Modern Digital Communication Techniques
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Lecture - 55
Channel Estimation and Equalization (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. So, we have discussed about Inter Symbol Interference and we wanted to transmit at the maximum rate of signalling without having a ISI or 0 ISI, but the constraint that we found is realizability of the filter. Because of which, we required axis bandwidth, so since that causes some wastage of bandwidth; we wanted to investigate some methods, by which we could restrict ourselves within the nyquist bandwidth and yet have communications at the nyquist state that is 2 W symbols per second. For which we introduced the kind of signalling techniques known as partial response signal and we are introducing the pulse shapes or the signalling scheme known as duo binary signals.

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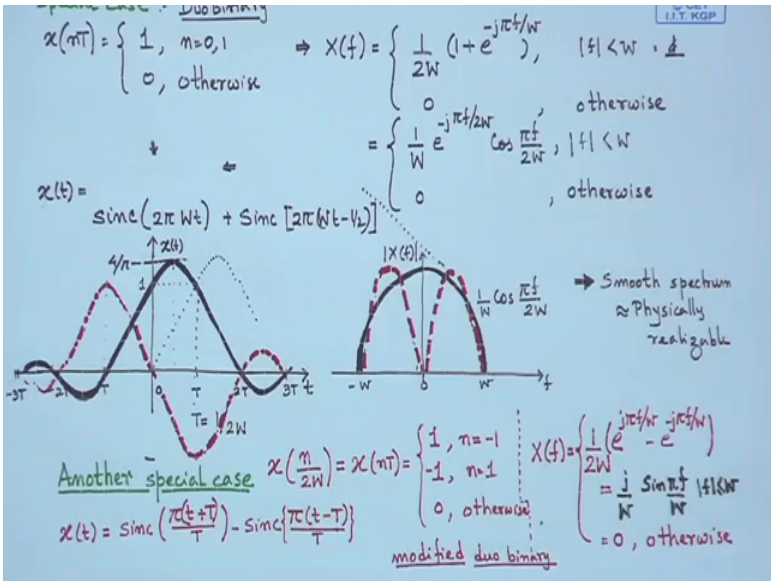

So, we explained briefly how the pulse shape would be; so, the pulse shape that we have is the solid over here; which has nonzero values at two sampling instants. So, in this situation there will be another pulse, which would begin here and it will continue in a similar fashion. So, it will cause inert symbol interference, but since it is controlled; we should be able to handle this inter symbol interference. The other pulse shape is mentioned here which also allows some kind of ISI in a similar manner.

The x n T that is when described at sampling instants is mentioned as if it is 1 at n equals to 0 and 1 and it is 0 otherwise; that means, in other situations. So, if you take the fourier transform for this, the spectrum would appear as described over here for frequency less than W and 0 otherwise. And this can also be viewed, if you expand this; you could view it in terms of e to the power of j 2 pi by W f; that means, if I take e to the power of j 2 pi W f out, you are going to left with a; e to the power of j 2 pi; W, 2 pi f by 2 W plus e to the power of minus j 2 pi f by 2 W; which works out to be 2 cos of pi; f by 2 W, which we have over here and 2 and 2 cancels out; this is the common factor.

So, which has an expression which looks like this; there is a phase term and there is a cosine kind of role of instead of rectangle role of. So, if you look at the spectrum; we will find the spectrum takes a form from these expressions above as given here. So, for the duo binary of the solid black impulse that we shown on the left, you have the spectrum which is cosine as you are seeing over here. And there is a phase which is e to the power of j 2 pi; f by 2 W and for the other one you have the spectrum which looks like this.

So, if we focus on the spectrum it is much more realizable; it falls off smoothly and beyond that it is 0. So, this is much more realizable than the nyquist filter and we would restrict our self within the W spectrum that we need and yet we are able to signal at nyquist state.

So, the other case which is the red one is indicated by the corresponding expressions of x t in the red colour over here, which has two sync; the plus and a minus, as you can see plus and a minus. There is an offset of plus T and minus T; so, you have this offsets. And it can be described as 1 for n equals to minus 1; that means, at that sampling instant it is one at n equals to minus 1. And it is minus 1, if you look at this value it is minus 1; at n equals to 1 and 0 otherwise, it is 0 over here, 0 over here and 0 at all the places.

And in terms of frequency response, it is e to the power of j something; minus e to the power of minus j and this is 2 j; sin pi f by W. So, 2 and 2 cancels out; you have j by W; sin pi f by W for f restricted within this and 0 elsewhere. So, these corresponds to the next set of pulse ships that we are looking at call the duo binary signals.

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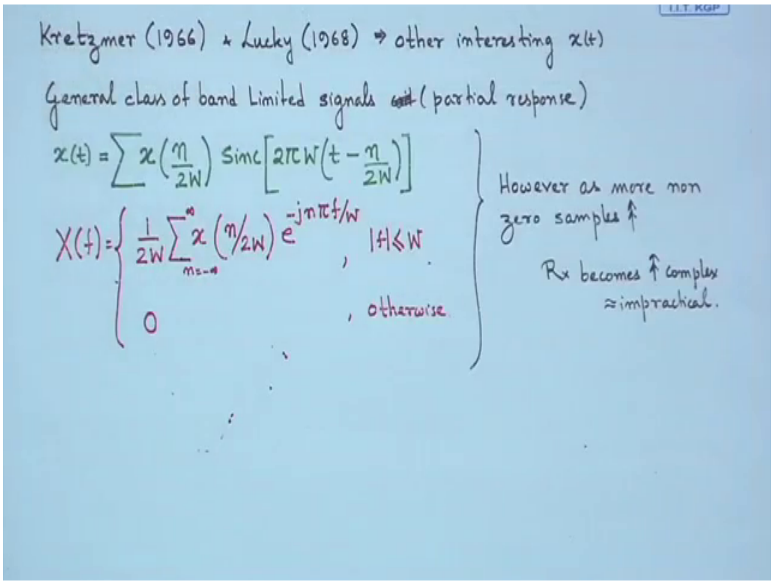

So, if we look at what happens; I mean before we proceed further just let us quickly take a look at the general class of band limited signals with partial response can be expressed in this form. So, if you compare it whatever we have there; there is a sinc plus a sinc, here there is a sinc minus a sinc. So, you have a summation of sincs and you have values of x n upon 2 W; which can be plus or minus. And in the frequency, you have again a form e to the power of j pi f and x can take different values, according to which you are going to get spectrums which look like this.

So, this is the general form and Kretzmer and Lucky they produced other interesting pulse shapes, but they are pretty complex. So, we restrict ourselves the study of duo binary signals as discussed in the previous picture.

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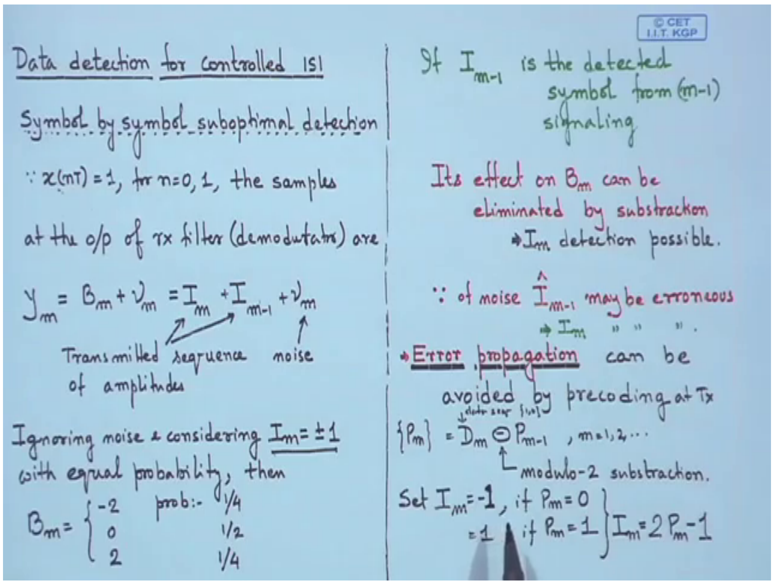

So, if we now focus on the data detection; so, there are several ways of detecting the data. Ideally speaking, since we have memory in the system; so, we should use some kind of a memory based demodulation technique. So; that means, there is a sequence that we should concentrate on and because of the sequence, we can use the maximum likelihood sequence detector; because there is memory involved, there is a sequence not symbol by symbol. However that is very complicated, so we will focus on a symbol by symbol detection all though it is suboptimal, but still it serves a great purpose and it also gives us insight into realizibility subsystems.

So, looking at symbol by symbol suboptimal detection; so, since x n of t is 1 for these values; that means, at two sampling instants it is non-zero. The samples at the output of the received demodulator filter are Y; m which is some B m plus noise and B m is Ith amplitude; plus the I minus oneth symbol. So, these are the transmitted sequences; so, because of inert symbol interference; we are seeing controlled inter symbol interference because of the symbol and the previous symbol which we had drawn in the picture here. So, there is and interference here, so that is what we had depicted and this is the noise of course, we have stated.

So, when we start building the receiver; we will take a look at the noise free received samples. So, as to construct the receiver algorithm and whatever noise is present would cause in payments in the performance. So going ahead so ignoring noise and considering

I m; to be plus minus 1. So, we are taking limiting ourselves to the situation where it takes binary values and we also say that let it take these values with equal probability, if the probability changes or calculations would be different and generally equal probability is valid.

Our B m over here can take minus 2 or plus 2 when both of them are minus 1 or both of them are plus 1 simultaneously. So, if each has a probability of half probability of getting 1 and 1 is half multiplied by half; that is one fourth and probability of getting minus 1 and minus 1 is similarly probability of getting minus 1 half multiplied by that of half. So, getting the joint and considering these to be independent sequences is one half and 50 percent probability of other situation that is 0; that means, a plus 1; a minus 1 or minus 1 and a plus 1. So, there are two conditions each with one fourth probability adding to give half probability which we get as 0.

So, given this we consider at this point that I m minus 1 is detected symbol from the m minus oneth signalling instants. So, suppose we begin with that I m is detected correctly and that can help us in the decoding procedure. So, if this confuses we can begin with the state where there is no transmission that is 0 amplitude, when detected the first detected symbol is let us say I 0. So, its effect on B m can be eliminated by subtraction because you can see B m is given by this.

So, if I know I m; I can take away I m minus 1, if I know this I can take away I m minus 1 and I will be left with I m. So, if I have detected this correctly I would be left with only the current signal. So, if I can do this; that means, if I can eliminate I m minus 1, I can detect I m. Now, because of I m minus 1; error in detection because of noise because there is noise present and we have discussed how noise affects the detection and there is a detection probability and probability of error. Therefore, I m may also be erroneous; so once I m is erroneous, the next symbol may also be erroneous and so on and so forth; so, this procedure is known as or this situation is known as error propagation.

So, we would like to see if it is possible to remove this error propagation. So, error propagation can be avoided by precoding techniques, so if we are using certain precoding techniques we can remove the error. So, what we can do is at the transmitter we can produce P m as D m; modulo two subtraction of P m minus 1, where P m is the output and D m is the data sequence of 0’s and 1. So, P m would be 0’s and 1’s using a memory of the previous output.

So, this since it is modulo two operation; we are going to get P m as 0’s and 1’s, this is the incoming bitstream; so, there is some kind of memory involved. So, there is some process similar to the n r; z I that we had seen before. So, if we set I m equals to minus 1; in that case rpm would be 0 and if we can set I m as 1; if P m is 0; that means, whenever P m is 0; I would send a minus amplitude level and when P m is 1; I would send a plus amplitude level. So, this could be minus d plus d or it is represented as minus 1 plus 1.

So, this is the pulse amplitude modulated; binary pulse amplitude, so these are the two values inconsistent with this description over here. So, based on the bits sequence here; I get these amplitudes. So, hence we could write I m as 2 times P m minus 1 because P m takes a value of 0; I m takes a value of minus 1, which is depicted here and P m takes a value of 1; I m takes a value of 2 times 1 minus 1 which is 1, which is depicted here.

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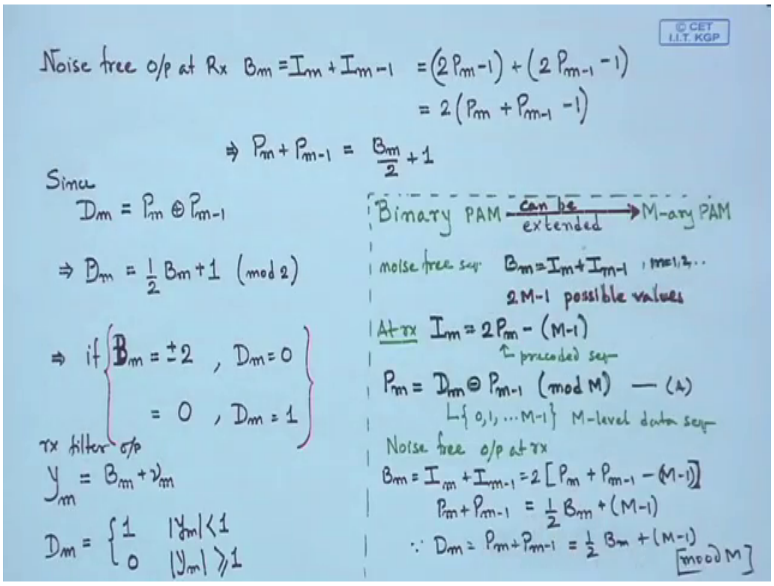

So, the noise free output at the receiver; so you would recall that the received signal is this. So, noise free means I am looking at removing this only at B m; so that means, I am looking at this and not at this. As I said, we want to construct the receiver by focusing on this and not on the noise and then the noise would affect our decisions of course.

So, B m is equal to I m plus I minus 1, I minus 1; so, we have all the necessary expressions visible now; B m as this that is what we have written. I m is equal to 2 P m minus 1; so, use here 2 P m minus 1; I m minus 1 is 2 times P m minus 1. So, we have this result 2 times; P m plus P m minus 1 minus 1. So, what we get is P m plus P m minus 1 is; so, we have B m is B m by 2; half of B m, plus of 1 that is what we have over here.

Now, since we know that by this expression; we have P m is equal to D m modulo 2 subtraction P m minus 1. Therefore, D m from this we can say D m is equal to P m modulo 2 addition of P m minus 1. So, proceeding with this we can write P m plus P m minus 1 that we have over here is B m by 2 plus 1; we use this over here. So, we have D m; another variable which is basically given here. The same variable is continued here is half B m plus 1 with modulo 2 operation; modulo 2 because it is received 2 value of 0 and 1.

So, that would mean that if B m takes a value of plus minus 2; we have seen that B m takes a value of plus minus 2; we would choose D m to be 0 and if B m takes a value of 0, we would choose D m to be 1. So; that means, we have recovered our data sequence at the receiver by simply comparing B m to a threshold; which if it is near 0, we would set it to be 1, if it is closer to plus minus 2; we would set it to be 0.

So, in that fashion we can recover the data sequence that has been transmitted by virtue of cancelling the interference right at the transmitter. So, the receive filter will be having Y m is equal to B m plus noise. So, we would said that if because it is 0 and you can say there is a 0, there is a 2, there is a minus 2. So, decision point would be somewhere in between which is 1 and minus 1. So, if B m so if I receive Y m which is B m plus noise; that means, B m is plus minus 2 or 0 plus noise.

So, if it falls within this range I would declare that data received is 1; that means, if this is 0, it is 1. So, going by this if the received signal with noise is lying within this range plus minus 1, I would decode it as 1. Whereas, if it lies in this range; I would decode it as 0 going by this, look at this. This connected with this range and which is depicted that Y m is greater than 1, so this is midway between and we get D m. So, this is how we would build our receiver; so we can extend this technique to M-ary PAM. In case of M-ary PAM; we have the noise free sequence B m as which is as did before, but these are 2 m minus 1 possible values; because now this not only take plus minus 1, but it takes larger values up to 2 M minus 1.

So, at the receiver you would still have I m equals to 2 P m minus M minus 1 and this is the precoded sequence, going by the previous notations. And P m would be modulo m operation, so this is the precoded operation as you do at the transmitter. So, the noise free output at the receiver in a similar fashion can be taken as I m plus I m minus 1, which is the same expression as we did before, but here there is an m minus 1 previously we had one. So, there m was 2; so, 2 minus 1 is 1; so, minus 1 is what we had for binary level and hence P m plus P m minus 1, which is again a similar expression as we had seen before and there M was 2 and here M is general and D m can be P m plus P m minus 1; there is the data sequence is what we have over here, which is half of B m plus m minus 1 in modulo manner. That means, we take B m and we would decide on the value as we did for the binary situation. So, that way we can easily extend this to M-ary PAM and get our received signal.

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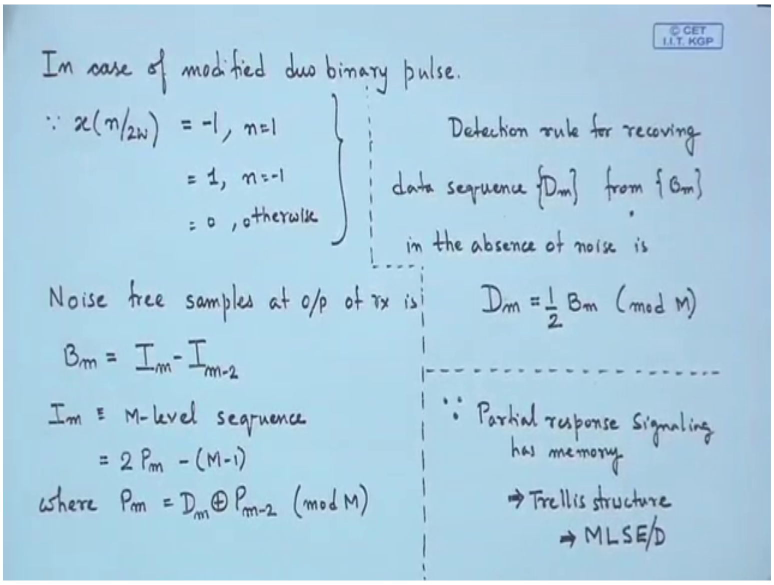

So, in case of the modified that is the red figure that we had shown; that means, if we had modified this is known as the modified duo binary. If we use the modified duo binary, so we have minus 1 at n equals to 1 and 1 at n equals to minus 1. So, at the negative side you had a plus 1, at the positive side you had a minus 1 and your demodulation would be a minus over here instead of plus because of this minus sign so we had a plus earlier.

So, I m would be the M-level sequence, so in general it looks like P m minus m minus 1 and P m is this one; which was earlier a modulo two subtraction, here it is modulo two addition and the detection rule for recovering the data sequence would be in absence of noise is of course, this with noise there will be thresholds like we considered here. So, similar to the threshold that we had considered; we must consider thresholds in this case and would decode the received signal.

So, as we stated in the beginning that all these methods are suboptimal because we are looking at symbol by symbol. And the other way of recovering the signal would be to consider maximum likelihood sequence detection or maximum likelihood sequence estimation which we had described in an earlier lecture.

So, by constructing the chain of received sequences; we should be able to compare the probability of one sequence with another and another given a particular transmit sequence; that means, we will change the transmit sequences and compare the posterior probabilities, whichever transmit sequences maximizes this probability we are going to select that in case of maximum likelihood sequence detector.

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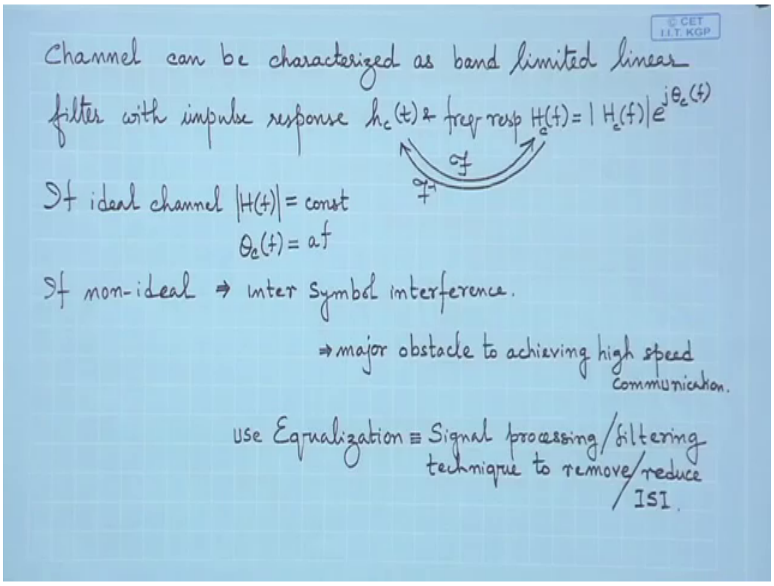

So, with this we cover our discussion on techniques by which we can send signals through band limited channels. So, just a quick summary that we could restrict our signals to a bandwidth of 2 W, but in that case we would stretch our symbol duration to be longer than the typical nyquist. And the excess bandwidth could be used for a smooth role of, but since we have stretched the bandwidth; it causes inter symbol interference.

And this inter symbol interference is introduced consciously, so it is controlled inter symbol interference which could be address at the receiver. And then we saw that one could precode at the transmitter and could decode symbol by symbol at the receiver. Just a reminder, we developed the receiver algorithms by considering no noise and noise would definitely affect the performance and finally, as the last word for the best performance one should use the maximum likelihood sequence detector because there is memory in the transmitted signal.

With this we are now geared to move into the channels which are non-ideal. So, let us proceed to the discussion of channels which are non-ideal. So, in case of non-ideal channels, so channel can be characterized as a band limited linear filter; which we have already said. With an impulse response h c of t and frequency response H f; which is having a gain and a phase term and these are related by fourier transform relationships.

If it is the ideal channel, we would say that the gain is constant and there is a linear phase associated with that and if it is non-ideal, we would have inter symbol interference which we described through the impulse response diagram earlier. So, this inter symbol interference is a major obstacle to achieving higher bit rate as we have said because of complexity in calculations.

So, to reduce or improve the situation we can use equalization or signal processing at the receiver. So, by signal processing we mean that we are going to use some form of filtering. Because at some point we stated that the complete transfer function of the system; which is the transmit filter, the channel transfer function followed by received filter as well as equaliser together should give us a 0; ISI situation.

Since we said the channel is not known a priori; we would restrict the raised cosine pulse shape to the transmitter and receiver filters and we would restricted to route raised cosine. Then we said that whatever is remaining with the channel could be compensated by the equaliser filter so that the channel and the equaliser filter could produce an ideal response. So, when we take together the whole set we would produce a 0 ISI received signal sequence.

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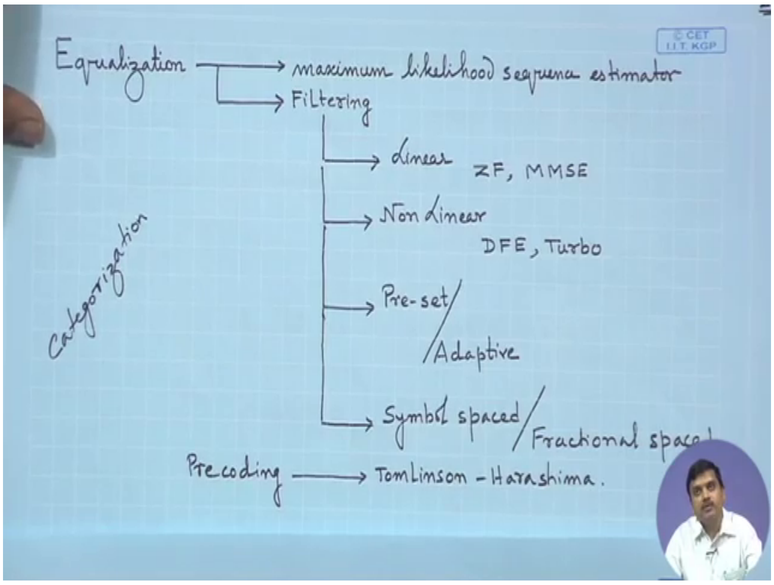

So, if we look into the study of equalization techniques; there are several equalization techniques which are known. One is the maximum likelihood sequence estimator; we have seen the general structure of it; that is extended to apply in equalization. In the filtering techniques, we can classify it as linear which could be 0 forcing or MMSE; we will see them. It could be non-linear; that means, decision feedback equaliser or turbo equalizers which we will not see. It could be categorized as free set; that means, fixed or adoptive, we will partially see some form of adoptive equalization and it could be symbol spaced or fractionally spaced.

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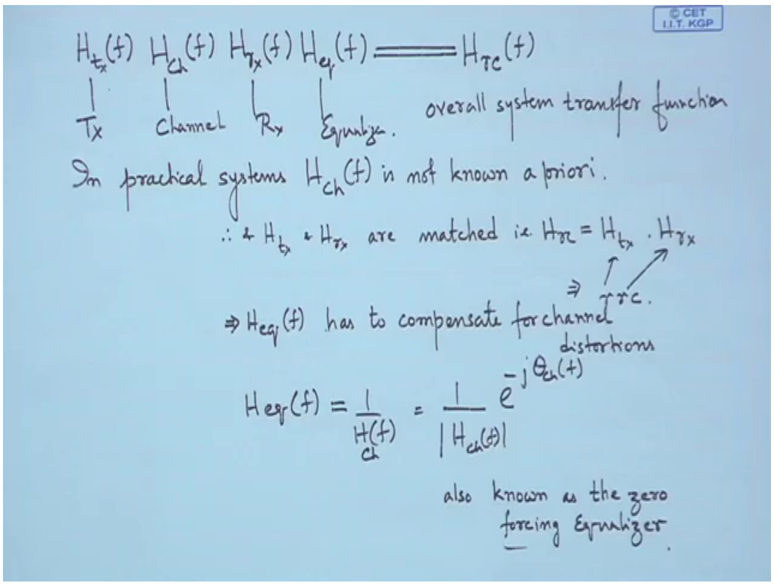

Further, we could also use precoding techniques; one famous precoding technique is the Tomlinson-Harashima precoding. We have already seen one precoding technique in the earlier discussion. So, here we have the overall system transfer function which includes the transmitter filter, channel, receiver, equaliser. So, these two we have already assigned let us say route raised cosine. Of course, we should remember that the receiver knows the kind of signalling the transmitter is going to use; this is part of a protocol.

So, only a transmitter and receiver pair which has been designed together, following a particular technology or standard arrival to decode each other. So, suppose I use full response signalling at the transmitter and I use the filter for partial response signalling; things are going to go away. So, you must use the corresponding pair of transmitter receiver filters at the both transmitter and receiver respectively.

So, now we focus on the channel and equaliser; this we have already seen are route raised cosine, which produces a raised cosine. Now, whatever should be the channel; the equaliser should be able to cancel the effect of the channel and that is why the name equalizers the channel effects.

So, in practical systems as we said channel is not known a priory and this have been converted to route raised cosine so that overall it is raised cosine and as we just mentioned that H e q has to compensate for the channel. So, now a quick description of things is that as we said that; let us take a look at the channel impulse response which we had seen before alright.

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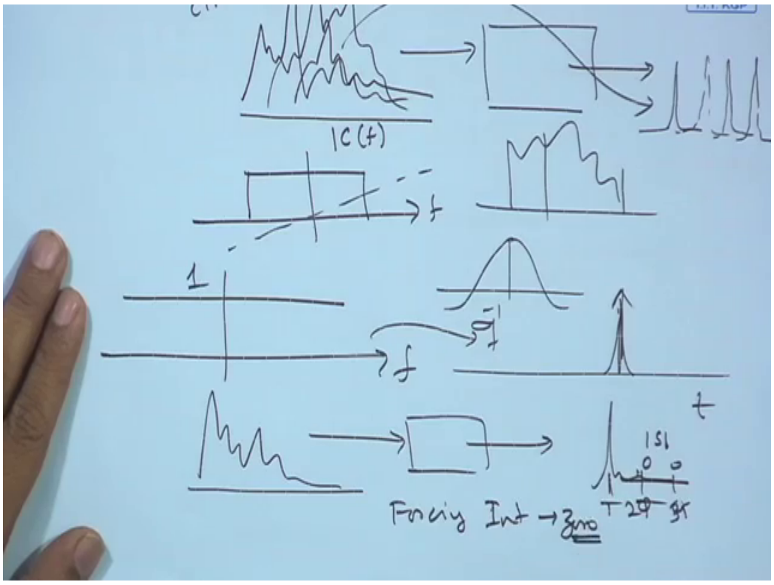

Suppose, the channel has an impulse response which is like this and then there is another echo and continuous signals are sent; so, there is ISI. So, you want to pass this through a filter so that we get a raised cosine. Overall, we have seen that these two combined to produce the raised cosine. So that means, these two together remaining should produce an ideal filter. In other words, in the frequency domain there response should be flat in terms of gain and there should be some linear phase.

So, whatever is the channel transfer function it should equalise the channel transfer function to produce. If this is the impulse response, the channel transfer function would be something like this. So, if we have to produce a channel transfer function which is in this manner, we should have a sinc response composite. If we think of infinite bandwidth situation, in that case this should be converted back to an impulse. If the response is converted back to an impulse, we will get consecutive trains which are in pulses and there would be no inter symbol interference.

So, this leads us to having the equalising filter made equal to 1 upon the channel transfer function; that means, when these two are multiplied; it produces a 1; living these two products to give a raised cosine; that means, the gain is inverse of the gain of the channel transfer function and the phase is the conjugate of the phase of the channel transfer function. So, together if it produces a 1 as the frequency response; it produces a value of 1; if you take the inverse fourier transform, it should produce and impulse in the time domain. So; that means, if the channel produced an impulse response in this fashion; when it passes through a equaliser filter, it should produce an impulse at the sampling instants and 0 at all other sampling instance. So that means, it is not allowing any ISI or it is forcing the interference to 0.

So, these kind of filters as described by the expression here are also known as 0 forcing equalisers because the force, the impulse response after passing through the equaliser filter, composite impulse response to be the impulse. And impulse means, it is unity at the first sampling instant and it is 0 at all other sampling instants.

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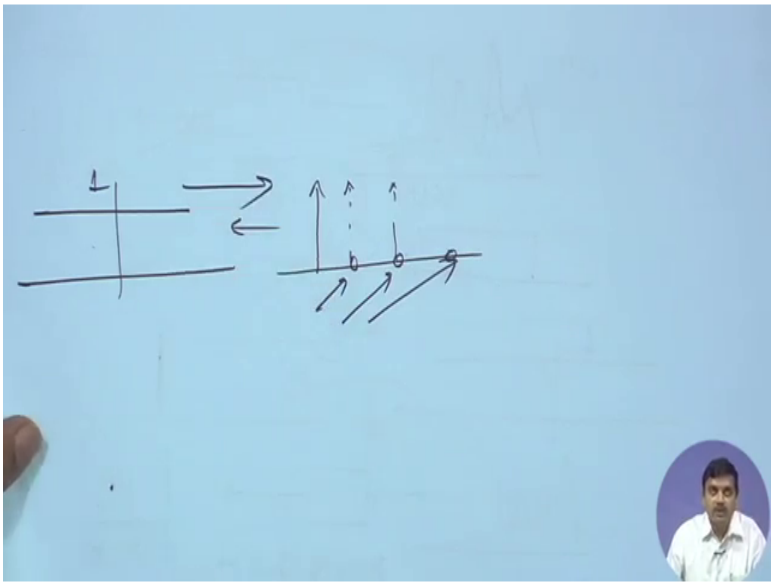

So, which means that if you have an impulse and 0’s elsewhere; if I send another signal, there will be no interference; so, it is forcing all interference components to 0 because in the frequency domain, you have a composite respond which is flat N l; in the time domain it is an impulse and therefore, this kind of an equaliser filter is known as a 0 forcing equaliser.

In the next lecture, we will see some other kinds of equalizers and thereafter we will proceed to design of other important algorithms of the communication system such as synchronisation.

Thank you.