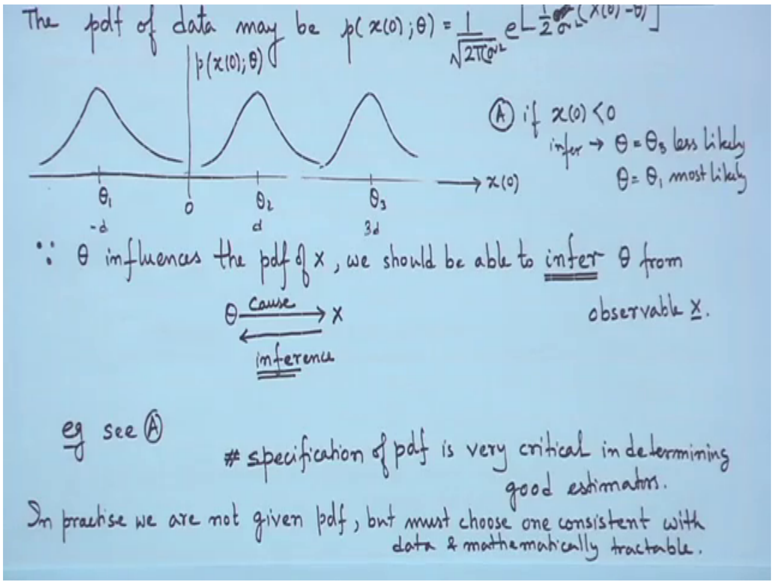

And there is some unknown parameter theta it will be clear shortly what we mean by this particular description. So, suppose our PDF is Gaussian again which is a very familiar to us because we have Gaussian noise.

So, we receive a particular sample that sample contains a data point as well as there is noise. So, the conditional PDF of the observed signal condition on a particular data is the Gaussian distribution with the mean value of that of the signal. So, the PDF of the data can be expressed in this form were theta is the mean and x 0 because we have a single observation. So, this PDF can be plotted along the x 0. So, if put a 0 and we parameters is this by different values of theta; so, what we find that if I have received x of 0 which is negative on this site most likely value of theta would be theta 1 and not theta 2 and theta 3, right and this kind of description, we have given we talked about the maximum likelihood detector and we have used the PDF and we have you some form of philosophy of destination detection theory which will be a parent very soon.

So, what we see from this is theta influences the PDF of E x. So, for example, if this is d this is 3 d, this is minus d, all mean to say if I receive x 0 somewhere here the most likely function of the most likely signal that must have been sent is minus t which is represented as theta one over here. So, the likelihood of that parameter being theta 3 is much less because if we take the PDF, the area under the curve here is much less than the area under the cove over here. So, we could plot like this that theta that is a parameter causes x means we have chosen minus d to be transmitted and that would influence what I observe of course, theta is perturbed by noise to produce x.

What we want to do? We observed x, we want to in for what we want 2 inform what theta was. So, this inference problem that is what very interested in the reverse direction that is what we going. So, if we see this picture we will find that specification of the PDF is very critical in determining a good estimator; that means, if we do not choose the appropriate PDF or the PDF itself affects the choice of estimation right and in practice we are not given the PDF, but must choose one that is consistent with the data what is means is that in practical systems its hardly possible to have the exact PDF of the signal.

So, we generally choose a PDF which is which closely matches that of the data that is all that it means and the other important criteria we should have is that it is mathematical attractable otherwise not viewed to derive the algorithms.

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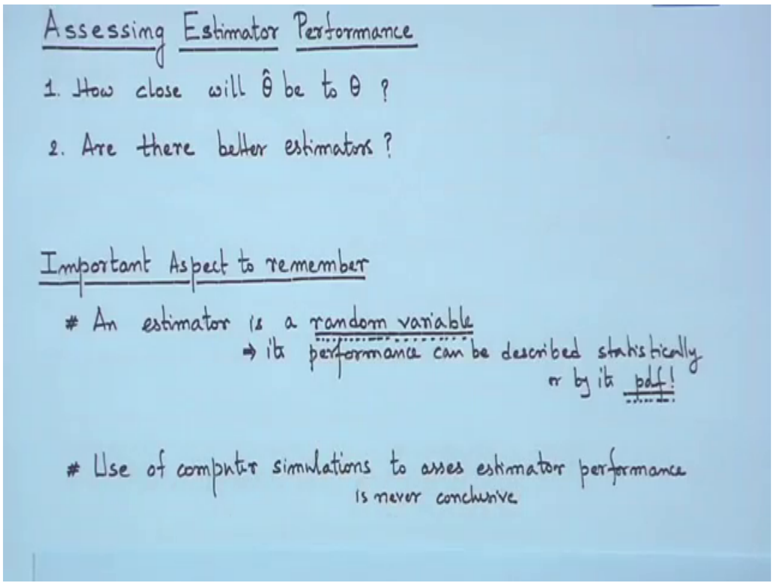

So, when we have estimated; we would like to evaluate the performance of the estimator some of the questions which we ask how close the estimated output will be that to the original parameters that we want to design that you want to estimate and are there better estimators available. So, before we proceed, we must remember that the estimator is a random variable this is a very important concept because the estimator some function of the received signals; receive signals are random, therefore, the function also should produce a random outcome and therefore, we can say that this function represent a random variable which is the estimator and hence is performance is described statistical by its probability density function since the estimator is random variable.

We use PDF to describe its behaviour and we should also not that use of computer simulations to access the estimated performance is never conclusive. So, we should try to find the PDF and there are other techniques by which we could estimate whether these 2 are achieved or not. So, one way to do that is to find whether the estimator is unbiased; that means, on an average it produces the parameters which we required and whether it produces the minimum variance. So, we should be able to analytically describe this behaviour of the random variable of interest.

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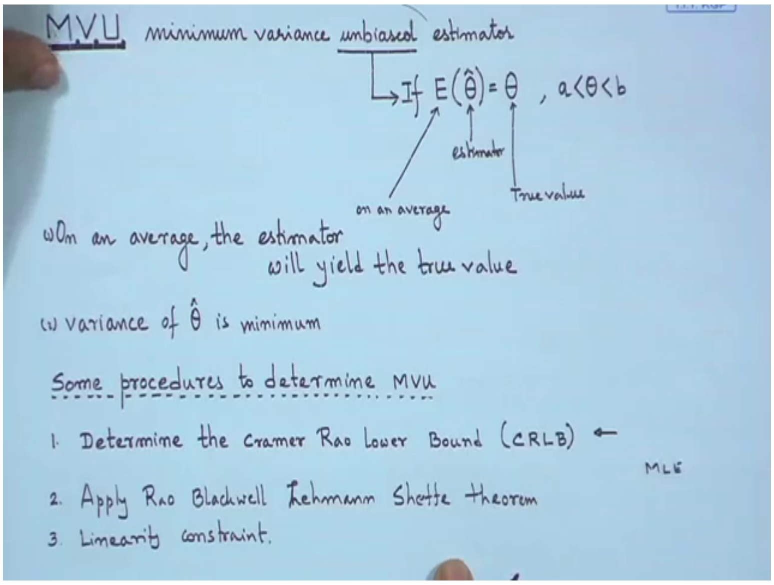

And then we have the minimum variance unbiased estimator as the status un bias means on an average, the estimated or the random variable reaches the parameter to be identified and it is minimum variance; that means, no other estimator is found whose variance is lesser than that what we are found. So, some of the procedure to determine the minimum unwired minimum variance unbiased estimator are through the camarilla by the lower bound which will be see shortly and through the Rao Blackwell Lehmann Shettle theorem or with linear constant will restrict also itself to this.

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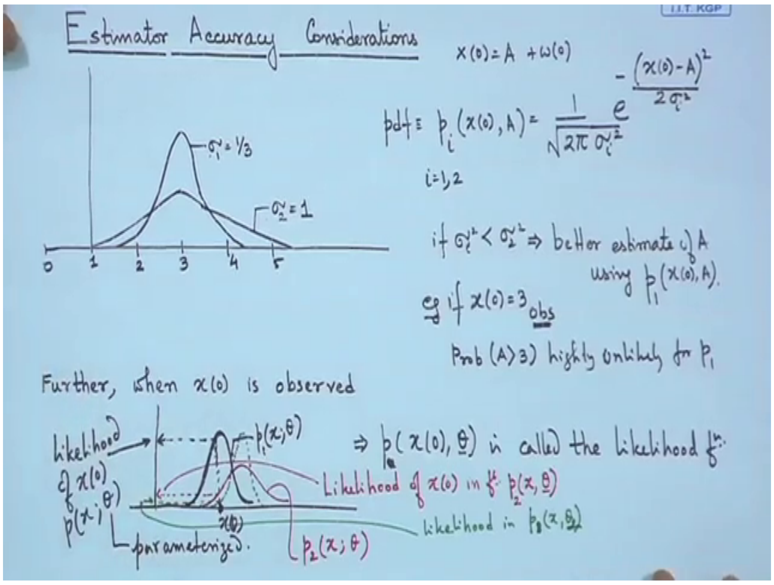

And then will move bring to the maximum likelihood estimator just if you note before we are actually able to produce the estimator is that if we consider PDF with 2 different variances one with one third another was one this is just for representative sake.

What we can make an observation is that the smaller the variance tighter the PDF, better is the estimator accuracy because if the PDF is spread larger, we cover a larger region in this axis. So, it is less accurate that is what people say. So, for example, of course, we call this as the likelihood function as you mentioned before and I would like to consider the 0 that is the observed variable and this is the PDF parameterized by the unknown parameter along with the observed data. So, I have the PDF function with this I observe this I will put this year and is to be an estimated which is given as theta over here.

So, what happens if I consider this particular PDF which is parameterized by theta and suppose I have observed x at this point. So, when I put the value of x in this PDF, right, I could find. So, of course, this PDF is parameterised by A or theta over here I would find that this function yields a value which is here. So, this tells the likelihood of receiving x 0, when I put into this PDF for a choice of A or theta as it may be, if we considered this dashed PDF which is green in colour, then will find the likelihood to get the signal is almost 0 which is very small. So, this is the shifted PDF, right, where is the black one is nearly added mean. Now if we compare the one with this red PDF which has a larger variance right what be fine is the likelihood would be more than that of the green, but it is significantly less than that of the black.

So, all what we try to mean is that the estimator accuracy the more accurate estimator will be the one which has a lower variance and as well as it is parameterized by the unknown parameter theta. So, our job would be to find the parameter using the likelihood function and this description also explains; why shall we call the PDF when it uses the data set and the unknown parameter has the likelihood function.

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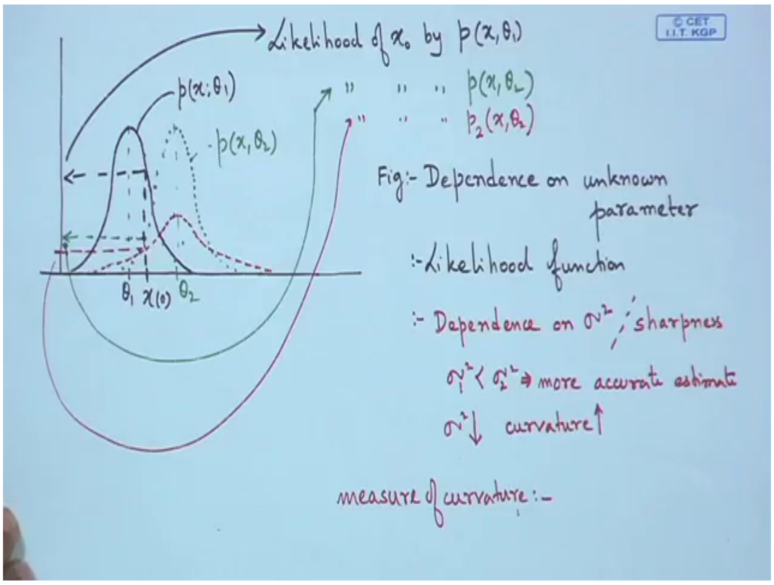

So, that picture is drawn here, it is a bigger version of the picture. So, that visibility is there and of course, we have if sigma 1 squared is less than sigma 2 squared; that means, as sigma decreases it becomes smaller the curvature increases.

So, it becomes shorter and sharper and the sharper it is the better is the estimator that is point stated at these points the likelihood values of x 0 driven by these different PDF noted in different colours. So, what we say that estimate accuracy somewhat related to the curvature. So, will discuss more about or tell you about the curvature once again; whatever we are discussing at this instant of time R for sake of information; however, the final results that we get will be the ones that are important and the ones which you may remember; however, if you are interested in designing receivers one should learn these methods away from a examination point of view, one did not exactly remember all the detailed steps of derivation. So, instead of taking the likelihood function directly.

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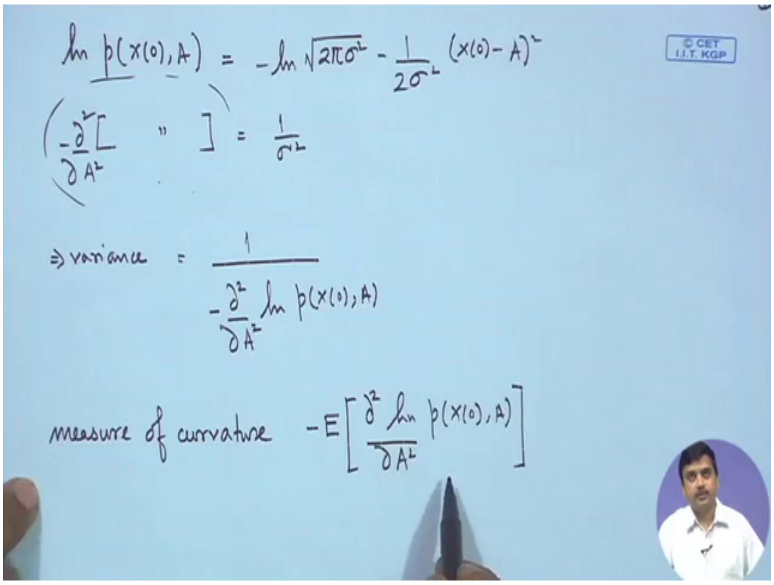

We consider the log likelihood function, the reason to take log likelihood function is when we take the logic expression becomes easier because instead of exponent, we get it directly here and we take the derivative. So, what we have. So, for the Gaussian PDF when we take the double derivative over here, we see that this particular expression is inverse related to the variance; that means, we can conclude that somehow curvature could be connected to this parameter. So, we define that the curvature in general has a negative of expectation of this particular function will see how is an important role.

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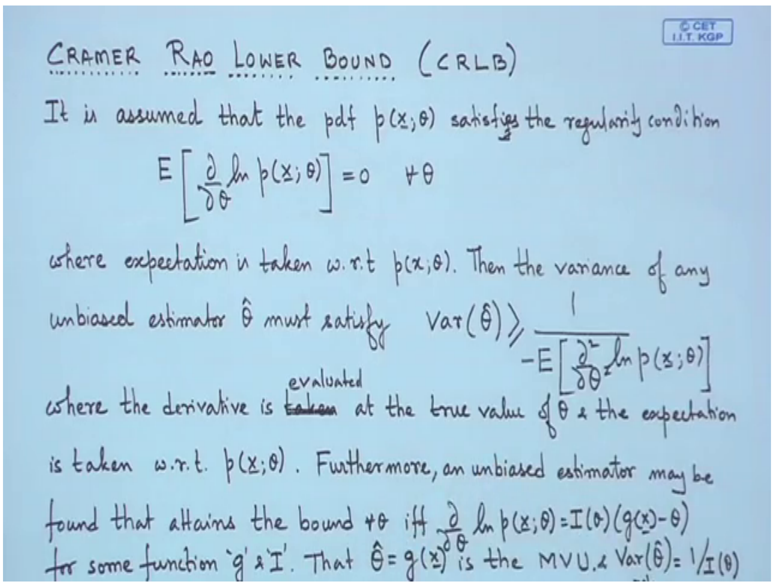

Next we define the very important Cramer Rao Lower bound.

So, in this it is assumed that the PDF P of x theta; x is the observed is the parameter to be observed to be to be found out satisfies the regularity condition; that means, the first derivative of the log likelihood function and expectation of it goes to 0, right where the expectation is taken with respect to the PDF this, then we can state that if this condition is satisfied, the variance of any unbiased estimator which you had describe before must satisfy the condition that variance is bounded by this which is the curvature; that means, the lower bound of an unbiased estimator is given by the Cramer Rao Lower bound through this expectation through this expression where e is the expectation used computed using P of x theta.

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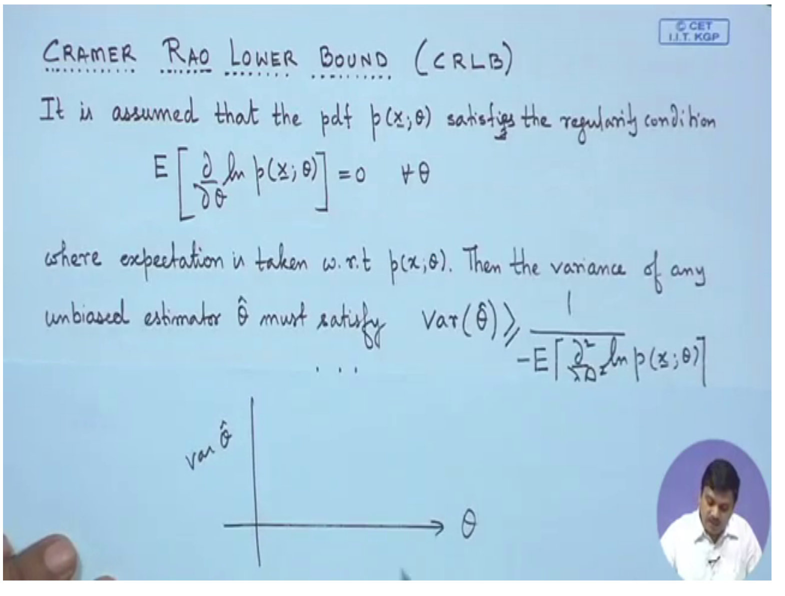

So, where the derivative is evaluated at the true value of theta, so, this is calculated at exact value of theta because we want to calculate the variance for a certain value of theta, for instance, if this is theta and this is the variance of the estimator, we would like to calculate at a particular value of theta what is this. So, then we can calculate the variance of the estimator. So, what it says that if this condition is satisfied then the variance of any unbiased estimator is lower bounded by this expression furthermore it also says so, very very important thing that an unbiased estimator may be found. So, it not only tells the lower bound of the estimated, it also gives an opportunity to find the unbiased estimator if and only if the derivative of the log likelihood function can be factored into i of theta;

that means, it has to be factored in such a way that there is some function i of theta and g of x minus theta g is some function which has only the observed data minus theta.

So, if you can factor the first derivative of the log likelihood function in that case you could claim that the estimator is given by this function g of x and it is the MVU; that is the minimum variance unbiased estimator while the variance of this theta is equal to 1 upon i theta and i theta is equal to nothing, but this expression, right so; that means, tells you that if one can factor it in this form, one would always find the estimator given by this. So, it is very simple to understand that it is a function of observe data this function produces theta cap which estimates theta and the variance is set equal to; that means, achieves the lower bound right just to summarise.

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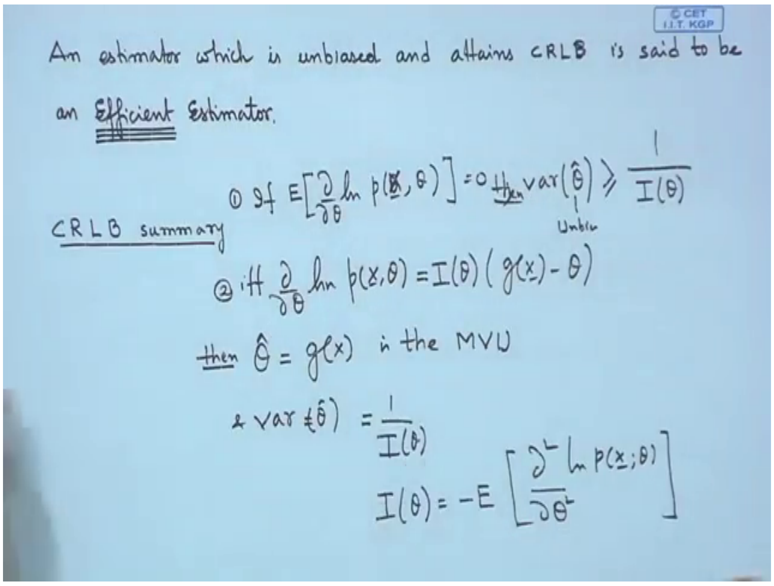

So, an estimator which is unbiased and attains the Carmer Rao Lower bound is said to be an efficient estimator because it you can find the estimated directly and on an average, it will produce theta while the variance is the minimum lower bound.

So, it will be unbiased as well as it will be lower bound and therefore, you call it the efficient estimator. So, what the Cramer Rao Lower bound give that is the lower bound of any estimator if the regulating condition is satisfied and if you can factor the first derivative of the log likelihood function you can directly get the MU estimator.

Thank you.