Modern Digital Communication Techniques

Prof. Suvra Sekhar Das
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Lecture - 53
Channel Estimation and Equalization

Welcome to the lectures on Modern Digital Communication Techniques, so till now we have been able to cover transmission of signal and deception signal processing and the receiver has been covered; by which we have been able to detect the signals. We have also seen how to do performance analysis of digital communication systems, as well as we have also compared the performance of digital communication systems with a bench mark known as the channel capacity.

So, we have done all these things; mostly considering AWGN channel, which is again a baseline channel; although it has practical implications. So, once we have covered these now we are ready to move into the next stage of communication system design, where we slowly remove one ideal condition after another. And then we will be faced with a (Refer Time: 01:10) task of designing transmission of signals, as well as processing of signals at the receiver whereby we face our non ideal conditions at the receiver.

The first thing that we are going to look at is communication through band limited channels. So, till now we have seen AWGN channel; and in AWGN there is no bandwidth constraint on the signal design.

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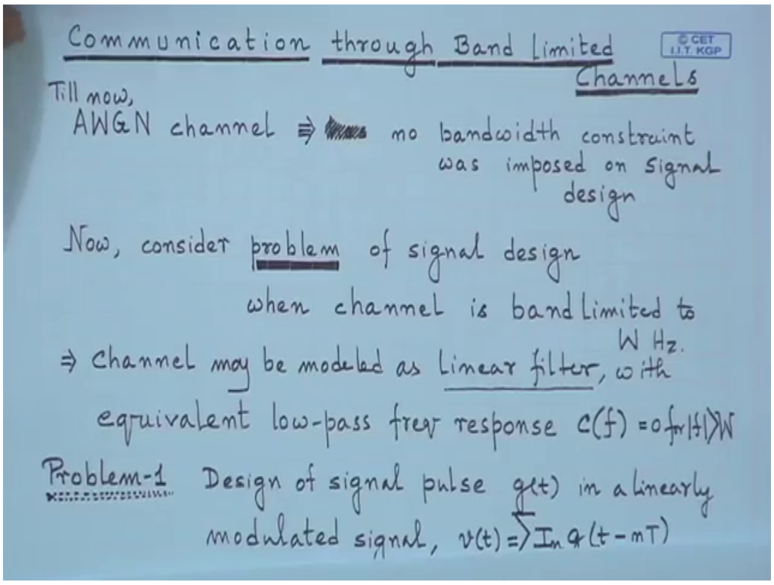

So, as we move on we consider the problem of signal design when channel is band limited to W hertz; now this is pretty practical because most of the communication channels have available amount of bandwidth through which signals have to be sent. Even if you might encounter channels, which have a huge amount of pass band or available bandwidth; still we would be generally be interested in confining our signals to a certain bandwidth which could be specified in terms of the parameter W.

For example, when we are doing multiplexing; for example, the one that you usually encounter is the f m signal, which is typically in case of analog communications. Similar thing we can think in terms of digital communications, a related situation would be the wireless communication or GSM; which are very familiar with in your daily use. There are several channels which go in parallel and in each channel, the signal is restricted to a certain bandwidth.

So, whatever we discuss would be restricted to that class where signal is limited to a certain bandwidth W. So, channel in this case may be modeled as a linear filter with equivalent low pass frequency response c of f is equal to 0, for frequencies greater than W. So, there are some important things here; we are talking about linear filter model; generally this is well excepted model. So, we need not be much worried about the channel being represented by a linear model. So, we will agree with this and we have to proceed with this for other situations things will be different, but since this is also a base line, it stands as a very very important platform for developing several situations as we will encounter.

So, the first problem that we will encounter is to design the signal pulse g t; which we have not paid much attention to other than being given a particular signal pulse. So, if you remember we had some point stated that let the pulse shape be rectangular; at some other places, we said that; we consider half sinusoid generally the m S k situation. So, here we say that given certain restrictions can be find the pulse shape g t.

So, the modulated signal could be represented as v t; which is the baseband, as I; n which is the sequence of data amplitudes and pulse shape g t minus n t; where g t is non zero in the interval 0 to capital T. Of course, we will see certain variations of that, so that is what we have been generally used to.

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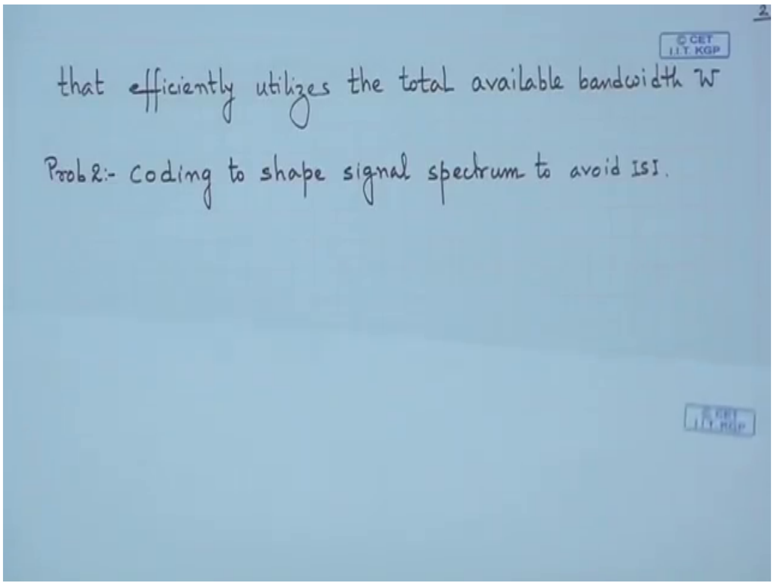

So, we would like to find the pulse shape g t; that efficiently utilizes the bandwidth W, so that is the first case that what we have to encounter. And the second problem we will also look at is coding to shape signal spectrum to avoid inter symbol interference. We have seen some things of it, when we are studying the spectrum of digitally modulated signals; we will see something more here. So, here we are restricting the signal to W and we want to find g t; that will provide us efficient utilization of the bandwidth.

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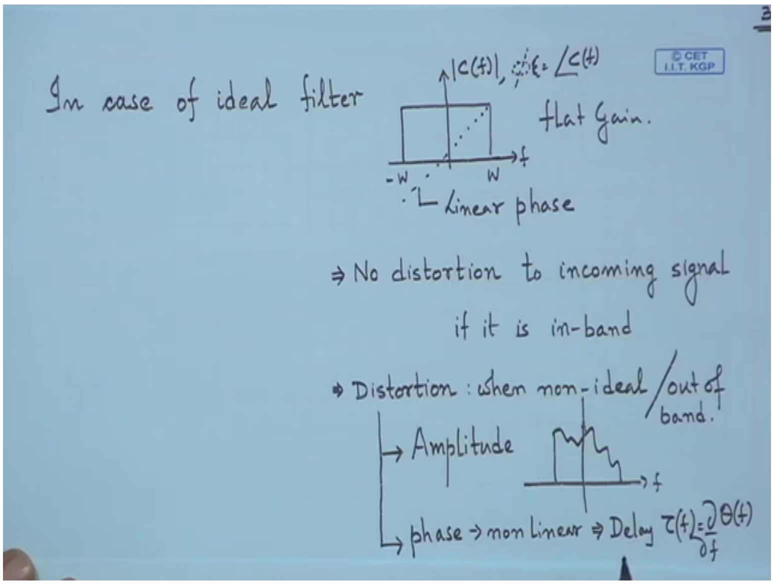

So, in case of an ideal filter; so let us start with the ideal filter because that is the simplest thing that we can begin with. So, here the channel amplitude is flat over the desired frequency that is W and we have stated previously that beyond W; it is 0. So, the channel; not the pulse, the channel is said to be 1 which is ideal; that means, no amplitude distortion and there is linearity of phase.

Linearity of phase ensures that group delay is constant; if the phase is not linear, the group delay which is the derivative of this phase with respect to frequency, which tells us what is the speed of propagation of different frequency components, what we will find; if they are not the same, some frequency components will proceed faster compared to others.

So, when the signal is received at the receiver; we will find that the some portions of the signal is received earlier, while some portions of the signal will be received at a later time. So, that will cause speeding of the signal; to avoid this we would like the phase delay to be constant; that means, all the frequency components arrive at the same instant of time.

So, if there are distortions; so, typically channel distortions can be thought of as amplitude distortions where instead of it being constant, it could be fluctuating with f and there could be non-linear phase. So, as we just mentioned that in case of non-linear phase, the group delays which is the derivative of phase with respect to frequencies is not constant; this we have just explained and this is what we get. So, distortions can appear in this manner and ideal would be in the situation.

So, as well as in ideal case there is no distortion and the signal is within the band of the pass band. So, if signal is outside the pass band; in that case you do get distortions. So, we do not consider those situations under the ideal condition.

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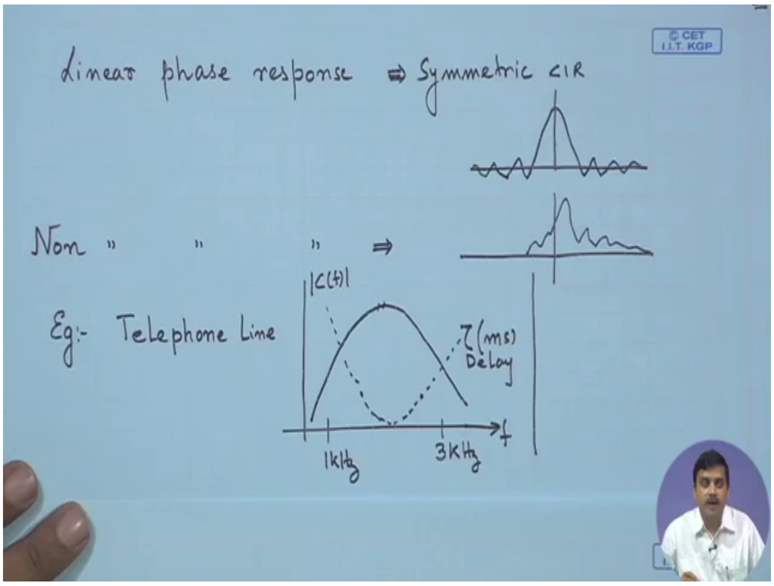

So, the linear phase response as we just mentioned results in a symmetric channel in pulse response. We will quickly see what is meant by the channel impulse response; however, if the phase is non-linear, the impulse response would be asymmetric and it could take; for example, a situation which is like this.

One particular example is the telephone line; so, if you are using the telephone line; the plain old telephone line system; the pots, the frequency response with f on this axis and the gain of the channel in this axis; has a response which typically is depicted by the solid line.

On the same figure, if we plot the phase delay on Y axis verses the frequency; we get the one represented by the dashed line. So, what it represents that the telephone line is not an ideal channel; however, just a side note; if we consider a small fraction of the bandwidth, it may be considered as flat and in that portion the phase may be linear. So, this is like part wise breaking the whole thing and taking piecewise linear model; we will not be using that, but this is just for your reference in case you may encounter such situations in certain references.

So, as we said we are going to look at the channel impulse response, but before we do that; since we have talked about the telephone channel, we would also like to mention in the passing about the wireless channel.

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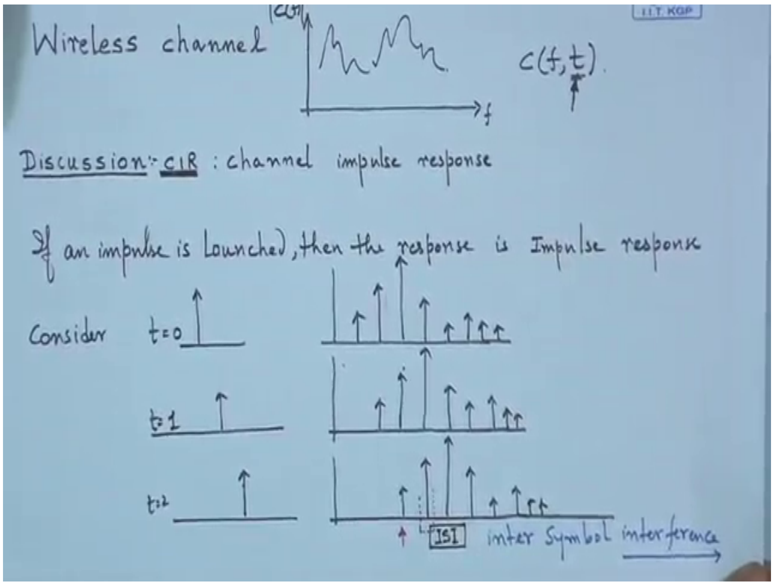

In case of wireless channel what we have is; if this is a frequency and this is the frequency response the gain of it, you get absolutely fluctuating gains and hence it is appears to be a random gain channel. And this is even difficult to handle and these are not discussed in this particular course, but whatever is studied in this course can be extended to with advance signal processing, when one studies a subject on wireless communications.

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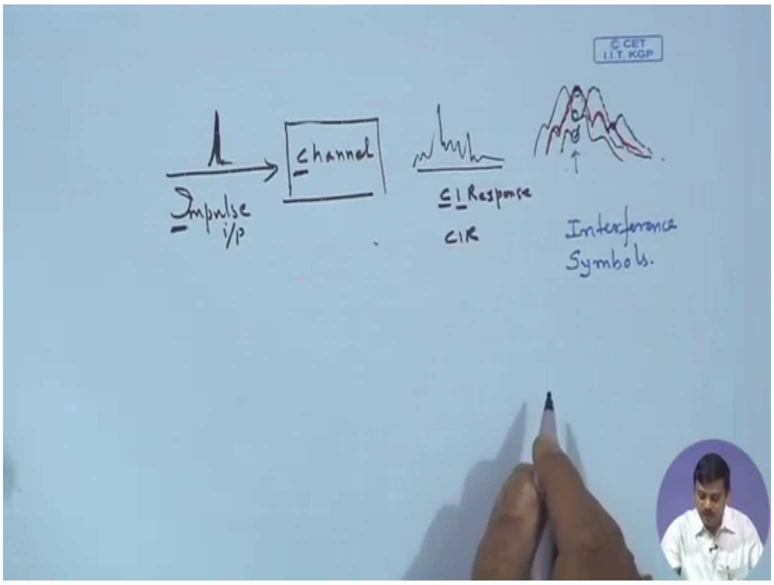

So, when we talk about the channel impulse response; so, the channel impulse response is defined as the response of the channel, when an impulse is launched into it. So, one can think of the channel as a system or as a filter as we said and if you launch an impulse into this; if I am going to launch an impulse into the channel whatever is the response I am going to get out of the channel would be the channel impulse response. So, channel impulse response and that is known as the C I R.

So, if the channel impulse response is something of the shape, similar to the shape; let us say something like this and we sample it in the discrete time because we are looking at discrete time processing. If we launch an impulse, let us say at time t equals to 0; one would get an impulse at a certain time followed by echoes of the signal which is available. So, there by what we get is if the channel impulse response appears as this or as something like this, what we can effectively read is that an impulse gets spread in time; that means, it becomes wider in time.

So, now let us consider the situation that at a time separation marked as 1; which is in the discrete time. If we launch another impulse, what do we get? We get a similar impulse due to this kind of a impulse launching to the channel. However, since we have said it is linear; we can apply the super position principle and then we get an output as this.

At the next instant of time, if we lunch another impulse; we are going to get the impulse response due to this from the channel. So, if we consider all the situations and suppose the receiver is observing the output due to this impulse; which has been the third impulse launched into the channel at a time which is here.

So, what it sees is when the first response of the channel appears; at that time the second response of the earlier response; of the earlier impulse is still available in the channel. While the third response of the third earlier impulse launch is still available; so, if the receiver is deciding, what has been transmitted based on this observation what it finds; that along with this signal also present are this information which has been transmitted at earlier instance of time.

In other words, there is echo of the earlier transmitted signals. So, if we look at the continuous version, so we might get this due to the first impulse. The second impulse might produce this; we are assuming that the channel response is remaining constant with time and the third impulse launched into it would produce something like this.

So, we have this three impulses; so when the receiver processes this signal, what it gets is that at any instant of time; there is availability of one of the signals, along with the other signal as well as the other signal. So, what we see there is interference from the symbols and we generally term it; inter symbol interference is the terminology that we use for the situation. So, what we see is that under non ideal conditions when the channel is not ideal, in those cases you generally encounter inter symbol interference; as could be understood from this pictures which we have drawn here

Now, this definitely leads to a huge problem at the receiver and we are interested in resolving this crisis and trying to handle a situation or create a situation which is better than this condition so that we can decode the signals, which are clean from interference so that our performance would be as good as that of the AWGN channel.

So, the question that is what we are trying to address is; what is the maximum signaling rate when the channel is ideal? So, even if the channel is ideal; that means, it is restricted to finite bandwidth; we will see that it can still result in inter symbol interference. Whereas, when you have infinite bandwidth; the situation is different; so, to do this let us proceed further and figure out what we have.

The next question which we are also interested in is; in case of non ideal channel as we have just defected, what pulse shape is to be used so that we can avoid the situation. So, these are some of the interesting things that is what we can look at today. So, first let us focus on the situation when the channel is ideal. So, when the channel is ideal; what we have here is the frequency response of the channel.

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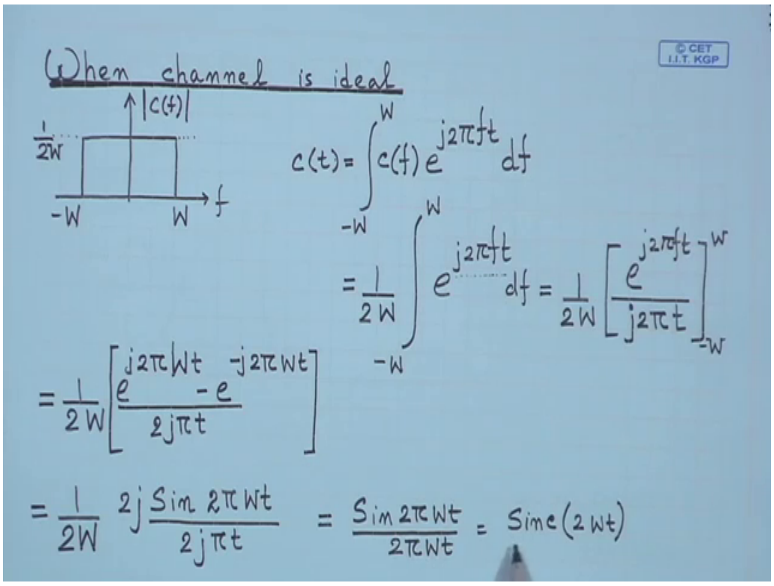

So, we would like to see; what is the time response or the impulse response of the channel for which one can take the inverse Fourier transform of the channel transfer function or the channel frequency response. Because that is how we usually define things; so, if you have studied signal processing and filters and signal system, you would be familiar with the things that the signals of fourier transform gives the frequency response and inverse fourier transform gives the time response.

So, if you are interested in a filter; we are generally looking at the frequency transfer function or the filter transfer function. And if we take the inverse fourier transform of it, we get the impulse response and vice versa if we take the fourier transform of the impulse response; we do get, the frequency transfer function or the frequency response of the filter. So, we use the same philosophy and we get c of t is the inverse fourier transform of the filter transfer function or the channel transform function.

So, if we proceed with the channel transfer function; what we get is within minus W to plus W, what we have is 1 over 2 W; so, we have one over 2 W over here and e to the power of j 2 pi f t. So, which works out to be 1 upon 2 W; e to the power of j 2 pi; f t upon j 2 pi f t; this is due to the integral of e to the power of a t; d t. So, if we look at the of course, we have d f over here. So, integral e to the power of a x is e to the power of a x upon a; so, that is what we have used over here.

So, this is the argument that is what we are using and this has to be evaluated at W and taken away at minus W; that is the limit of it. So, if you do it; you can easily see that we have e to the power of j 2 pi; W t minus e to the power of j 2 pi minus W t with the j 2 pi. So, e to the power of theta minus; e to the power of j theta minus e to the power of minus j theta is equal to j 2 sin theta, j times 2 times sin theta.

So, what we have in the denominator is j and 2; so, this j would cancel out and that is what we have over here; this is a next step immediate next step which we just described. And hence if you work this out it, turns out to be sin 2 pi W t; this is straight forward and 2 because there is a cos plus j sin, there is a cos minus j sin; minus sin and minus becomes a plus and cos with a minus of cos from this cancels out, so you are left with this term.

And if you would proceed further with this, you will be left with sin 2 pi W t upon this; which is straight forward which can be represented as a sinc. Generally in many occasions when you write sinc, we do not represent the p; but in some cases you can represent the pi. So, what we see over here is although; the signal appears to be ideal; that means, the channel appears to be ideal; we find that the impulse response is sinc. So, one quick hint what you can get from this is; if you know the response of sinc, we can draw the response of sinc briefly here.

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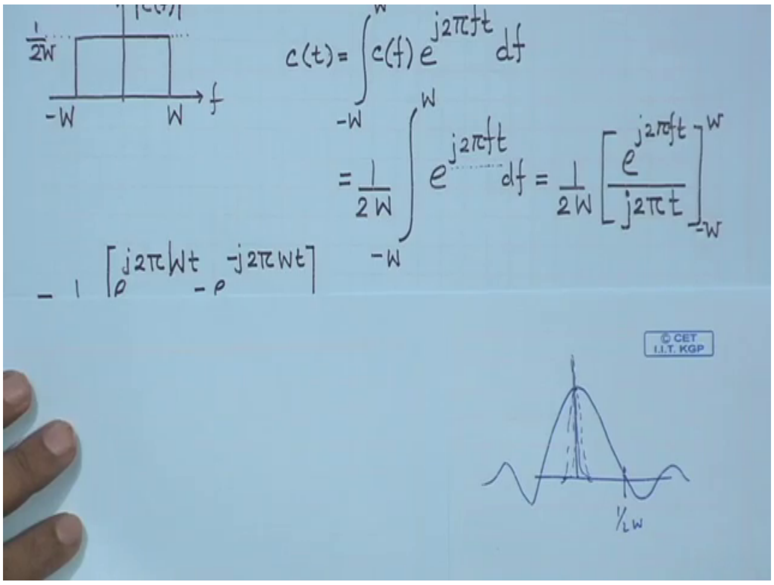

And this is the 0 crossing at 1 by W; so, in this case it is 1 by 2 W; so, what happens is as you increase W as I increase W; this is 1 upon 2 W, for our case over here as I increase W; we have the figures more or less visible.

As I increase W this denominator term grows, if the denominator term grows your sinc is going to come closer to origin. And as you make W tend to infinity, this will be approximating an impulse over here; which matches with our typical understanding. So, if the W is infinite, which is usually the case of AWGN channel; generally that is how we start with. So, the impulse response is ideal, the impulse in produces an impulse out whereas, if you start restricting the W to finite values; what you will get, is a sinc with a certain particular description or a function as can be seen in this particular figure.

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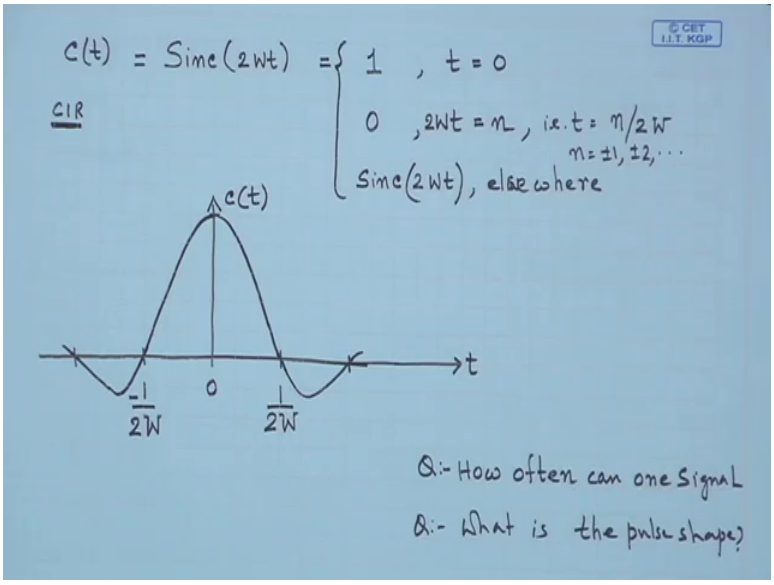

So, moving ahead c of t as we have written here is sinc, so sinc as you know has a 1 at t equals to 0 and 0 at all other sampling instance; at all other locations where 2 W t equals to n; this is well known. And elsewhere it follows the sinc function, so if you plot it; this looks over here because we have a 2 W; it turns out to be 1 by 2 W.

So, if I launch an impulse into a finite bandwidth ideal channel the response will be like this. So, now, the question at hand is how often can one signal; that means, send a signal and the next question is what is the possible pulse shape whereby we can avoid inter symbol interference.

So, before you proceed we would like to highlight that if you are considering AWGN channel; that means, we are taking infinite bandwidth; generally we have taken the power spectral density be N naught by 2, but of course, we have also studied situations where we consider the pass band and there is a certain bandwidth W and we did consider narrow band, band pass process; so, do not get confused with that. So, generally when we take AWGN; we mean that it is a signal with infinite bandwidth otherwise we would specify the situation.

So, what we have seen here is now instead of taking infinite bandwidth; the moment with restrict the bandwidth, we have an impulse response which is not an impulse, but it is a sinc. So, trying to answer the first question; how often can one signal what we see is that has described by this, as well as pi the situation by the figure here.

If we launch another impulse here, we will get 0 crossing here. That means, the receiver will not encounter any inter symbol interference at the sampling instance and that is what we depict in the figure in the next page. So, although this is a bit small, but this is what we understand.

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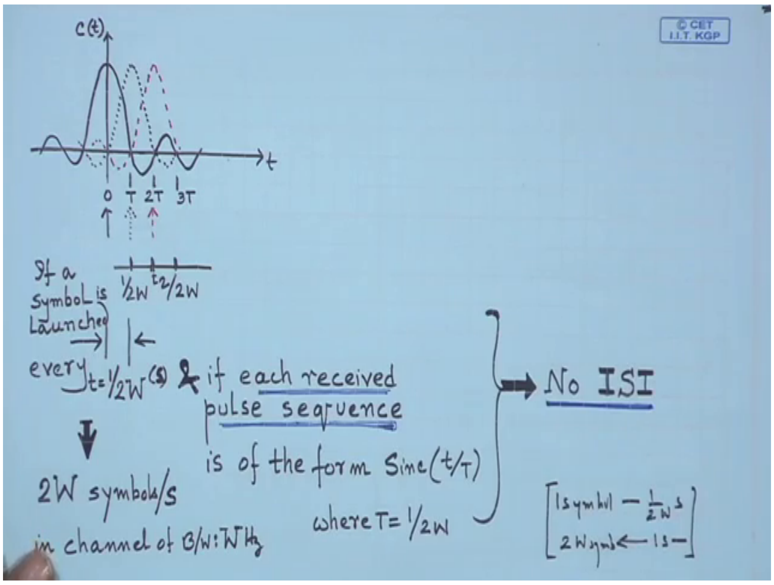

So, if we are launching signals every T duration; so, if we launching a T, 2 T, 3 T such that T is equal to 1 by 2 W, what we find at sampling instance; all other echoes go to 0. So, that clearly means that we can launch signals at every 1 by 2 W seconds.

So, now if we can launch a signal at every 1 by 2 W seconds; the question is how many signals can you launch per second; simply by unitary techniques, you can get; you can launch 2 W symbols per second. And this is the best rate of signaling, if the channel bandwidth is restricted to W hertz.

So; that means, if each received pulse sequence is of the form sinc t upon; so please note what we have written. We are saying that; if each received pulse sequence is of the form; so please note I repeat again; if each received pulse sequence. So, we have not said anything about the channel over here, we said whatever you do; what you receive, if it is in this form. I do not care what has happened before; if I receive in this form then I do not see any inter symbol interference. And this is the maximum rate at which I can send such signals, so when these two go together; we can say the maximum rate of signaling is 2 W symbols per second and there is no ISI or inter symbol interference; that is encountered, which was one of the problem that we begin with.

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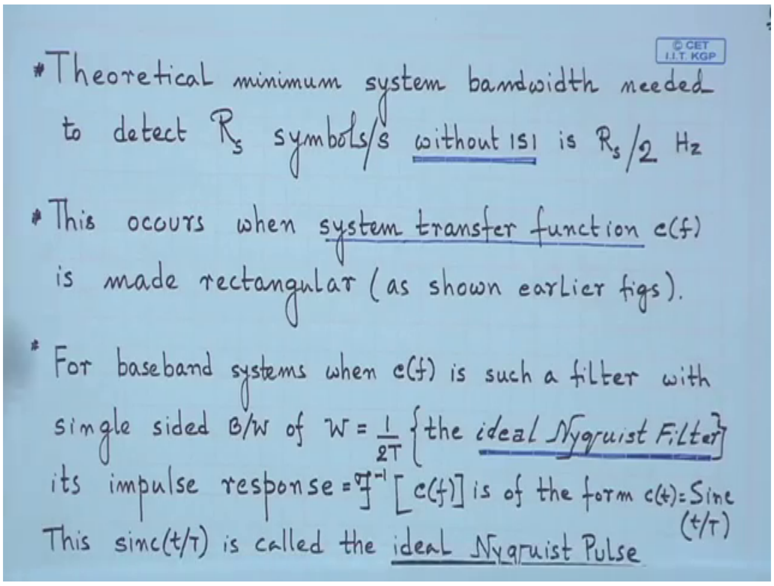

So, hence what we have is the theoretical minimum system bandwidth needed to send R S symbols per second without ISI is R S by 2 hertz. So, we have just stated the same thing in a little bit better way because we started off with 2 W symbols per second, which sounds bit odd. So, it is rather better to say that if I want to send R S symbols per second; what is a bandwidth required? So, both are valid, but this is just another important view.

So, we go ahead with what is the theoretical minimum system bandwidth needed to detect R S symbols without inter symbol interface; so this is critical, I am specifying without inter symbol interference is R S by 2 hertz.

Now, this occurs when the system transfer function; so, please note we are talking about the system transfer function. Again to remind you, we are seeing at the receiver over all. So, when the system transfer function is made rectangular as shown in the earlier figures. For baseband systems when c f is such a filter with single sided bandwidth W equals to one by T; it is known as the ideal Nyquist Filter. So, we have arrived at a very very important term. So, if you have done digital communications if somebody asks you what is an ideal nyquist filter. So, you now have the description of the ideal nyquist filter and its impulse response is given by; the inverse fourier transform of c f, which is of the form sinc of t upon T and of course, we do not have to mention that there is a pi involved in it.

This sinc t upon T is called the ideal nyquist pulse; so, what we have over here is if we are using the ideal nyquist filter; that means, we are using the ideal nyquist pulse, then we can send at the maximum rate of signaling without inter symbol interference.

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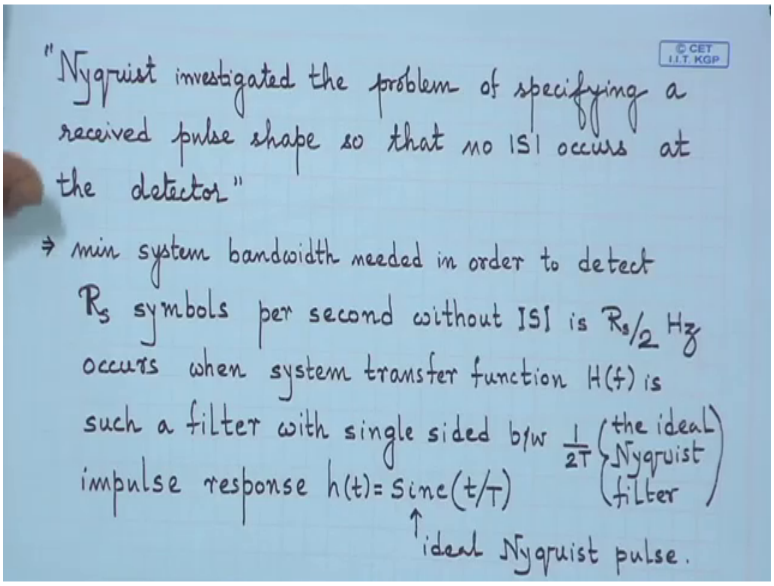

So, we put forward the statement that the nyquist investigated the problem of specifying the received pulse shape so that no ISI occurs at the detector. And what you said is that the minimum system bandwidth needed in order to detect R S symbols per second without ISI is R S by 2 hertz and occurs when the system transfer function, we have changed the notation a bit; H f is such a filter with single sided bandwidth 1 by 2 T; we have described this and the impulse response H t; while these are known as the ideal nyquist filter and the ideal nyquist response.

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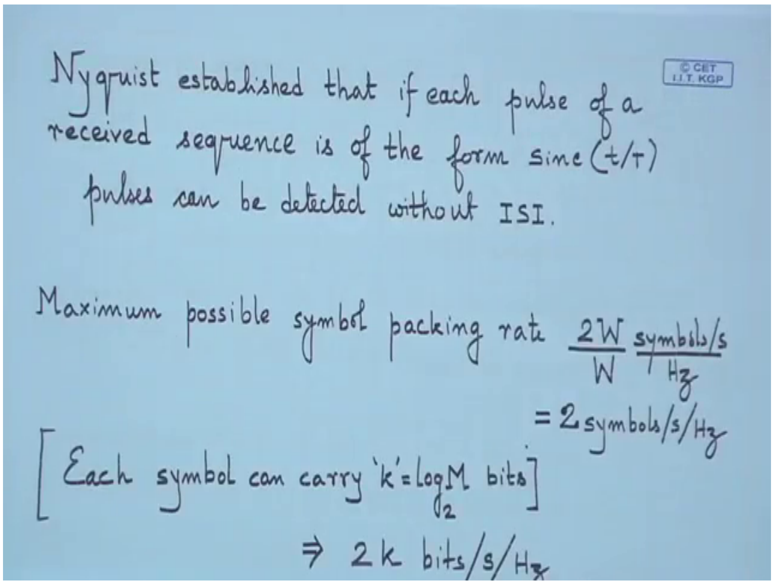

So, he established that if pulse of a received sequence; again we are talking about pulse of the received sequence is of the form sinc of t upon T; pulses can be detected without ISI.

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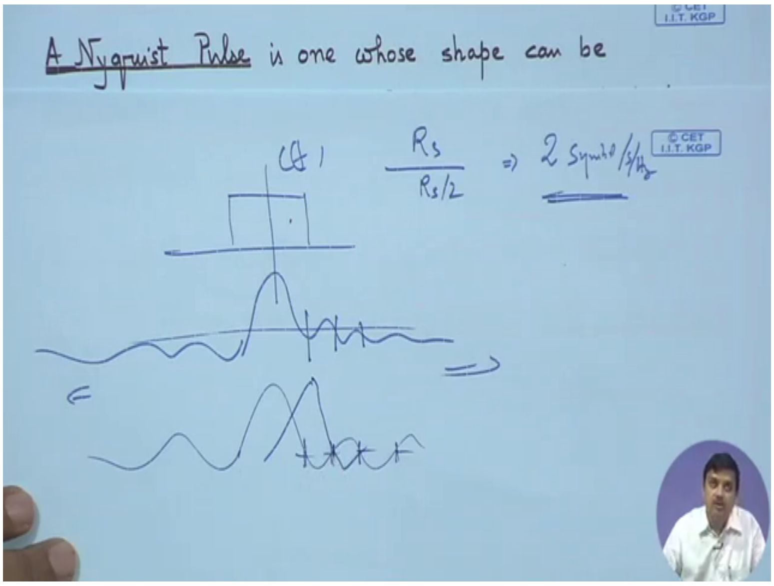

And the maximum possible symbol packing rate, 2 W symbols per second over a bandwidth of W or you can think of R S symbols per second over a bandwidth of R S by 2, it results in the same thing. You have R S symbols per second over bandwidth of r s by

2 both would result in 2 symbols per second per hertz. So, this is some kind of a symbol efficiency symbol spectral efficiency.

Further, if we say that each symbol carries k; which is log M base to bits because we have talked about only symbols, we have not said about the modulation techniques. So, each symbol would carry a few bits; if it is binary pam it will carry 2 bits, if it is m ary pam; it will carry log base 2 of m bits so; that means, what you have is 2 k bits per second per hertz; so, this is the maximum symbol packing.

So, as we increase k; we get higher and higher spectral efficiency and what we find is that in case of pam the spectral efficiency is achieved by spending power or energy at the transmitter; this is what we have seen already.

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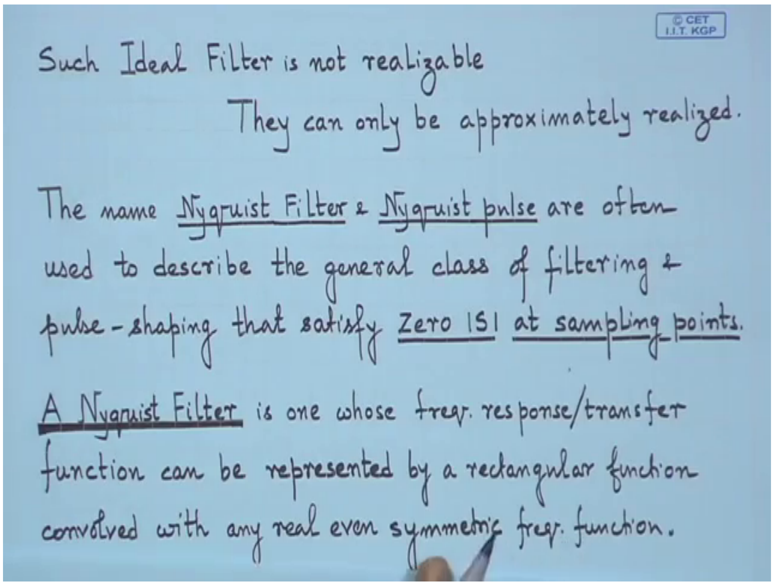

Such ideal filter, you can easily note it is not realizable because of the rectangular shape because you have in the frequency or rectangular shape. And it is not possible to realize for all reasons because it is non causal it causes; this is the frequency response, it causes an impulse response with the sinc, which tends to infinity. So, hence it is non causal, it is not realizable or it can be realizable only approximately and we have the definition that the name the nyquist filter and nyquist pulse are often used to describe the general class of filtering and pulse shaping that satisfy 0; ISI at sampling points.

So, what we are saying; we not only talk about this, but what we are interested in are pulse shapes which will produce 0; ISI. So, all those pulse shapes which produce; 0 ISI at the sampling point would be called the nyquist pulse and correspondingly the nyquist filter. So, you can find the nyquist filter; whose frequency response or transfer function can be represented as a rectangular function, which is the ideal. Convolved with any real even symmetric frequency function; this how you generate the whole family of nyquist filters.

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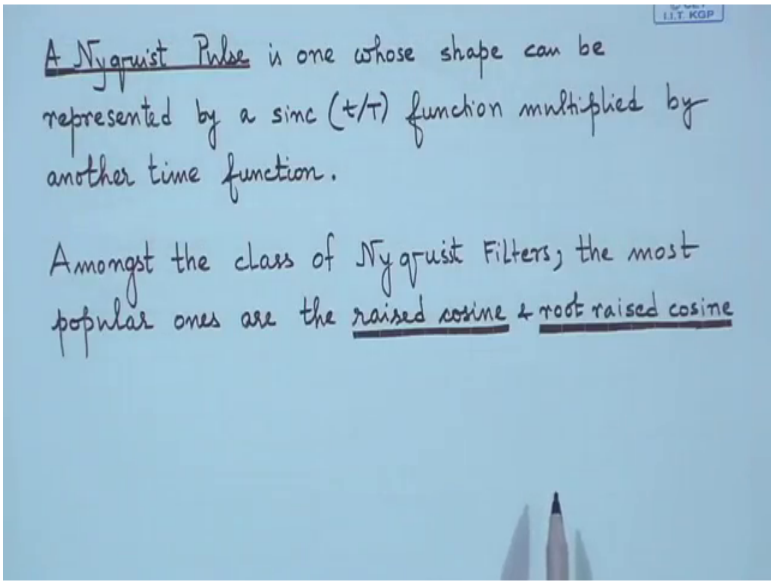

In the pulse domain, if you would multiply another time function with sinc t. So, if I multiply any with this, then I am definitely going to get 0 crossing. So, if I multiply any time function; my 0 crossing will not change. So, I going to get additional 0 crossing, but these 0 crossings will remain the same and when I launch another pulse I am going to get no ISI. So, this is how you would generate the family of nyquist pulses and nyquist filters in order to send maximum 2 W symbols per second over a bandwidth of W hertz.

Thank you.