Modern Digital Communication Techniques
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Lecture - 54
Channel Estimation and Equalization (Contd.)

Welcome to the lectures on modern digital communication techniques in the previous lecture we have discussed the Nyquist pulse and the Nyquist filter by virtue of which given a bandwidth W, one should be able to send 2 W symbols per second which produces the maximum symbol packing which is possible.

If we consider K bits per symbol; that means, if you are doing PAM with K equals to log base 2 of M, in that case, we would be getting 2 K symbols per second per hertz that is the maximum limit that one can achieve.

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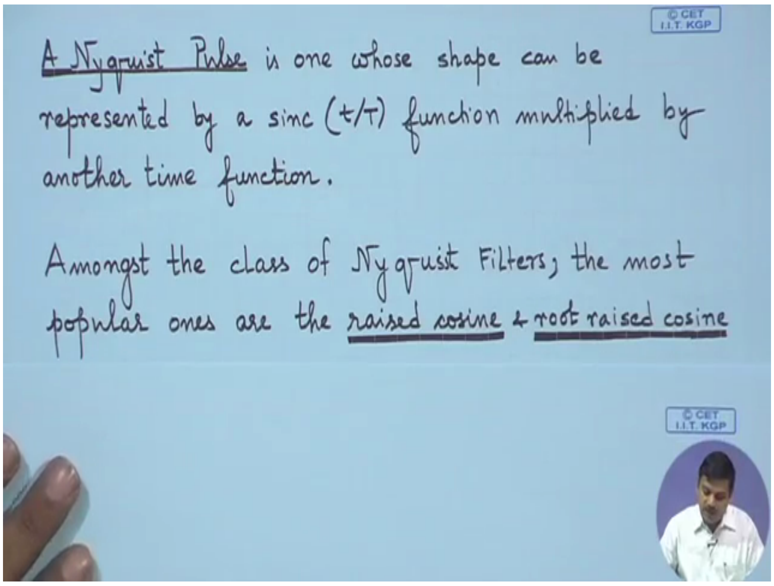

So, we revisit that the Nyquist pulse is one whose pulse shape can be represented by a sinc function multiplied by other time functions and we just explained in the previous lecture that if we consider this as the sinc, suppose I consider roughly speaking, a sinc goes like this and I consider another time function let us say this. So, when I multiply these 2 it is going to follow the 0 crossing at appropriate 0 crossing instance you cannot help it. So, this with multiplied by this is generally going to produce this.

If you are producing if you are using some other pulse shape let us say this in that case also you will not get a product of this. So, at 0 crossings this will turn to be 0s, this multiplication of 0 with this is going to produce a 0. So, you might get some shape which we do not know may be which we have to calculate, right.

So, in the frequency domain what happens it is a convolution of this produces in the frequency domain rectangular pulse and this will produce some frequency domain structure. So, time domain multiplication would be in frequency domain convolution, right. So, that is why we said its convolution in the frequency domain.

So, amongst the class of Nyquist filters when we say the class of Nyquist filters what we mean is all those filters for which the frequency response is convolution of the Nyquist frequency response convolved with any other frequency response of the choice which would been if I look at the time domain the Nyquist pulse multiplied by the pulse of that particular new pulse shape which I am concerned with this way we are going to produce 0 ISI at the sampling instance of time which is our desire.

So, amongst these several possible things the raised cosine and the root raised cosine are the most popular. So, we are going to take a look at the raised cosine pulse shape.

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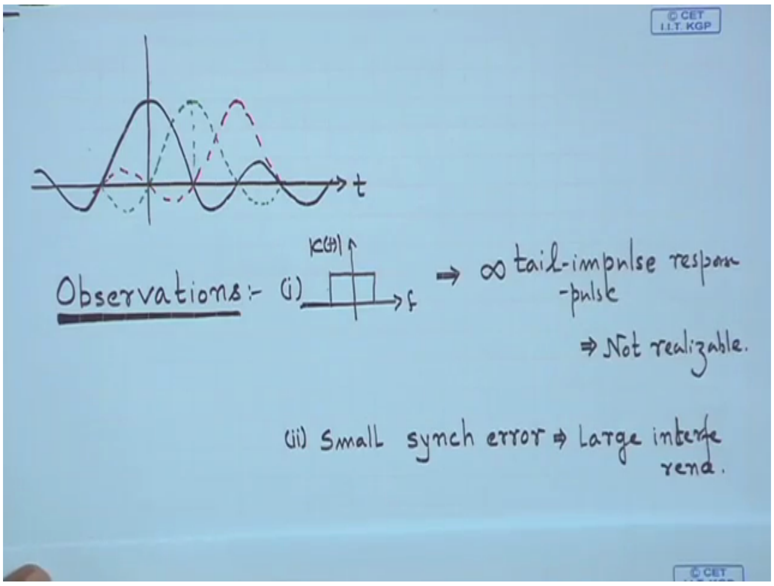

So, from this picture is just to remind us what is happening to ensure 0 ISI and for this particular spectrum, we know that the pulse shape would be this which has an infinite tale as we have just mentioned, since it is not realizable, we should look for other solutions.

Further, we can also note that if we have a pulse shape a strain of pulses arriving in this manner. So, we could redraw for a sketch I am trying to understand what goes on. So, if we say that there is a 0 cross here, right. So, roughly speaking we have done that and suppose our line goes here.

So, we are supposed to sample at this location at this location roughly at this location where this also goes to 0. Now, if the receiver clock makes an error and instead of sampling there, it samples at a little bit of set point. So, if it starts sampling at a little bit of set point, what we see that the desired signal has a lower value while there is contribution of other symbols from the neighborhood and we do get a lot of inter symbol interference.

So, this although produces orthogonal and the highest packing, but still it has some problems which we can visualize from a situation as depicted here. So, small synchronization errors can cause large interference because of the pulse shape.

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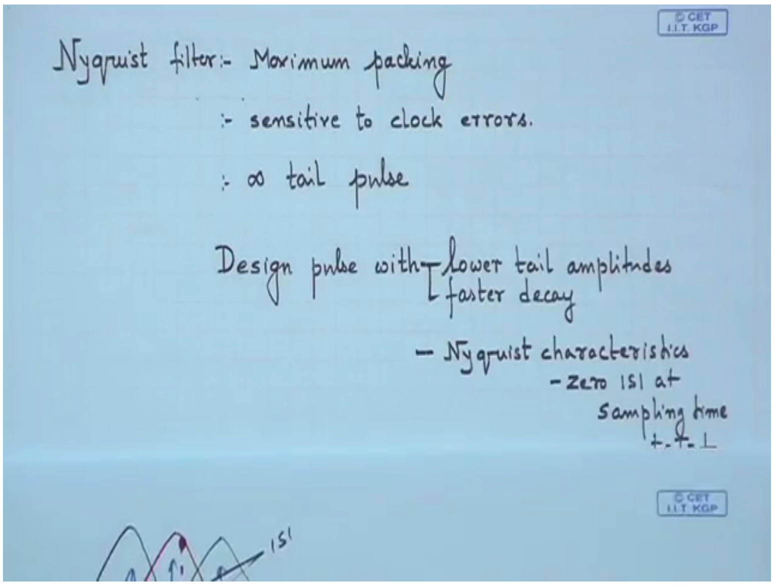

So the Nyquist filter as said it maximizes the packing, but it is sensitive to clock errors and it has infinite tale. So, although there is an advantage, these are some of the disadvantages. So, we would want to design a pulse with lower tale amplitude. So, if the tale amplitudes are lower; that means, if it rises slowly; that means, it of raising there if the rise would have been like this if the fall would have been like this then the amount of interference would have been less, right, we could have produced a lesser amount of interference if the thicker lines would produce be produced.

So, we are looking for a pulse; that means, we are looking for pulses with faster decades. So, there the decay is not. So, fast the Nyquist characterizes 0 ISI at sampling time. So, we must ensure this we cannot avoid to change this and the solution one of the famous solutions is raised cosine which provides us with the features that we are looking for, right, if we have the Nyquist property as well as the tails are going to die out much faster.

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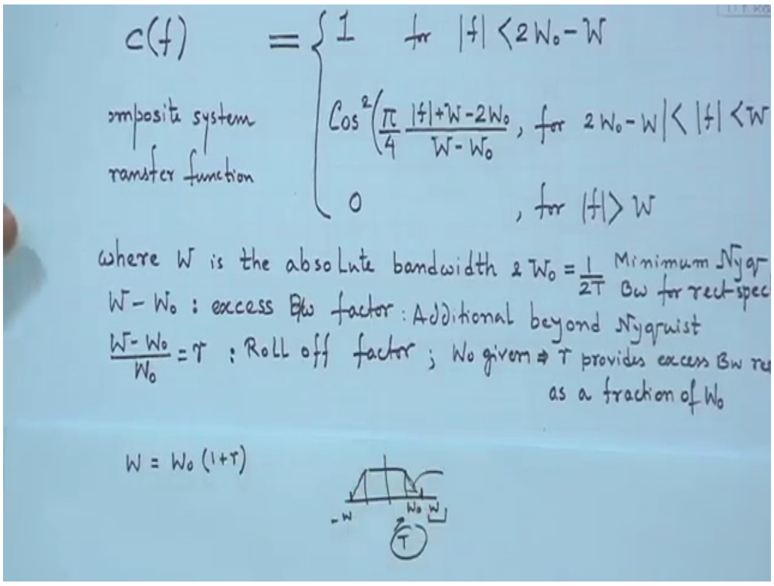

Such a filters channel transfer function is described it is given. So, we will take to be given out of design which is equal to 1 for certain duration of frequency, we would explain these terms shortly. So, it is flat over a certain duration of frequency and it is equal to cos squared pi by 4 and these expressions for another range of frequencies, right, we are going to depict the picture of this spectrum and things will be clearer and it is 0 outside W where W is our spectrum that is available to us.

So, where in this particular expression we have W is the absolute bandwidth and W 0 is 1 over 2 T that is the minimum Nyquist bandwidth required by the spectrum or the minimum Nyquist spectrum.

So, what it means is that if I am signaling if my symbol duration is t, the Nyquist bandwidth is W 0, but I use a practical bandwidth which is W, right. So, in these expressions W 0 to be used is one upon 2 T and W is something which is given and W minus W 0, this particular value is the excess bandwidth factor there is the additional beyond Nyquist and the ratio W minus W 0 upon W 0 is defined as the roll off factor. So, this is a parameter.

In some cases, you might find this particular expression and these frequency relationships are given in terms of roll off factor which you can also translate by bringing r e to the expression by using this relationship.

So, W 0; so, if W 0 is given that is the Nyquist bandwidth given r provides the excess bandwidth as a fraction of W 0 which we will clearly see if I give W 0; that means, if T is given the symbol duration is given then we can and r is given some fraction then we can calculate what is the excess amount of bandwidth that is required.

So, W is equal to r times W per W 0 from this you can see W is equal to W 0 into one plus r. So, this is r is W is greater than r. So, r is greater than 0 greater than or equal to 0 so; that means, W is greater than W 0. So, what we are trying to say before we get into further discussion a sinc, this is not realizable we give an excess bandwidth and then we would ask the filer to taper up slowly.

So, if we can ask the filter to taper up slowly this makes a more realizable filter because a filter with rectangular transfer function is not realizable for all reasons. So, if we can design a better filter which tails which die out faster we can truncate the tails at a certain point then in the frequency response we are going to get something like this. So, what is this excess ratio if we can specify given this, we can find the amount of W that is required? So, all this given a value of symbol duration that is a symbol rate.

So, why do we go in this manner because generally you will be given the bits per second to be transmitted bits per second per hertz is what we calculate. So, if you are given bits per second you know the modulation; that means, you know K all you are left with is to determine T. So, if you determine T you get the symbols per second.

So, once we get symbols per second what you can calculate is the Nyquist bandwidth now once you calculate Nyquist bandwidth and we know that Nyquist filters are not realizable you can allow an excess bandwidth with the certain roll off in the frequency domain which is more realizable and you can calculate the overall W that is required. So, that is what is being described by these expressions here.

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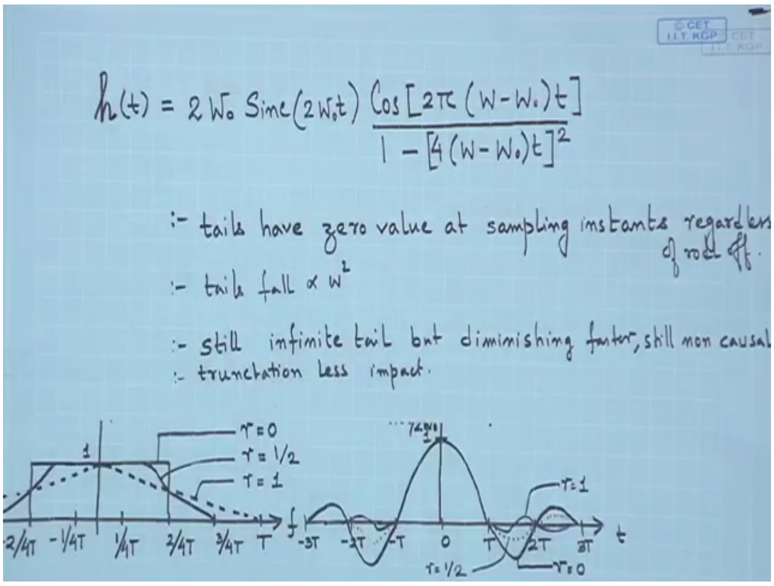

So, the channel impulse response how do you obtain simply by taking the Fourier transform or the inverse Fourier transform of C of f. So, if we take the inverse Fourier transform of this, we will be left with the impulse response which appears the expression, if you derive would get to something like this we cannot help, but use this expression because there is no point in doing the inverse Fourier transform which we are supposed to do by ourselves.

So, this has tails which go to 0 at sampling instance regardless of the roll off that we have already said because you are multiplying with the sinc as you can clearly see. So, if this goes to 0 at sampling whatever is there will go to 0 tails fall of as W squared what you can see the denominator there is W squared. So, tails would follow up the power falls up at W to the power 4 and still in, but still there is infinite tale, but since it diminishes faster you can do some truncation as we have pointed out.

So, we will just take a look at how things look like. So, what we have here well we will compare heat transfer function and yes, we compare the transfer function and the impulse response in this particular figure. So, what we see is I will bring to the center that if we have the roll off factor of 0, we get the rectangular ideal Nyquist filter. Now this is pulse. So, possible to understand by looking at this expression. So, if we make r equals to 0 W is equal to W 0 which is the ideal Nyquist bandwidth.

And if we let r equals to 1 if r is equal to 1 W is equal to 2 times W 0. So, when r is equal to 1 that is dashed line we see that it is a much more relaxed roll off for the filter, right; the pass band has sufficient space. So, you can design a good filter with this easy filter with this. So, it is spread in the bandwidth right and if we look at the corresponding pulse shape the solid line is for the one where r is equal to 0 that is corresponding to the rectangular pulse shape right and if we look at r equals to one is the one which is here.

So, what we clearly see that if I make the total available bandwidth as twice I can design a filter with very very fast decaying tails anything in between 0 and one anything between 0 and 1 would appear like this and here it is drawn for r is equal to half which requires 50 percent excess bandwidth and the pulse tale goes like this, right.

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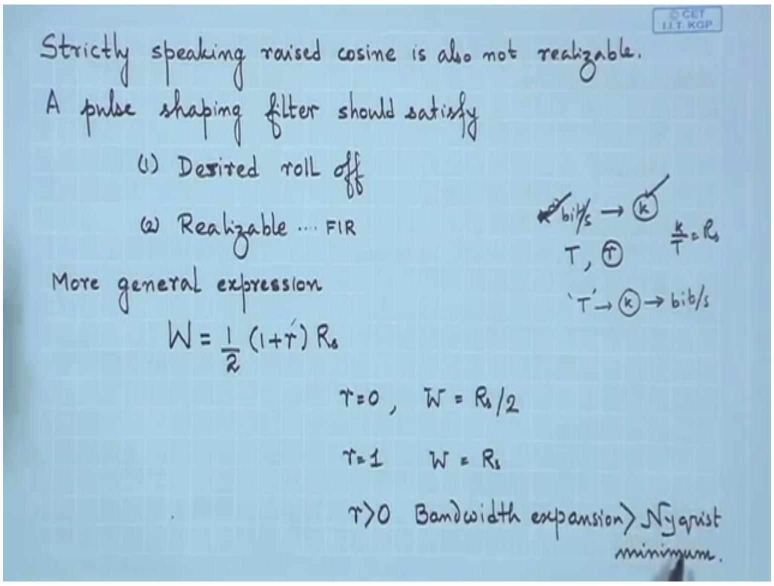

So, what we see by choosing an appropriate value of r one can design a suitable pulse and there would be a corresponding implication on the spectrum that is occupied. Now what you get is much more realizable situation; however, what you pay is excess bandwidth which is not required for communication, but you need it for practical realization.

So, if you could realize a filter which is very very close to the Nyquist, then we have a better communication system, but because of practical constraint you would set a value of r which is not equal to 0 neither you would you choose r is equal to one. So, you generally choose somewhere in between 0.5-0.3 and something like that which would give you a good roll off factor and a trade of between the bandwidth efficiency and realizability of the filter.

So, what we have is that the raised cosine filter is also not realizable in a strict sense simply because the tale stretched to infinity. So, if you have such a situation we would like to design a pulse with the desired roll off and a realizable FIR. So, a more general expression of W with respect to R s is given through r where we find that R s by 2 is the W because if R s is the signaling rate the bandwidth required is R s by 2.

So, now through this relationship you can calculate given a symbol rate; that means, if K is specified; that means, K bits per second if it specified this and you have K which is specified. So, you could calculate T if you know r right that is a reverse way of doing things; that means, I want to calculate the amount of bandwidth that is required given a certain bit rate and a roll off factor.

So, I must be choosing my modulation level right and given r right I would be able to calculate because if modulation level is fixed K upon T should be equal to R s that is fixed and we want to calculate W or in a reverse manner if W is given r is given; that means, the bandwidth and the roll off factor you would be able to calculate what is the symbols per second that you can send and once you calculate the symbols per second you have T with you. So, if you have T with you now you can adjust the value of K if you want to achieve certain bits per second data rate.

So, if you are confined with the certain symbol rate by increasing K you can increase the bits per second. So, what we have is if r is equal to 0, we have the Nyquist situation if r is equal to 1 we have W is equal to R s and r is equal to 0 r is greater than 0 is the bandwidth expansion factor which is beyond the Nyquist minimum.

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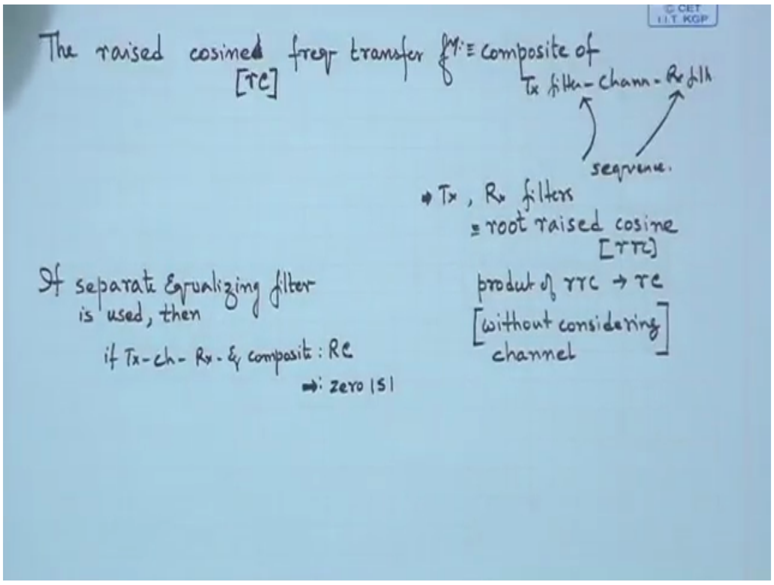

So, if we look at the raised cosine frequency transfer function, it is composite of the transmit filter the channel and the received filter why we say this because we have stated that we want to have this kind of a received pulse strain whatever happens if you receive this if you get this there is no, ISI right if you are sampling at the right points, there is no ISI.

So, this is the composite, we want to have the raised cosine. So, if you want to have composite as the raised cosine you have the transmit filter at the transmitter side you have the channel there is a filter and there is a received filter, right.

Generally the channel is not known a priory and it has to be estimated and compensated at the receiver which we will see shortly. So, in those cases the transmit and received filter together should give the raised cosine of course, we will see there is a equalizer which is present here together which would give an ideal transfer function.

So, under those conditions if the received if the raised cosine has to be split between the transmit and the receiver you get a root raised cosine because when it is root raised cosine when multiplied in the frequency level because you have the you have the filters in sequence cascaded. So, the overall composite transfer function is the multiplication of all this in the time domain it is the convolution of all the pulse shapes.

So, what we have is the root raised cosine. So, if it is the raised cosine over all square root square root when multiplied together in the frequency domain as a transfer function we get the raised cosine function. So, that is what we generally do.

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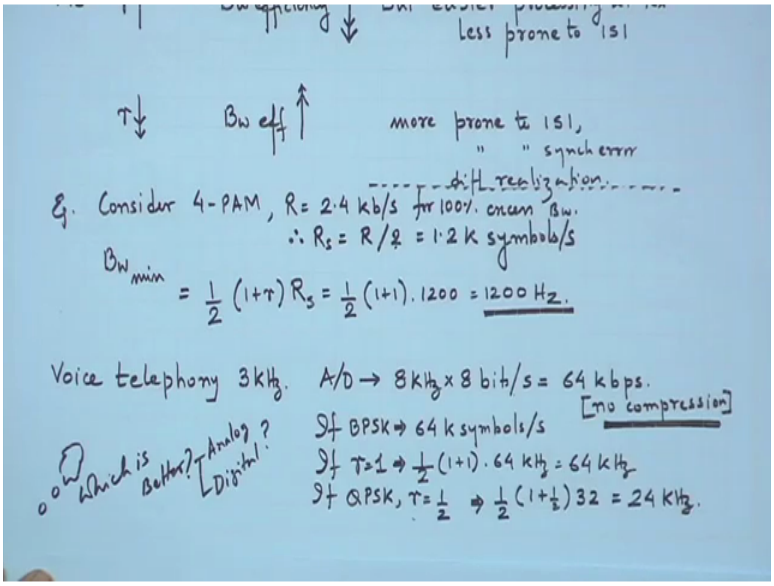

So, what we see from what we have discussed till now is as we increase the roll off factor right as we increase the roll off factor the W becomes larger we get excess bandwidth right r is the fraction of excess bandwidth or bandwidth efficiency decreases since what we have already stated whereas, in the reverse way if I reverse bandwidth efficiency increases, but it is more prone to ISI and here it is easier to receiver to do the receiver processing and it is less prone to ISI. So, these are some of the tradeoffs which we are usually concerned with.

So, as an example if you take 4 PAM and r is equal to 2.4 kilobits per second for 100 percent excess bandwidth; that means, r is equal to 1, we are going to get R s is equal to r by 2 because there are 2 bits per level. So, you have R s by 2. So, 1.2 kilo symbols per second. So, bits per second converted to symbols per second and the bandwidth minimum that is required is half 1 plus R sl r is 1 because it says 100 percent excess bandwidth available you can use 1.2 kilohertz to communicate 2.4 kilobits per second that is what we get over here.

So, in case of voice telephoning if you would extend the example to a voice telephoning the bandwidth of voice typically is 3 kilo hertz. So, if we are doing analog communication we will be requiring to send the signal over a bandwidth which is slightly more than 3 kilohertz.

If we do digital communication we will first sample it and convert to sample sequences at 8 kilohertz per second because more than twice is 6 which is 8 each sample, if it is encoded into 8 bits per second, we have 64 kilobits per second without any compression.

So, if BPSK is used; that means, if we use binary PAM then each bit is going to take a wave form and we will be using 64 kilo symbols per second stated at 1 bit per symbol if r is equal to 1; that means, the roll off factor is 1, we are going to use 64 kilohertz by the calculation if instead of using BPSK we go for QPSK what we get and we use r is equal to half we have 24 kilohertz.

So, the question that I would like to ask you or keep pondering over that whether transferring 3 kilohertz voice using analog communication is better than digital communications. So, why I ask this question because as you can clearly see even after using r half and higher order modulation we still require 24 kilohertz whereas, in case of analog signaling we would not require more than let us say 4 kilohertz at most or may be 5 kilo hertz. So, this is spectrally much less efficient in terms of message compare to analog at least in this pseudo example. So, whether it makes sense or not to use digital communication is something we should start thinking.

However, I would just like to give it as an exercise for you to think and discuss amongst your friends and come up with the right answer just a note a hint here we have said low compression. So, to find the answer think of all possible advantages that are digital communication system gives compare to analog communication system and the answer will be clear to you.

So, with this, we come to the next important kind of signaling where we will address some of the issues that we have discussed so far. So, one of the primary things that we have discussed is that if we are using the maximum signal packing then we could use Nyquist filters. Nyquist filters are not realizable because the spectrum is rectangular which is ideal it produces sinc pulse with large tails and that causes a non causal system. So, a better situation is if you could use a little bit of excess bandwidth provide a little bit of roll off in the filter transfer function instead of rectangular and then you would get pulse strains with smaller tails. So, if you would get pulse strains with smaller tails, then it dies out faster and you could truncate, it at a certain point and you could realize the transmission system.

But the penalty that is paid is in terms of excess bandwidth because you have used more the Nyquist and hence a spectral efficiency is reduced. So, the question which you can ask is it possible to communicate at 2 W symbols per second when the bandwidth available is W hertz. So, there is an answer to this. So, what you need to do is if you could avoid the constraint of 0 ISI; that means, in the previous case we said that is the limit if you have 0 ISI has the constraint.

So, if we reduce the constraint or relax the constraint that we do not want 0 ISI, there it is a possible answer. So, then we could be imagining that we could remove that constraint and we could go ahead and achieve very very fast signaling, but the problem is there is a limit to the amount of complexity there you can handle when you would like to cancel inter symbol interference.

So, at most 2 or 3 symbols interfering is a manageable complexity of course, things have become better these days, but this complexity that we are talking about is forcibly introduced by us and not by the channel.

So, what we must be aware of that there will be additional inter symbol interference because of the channel and we are introducing something more to it. So, we would like to restrict the amount of inter symbol interference that we can allow in a controlled manner and we get into the regime of controlled inter symbol interference.

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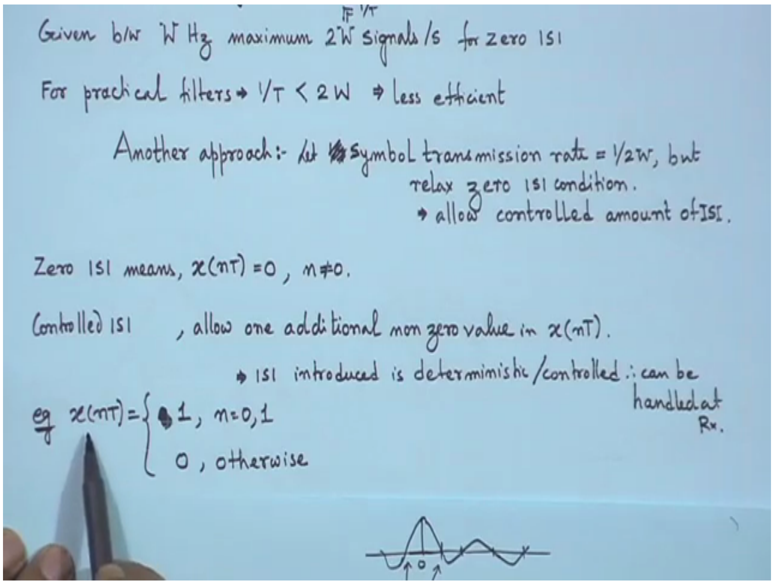

We achieve this kind of a communication system by means of partial response signaling and we have stated earlier the definition of partial response signaling which we will revisit once again in the current context.

So, given a bandwidth W the maximum 2 W signals per second is for 0 ISI which we have stated for practical filters this situation is less efficient. So, the different approach that we can use is to let the symbol transmission rate be 1 by 2 W by relaxing the 0 ISI condition and allow a certain amount of inter symbol interference.

So, 0 ISI would mean that x n T is equal to 0 for all n naught equal to 0; that means, at all positions we are going to get a 0 percent except at 0, right. So, controlled would allow at least 1 additional non 0 value in x and t. So, what it means it is that instead of having all 0s can we have a situations where there is a non 0 signal over here or may be a non 0 signal at some other point right only one is the thing that we are trying to introduce over here.

So, why what would we get we are going to get a symbol duration which is 2 T; that means, longer and hence we are restricting our self to a bandwidth which is much smaller than the Nyquist bandwidth and the remaining bandwidth that we have could be allowed for the roll off.

The problem is you could say that you are signaling at a much slower rate 1 by 2 T, but then if we send a signal at 1 by T what we get is inert symbol interface, but since there is only 2 symbols interfering we should be able to cancel and that is what we are going to see in this part of the lecture.

So, if we say that x n T is equal to one at n equals to 0 as well as n equals to one so; that means, instead of this we are trying for a situation where it is one over here and one over here all other places it is 0. So, it is right at all other paces it is 0. So, that is what we are planning to introduce.

So, what we have is the class of do binary signals, right, they do binary signals that we have is can be visually seen by the solid black curve as we have over here in this particular picture. So, I will cover up the rest of it to avoid confusion.

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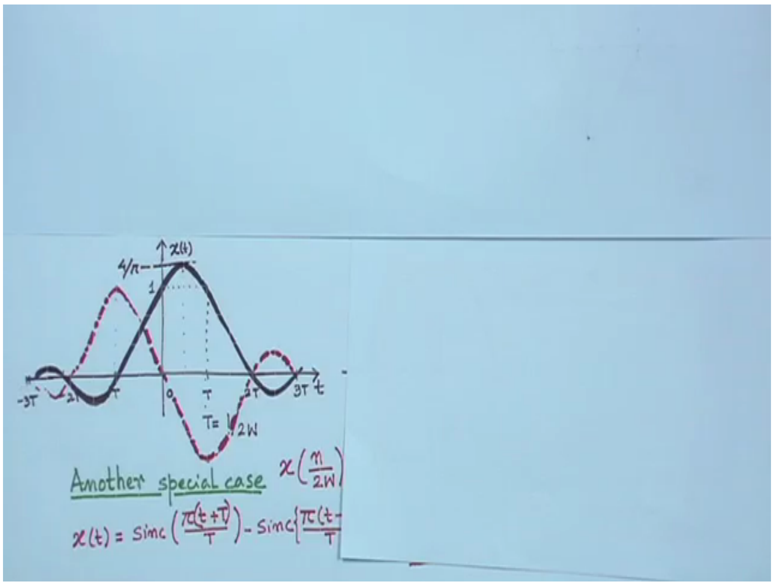

So, what we see is that in this case we have a non 0 value at 0 as well as we have a non 0 value at P; however, at all other places there is 0 value, right.

There is another signal which will also describe simultaneously is where it is non 0 at minus T and non 0 at T, but it is 0 over here. So, this also introduces 2 symbols with ISI because here, if I am sending a signal I am going to get ISI for this here if I send a signal I am going to I get ISI for the red one right so; that means, it allow certain amount of ISI, but since you can see that the span of the signal is more than the earlier situation, the bandwidth requirement may be smaller and smaller than the Nyquist or W 0 and if that is satisfied the excess bandwidth that we have up to Nyquist could be used for a smooth roll of which we are going to see shortly.

Thank you.