Modern Digital Communication Techniques
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Lecture - 56
Channel Estimation and Equalization (Contd.)

Welcome to the lectures on modern digital communication techniques. So, we are discussing signal processing at the receiver mainly the equalisation by means of which we are able to cancel the effect of the channel. So, we specifically choose the word cancel because we want to cancel out the echoes which the channel produces. So, just to remind, whenever we have an impulse into the channel, there is channel impulse response which can be viewed as multiple echoes of the transmitted signals at the receiver this echoes cause inter symbol interference.

So, if we pass the output of the channel through another filter which could cancel the echoes. So, that effectively we get the output as an impulse the effective output of passing the signal through the channel through the equalizer would be an impulse itself. So, the impulse response if it turns out to be an impulse then there is no ISI and we had seen such a filter in the previous lecture.

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So, when we consider channel equalization one of the important things, we should note is that it requires knowledge of channel we said that channel equalization coefficient should be equal to 1 upon the transfer function of the channel as one of the realisations. So, the natural question is; where do we get this channel coefficient from. So, this channel coefficient should be acquired by means of channel estimation techniques which we will explain shortly.

So, when we look at channel equalization and channel estimation the channel estimation part can be broadly categorised in 2 divided into 2 methodologies one is the data aided one is the data non aided; that means, no data aided the data aided means there is pilot or prior knowledge about the transmitted sequence where is a non data aided we do not use any prior knowledge about transmitted sequence with pilot what we mean by ideally impulse we mean to say that if the training sequence is an impulse the response of the channel is an impulse response.

So, if I send an impulse which is a non sequence I can easily capture the impulse response; however, an impulse has a very high peak to average power ratio and transmitting an impulse is a real problem. So, generally people work with pseudo noise sequence which has a channel transfer function whose gain spans uniformly across the available bandwidth or the bandwidth of interest.

So, we generally use a P N sequence and the same P N sequence is also used for synchronisation techniques we will see shortly. So, the design of training sequence is a very critical activity and it is a very important task and it helps in designing better communication systems.

Non data aided means there is no additional data requirement for channel estimation from whatever received signal is available, one could estimate the channel coefficients these techniques are naturally spectrally efficient because no time or spectrum is wasted for transmission of pilot; however, it has limited performance compared to a data aided and it is complex in nature.

Our focus on channel estimation; what we discussed would be primarily data aided; however, just a brief note when we move onto synchronisation another techniques we may take a look at it non data aided with things here bit clearer. So, in terms of equalization they can be various forms of equaliser and one of the forms of equaliser that we would be interested in is the linear transversal filter.

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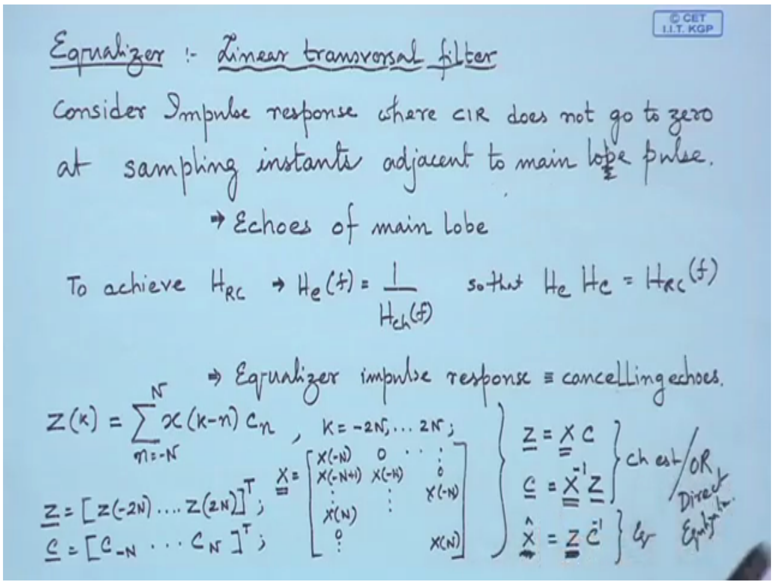

So, by transversal we kind of mean a tapped delay line model. So, we consider an impulse response where the channel impulse response does not go to 0 at the sampling instants adjacent to the main lobe. So, this causes inter symbol interference so; that means, the response if I launch an impulse into it, I can get responses something similar to that. So, at sampling instants this do not go to 0. So, these are basically echoes of the main lobe this is the same signal, but echoes coming at different instants of time and so on.

So, to achieve raised cosine overall transfer function if we keep H equaliser transfer function as inverse of the channel transfer function. So, that the equaliser times the channel gives a raised cosine we are satisfied, since we know as we have discussed the transmit filter can be route raised cosine the received filter can be route raised cosine. So, this would give a transfer function of one. So, the transmit along with the receive when taken together would give a raised cosine.

So, this equaliser basically cancels all the impulses. So, we briefly discuss this. So, if there is a channel if we send an impulse the response would be something like this and we want to fit a equaliser which will produce finally, an impulse. So, if it produces an impulse in that case all the echoes these echoes have been forced to 0, this echo has been forced to 0, this echo has been forced to 0 you are left with an impulse. So, combined transfer function that is when a H E F multiplied by C H F is equal to 1 in the frequency domain which is equivalent to impulse in the time domain since it pushes all the interfering pulses or echoes to 0, this is also known as the 0 forcing equaliser.

So, the output is typically convolution of a signal with the coefficients. So, you could find the coefficients as inverse of X time z and this procedure could be used for both the estimation of the channel coefficients as well as of the identification of the receiver received signals. So, when you estimate the channel coefficients we will have X as the known signal when we estimate the signal coefficient the signal values in that case in this model X would represent the channel estimates that we have already acquired.

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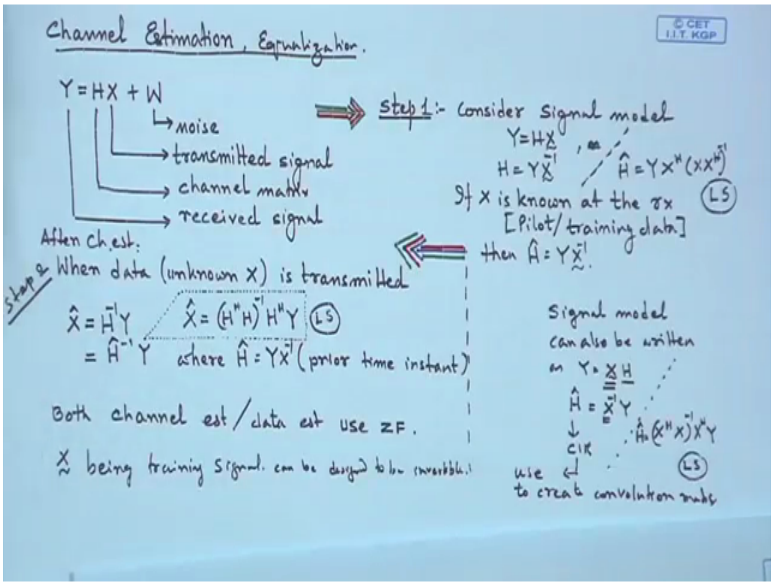

So, in the process of channel estimation and equalization we can model the received signal in the form Y is equal to H X plus W where W is represents the noise H represents the channel matrix which could be looked upon as the convolution matrix and X is the vector of transmitted signal and hence Y is an outcome of the convolution matrix times the transmitted signal. So, this ensures there is convolution.

As a first step we are supposed to estimate the channel. So, we can look at the noiseless signal which is Y equals to H X and one could estimate the channel coefficients in this form if X is known now since inevitability could be an issue in that situation one could use the pseudo inverse and one could acquire the channel coefficients. So, this X which is known at the receiver are basically the pilot or the training data which we discussed earlier when mentioning the data aided channel estimation.

So, once we have estimated the channel coefficients one could go to the next step when there is data transmission; that means, X is unknown in the previous case X was known and we could estimate channel now X is unknown and one would estimate X from observed Y using this expression when H here would be the impulse response and not the convolution matrix. So, depends upon how you write the expression in the least square sense the expression of the decoded or the estimated symbols would be this where H would be the channel impulse.

So, when we estimate symbols we use H estimate which has been acquired in the channel estimation process. So, it is a 2 procedure. First you estimate the channel followed by equalization. So, that you get back your signal in both the situations we see a lot of similarity in the way the expressions are used and hence one could think of using the 0 forcing algorithm for channel estimation as well as for equalization or one could use different algorithms in the channel estimation and equalization, we will see some more algorithms which we will talk about estimation of signal parameters which could be used in either channel estimation or in equalization.

A side note at this point is important one when we consider this kind of an equalization we have said that we will use one upon C flash equalization which is the 0 forcing equaliser because together when we see the transfer function, it produces a one in the frequency axis along the frequency axis. So, we are taking inversions of the channel transfer function.

So, if the channel transfer function has a response which goes like this we are definitely taking the equaliser coefficients wherever there is a high value, it is a low value getting multiplied with this. So, that you get a one and wherever there is a low value H E of F should be a high value because together it should produce a one over here.

So, if we look at the expression here; what we could guess is this kind of a multiplication with the received signal would also get multiplied by the noise so; that means, H E F would also multiply the noise so; that means, whenever H E F is high value noise would get enhanced because of high value of the channel transfer function at certain values of frequency and that happens wherever the channel has a transfer function with low values of frequency this phenomena is also known as noise enhancement, right.

So, with noise enhancement one; so, with this kind of equalization one encounters the problem of noise enhancement and this is primarily because we want to force all other echoes to 0 and keep only the impulse at the first place, if you would have relax this condition, then things could have been better and there are different receiver equalization algorithms which do not necessarily force the interference to 0, but allow a slight amount of information amount of interference where even then the communication achieves a good performance.

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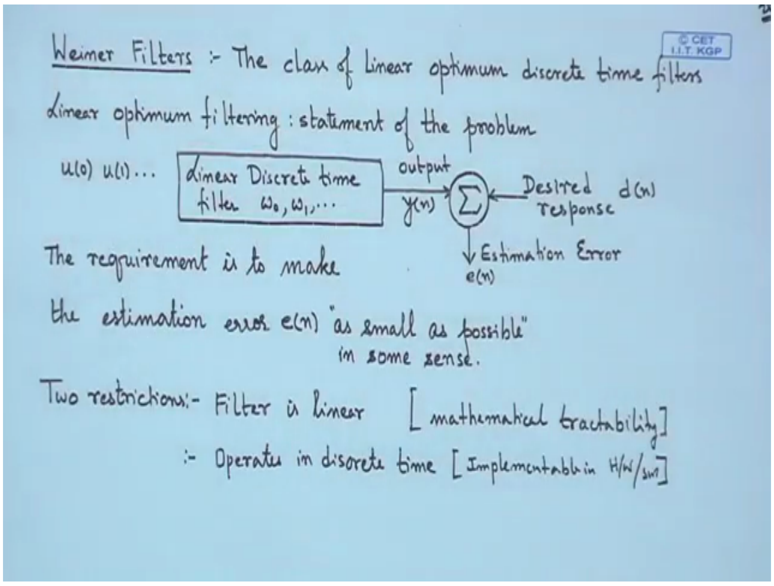

So to discuss channel equalization further, we need to look at the general class of optimum linear optimum discrete time filters which are known as Weiner filters in such a model we have sequence of input and there is a linear discrete time filter with weights W the output produced is Y and there is a desired response. So, we may desire an impulse out of it we can produce any sequence and it has therefore, an estimation error because these may not be exact which produces the desired response and the reason is these have noise in it. So, if it has noise we can only find these Ws up to a certain accuracy.

So, the requirement is to make the estimation error as small as possible in some sense; that means small in the sense of what is the question. So, generally we use the criteria that the mean squared error is minimised that is one of the major and important criteria that can be used.

So, there are 2 restrictions on the filter that we would like the filter to be linear and we would also like it to operate in discrete time the filter, if it is linear would help mathematical tractability and if it operates in discrete time then we will be able to implement it in hardware or software; that means, we have the power of DSP with us to implement the receiver there is another set of literature available which can help us in building such filters which is from the Yule Walker equations. So, there also we have r as the covariance matrix of Y and in that case our optimum filter wave coefficients could be calculated using this expression. So, we proceed with this knowledge.

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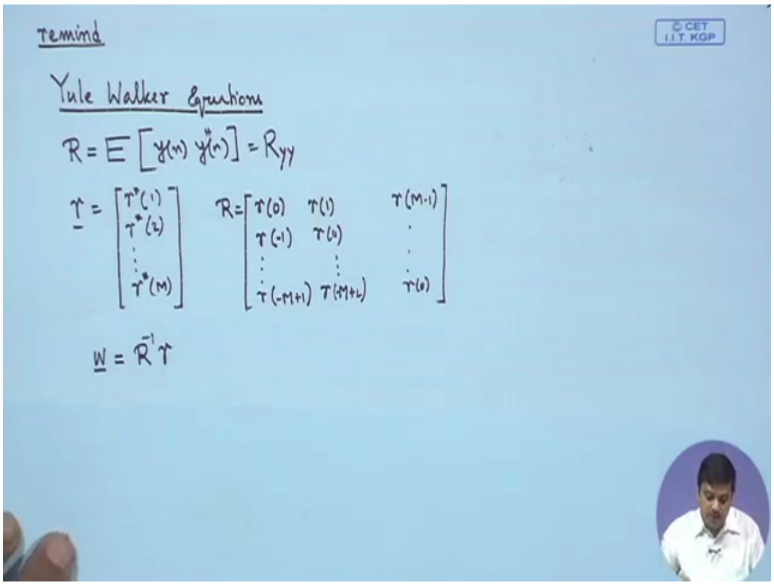

The next set of interesting relationships which matches our requirement are the; we Inerhopf equations for linear transversal filter. So, that says from orthogonally principle the input must be orthogonal to the errors we will see more of it, right. So, that the weights that we choose ensure that the inputs are orthogonal to the error; that means, the inner product would produce a 0.

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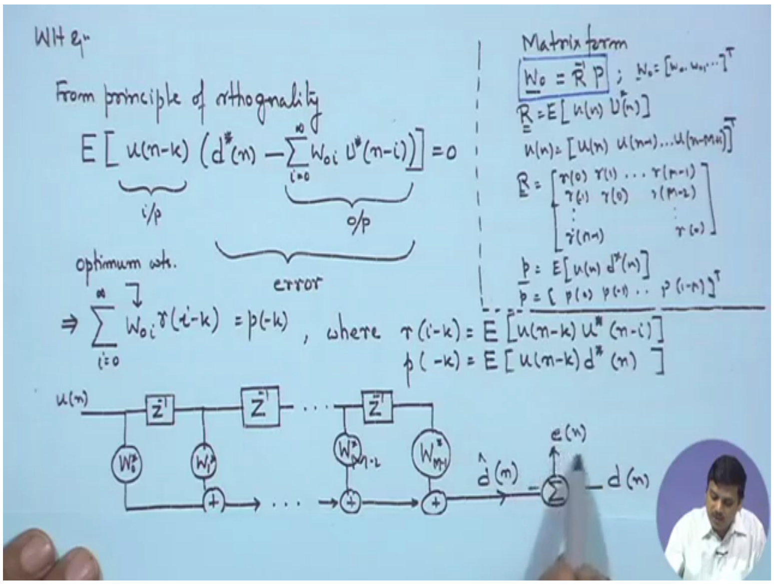

So it assumes that the input goes through linear transversal filter coefficients, we have to find. So, there again the coefficients can be formed using these relationships. So, they all give us the same kind of result which we can exploit in finding filters which will produce the desired response; that means, it will produce an error which is minimum in some sense.

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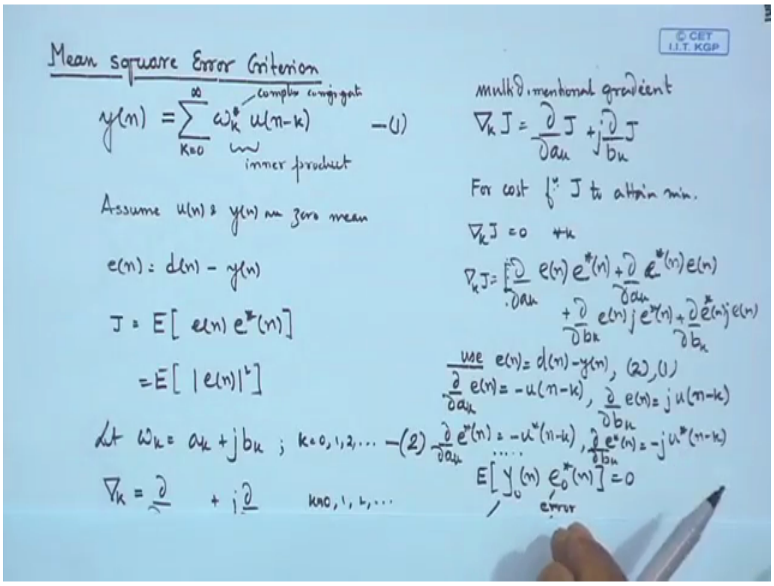

So, move ahead with this to discuss the mean squared error criteria. So, we have the output Y as per our model which is the convolution of the filter coefficients with the inputs that is what is expressed over here we want to find the filter coefficients that would produce Y which is as close as possible to the desired output.

So, we will assume that u and Y both are 0 mean which is reasonable in our case because noise is 0 mean and the constellation we have taken are mostly 0 mean if you take PAM it is 0 mean if it is QAM or QPS, we also take 0 mean and we have discussed when we talked about spectral characteristics that is desirable to have a 0 mean.

So, the error is d which is the desired minus the output of the filter and we use the cost function which is the expected value of the error squared. So, which is written over here we consider in general the weights or the filter taps to be complex that is a k plus J b k and in that case the del operator would have these expression which is again from calculus.

So, our objective is to find the weights which would minimise this objective function; that means, we should take the derivative of J and set it to 0 now weights are complex. So, therefore, we have this del operator in the complex notation. So, we should apply this on J set it to 0 and the result should be able to give us the weight coefficients. So, that is what we are stated over here. So, we take the del operator on j. So, that we want the weights which would find the minimum value of J and we set it to 0. So, after a few steps of operation which is stated here which is not necessary for us to describe in details at this point of time?

We can focus on the set of outcomes that we get what we get is that the output of the filter when it is optimised is orthogonal to the error that the filter produces or the outcome of Y with respect to d. So, we should be able to use this result in deriving the weights that we require these few steps one can derive or one could concentrate on the final outcome of W that we get.

So, when we talk about this orthogonally, we in other words mean that if we have an input and we have the output of the filter the error which is the difference between the desired and the output of the filter should be orthogonal to the output of the filter. So, we can use this geometry visualisation to get to what weights that we required.

So, if you work that out a solution which one reaches looks a bit combustion, but this is what we generally use as the MSC equaliser in most of the communication systems. So, we have suppose we write the expression as X equals to some H theta plus W where theta is something to be estimated. So, earlier we had written Y equals to H X plus W, but we have changed the notation because generally all estimation problems are described mostly with this theta as the parameter to be estimated. So, H is the set of coefficient or the matrix which relates or which maps these parameters and this together forms a signal model this is the noise.

So, we can think of this as the input data this as the convolution matrix and this as the outcome or the output at the receiver when this is the noise right. So, we have a similar expression there. So, the estimation outcome is the expected value of theta plus the auto covariance of theta times H Hermitian. So, if H is the channel matrix H Hermitian inside you again have H times the autocorrelation auto covariance matrix H Hermitian this is the Hermitian operation and then this is the covariance of noise which can be the noise power and the identity matrix inverse of that times Y minus E theta.

And for this one is referred to the book on statistical estimation theory by Steven N Kay there are lot of details about it given in that particular book. So, if you are very interested in pursuing the details as discussed in the previous few pages of course, we will not ask you to reproduce these derivations, but it is important to remember the results that it produces the notion or the kind of receivers that we get.

The details of this can we have can be found from Steven K and the books on adaptive filter theory by Simon Haykin as well as lot of lecture notes by John Cieffie. So, if we make E of theta equals to 0, then this term vanishes as well as this term vanishes and one would be left with E 1 could easily write this as expression given here which is the very well-known and celebrated expression of the mms C receiver.

So, what we have over here if we have r theta theta as identity, we have H H Hermitian. So, if we look at this matrix in a little more details and we say let us say this is identity what we have with us is H Hermitian H H Hermitian plus let us say I and of course, there is one upon gamma and Y. So, if we remove this term suppose we remove this term what we are left with is H we take the inverse operator H Hermitian inverse H inverse Y.

So, what it produces you see is that H Hermitian times H Hermitian inverse is identity. So, we are left with H inverse Y. So, we are left with H inverse Y so; that means, in an extreme case this boils down to a 0 forcing receiver. So, the difference from a 0 forcing receiver that we have over here is it introduces this vector this particular matrix which is the covariance matrix of noise so; that means, if we focus on the inside of the inversion we will find that there is some H invert invert invert available, but it is restricted by one upon SNR because there is a noise variance there is r theta theta which controls the power of the signal.

So, if I take r theta theta out I am going to get r theta the tain verse over here which would effectively cancel out with r theta theta and here we are going to have r theta theta in the denominator. So, if r theta is an identity matrix with some coefficient signal power that is variance of theta. So, we will have variance of noise upon variance of signal and an identity matrix.

So, we are going to have one upon gamma and i. So, this restricts the minimum value to be achieved; that means, we said that equalization is C of F which is equivalent to C inverse. So, here it is H both are identical. So, we are restricting this to some minimum value instead of saying C inverse, we are restricting this to a value one upon gamma inverse; that means, if C goes to 0; that means, when C is very very small the H E Q is very large. So, when C is very small, it goes to 0 the inversion becomes very large. So, we are restricting it by this parameter by this number over here.

So, if C is very small the inversion would not be very large because it is restricted by the one upon SNR parameters. So, it restricts the noise enhancement which was the typical problem in case of 0 forcing equaliser. So, we usually go for this kind of an MMSC receiver; however, a side note for those who are very keen with this kind of receivers this kind of receivers generally to have a bois. So, one may be required to remove the bois, but at very high SNRs, the performance of this receiver is comparable with that of 0 forcing when inversion of 0 forcing is not a problem if inversion of 0 forcing receiver is a problem then they do not compare at all because in case of 0 forcing the matrix does not get inverted well, just one point to remember if you are using the pseudo inverse, you can invert it, but there will be huge amount of noise enhancement.

The next method that we would like to look at is known as the method of steepest descent and this uses an adaptive technique instead of single shot. So, what we see in this receiver, it is one shot estimate of theta. So, in according to our earlier slides we get the weight vectors W directly by this computation. So, it requires one to wait till one receives good amount of Y, right or good amount of X over here.

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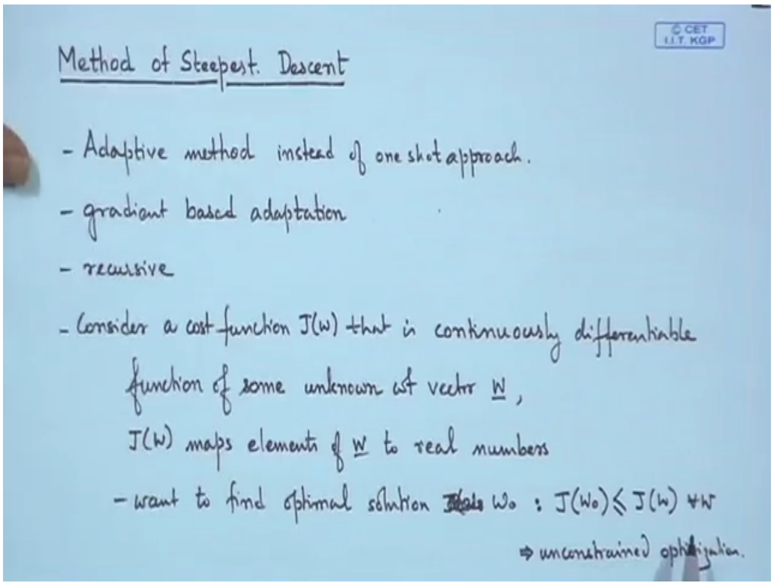

So, want to use adaptive methods because we could continuously improve the performance as well as we could continuously use tracking methodology to pursue the equalisation build up whereby we may reduce the demand on the pilot data that may be necessary it is a gradient based approach and it is recursive in nature.

So, we start with consideration of the cost function J of w; however, we consider J of w which is continuously differentiable. So, w is the weight vector. So, J is a function of w we have described before and J w maps the elements of w to real number because it is the mean squared error. So, W is complex, but still this is a real number and we want to find the optimal solution W 0 such that the cost or the mean squared error with this optimal weights or W o is less than any other weights that we decide we can clearly see this an unconstrained optimisation problem that we are handling.

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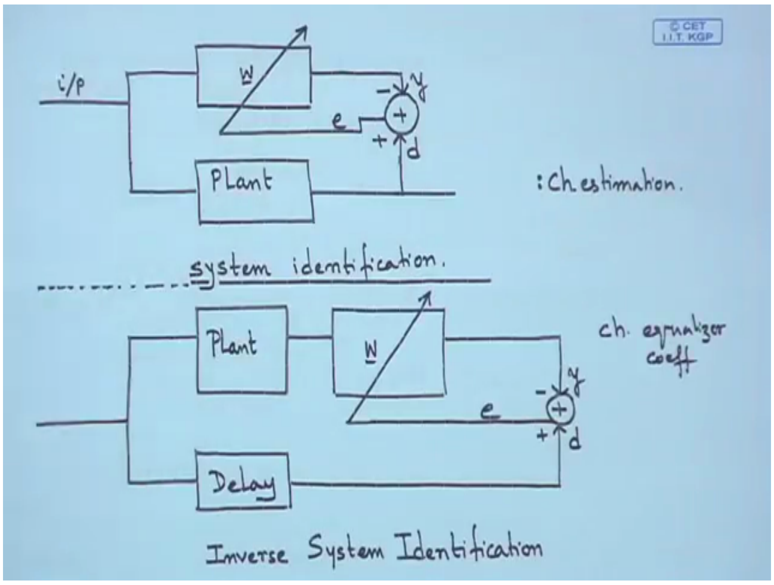

So, we can think of a situation where there is input and there is a plant, it is a typical model for channel parameter estimation. So, in case of plant we can think of a channel right which produces an output and we want to find the weights adaptively. So, that the signal when passes through this compared with the desired signal produces an error which is minimum and we will be able to drive these weights using these errors. So, that Y comes as close as possible to d. So, in this way we could estimate the channel coefficients in w.

The other version that we can have is the input signal goes through the channel passes through the equaliser filter and we have Y the other path would be the delay which means that I would be able to store the signal at my receiver which is d that is the pilot.

I will compare the output of the equaliser with the desired signal the errors that I get will be used to drive and find the weights of the equaliser filter directly, this kind of a system is known as inverse system identification because we would like to invert the effect would like to equalise the effect. So, that here we produce the desired output using whatever we discussed earlier we would like to get an impulse; that means, 0 ISI output here. So, with this we stop this particular discussion and we would like to continue with the kind of equalizer that gets build with this kind of configurations which are adaptive in nature.

Thank you.