Modern Digital Communication Techniques

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Lecture - 52
Performance of Digital Modulation Techniques (Contd.)

Welcome to the lectures on modern digital communication techniques. So, till the previous lecture, we have traversed quiet far in this particular subject and just to summarize although I have been saying the same thing in the earlier lectures that we have been able to transmit a signal receive it through an AWGN channel processor at the receiver and finally, detect the symbol or the signal that was sent and hence recover the bits.

However, we were interested in verifying or comparing the performance of the different digitally modulation schemes that we have been analyzing. To do this, we said we had to look at an important expression known as the channel capacity and in the previous 2 lectures we have given a background of how to proceed into the expression or how to arrive at the expression of channel capacity it may have been done in a quick way and I may recommend you to look further into text prescribes in this particular course or any other important references which you maybe more comfortable to with to arrive at those expressions; however, it is not necessary to derive through this expression as we have briefly given you some guidelines in the previous lectures.

What is rather more critical or more important for us is to understand the expression, it is implications which we are going to do in this particular lecture and use it in order to compare the performance of other digitally modulated transmission schemes.

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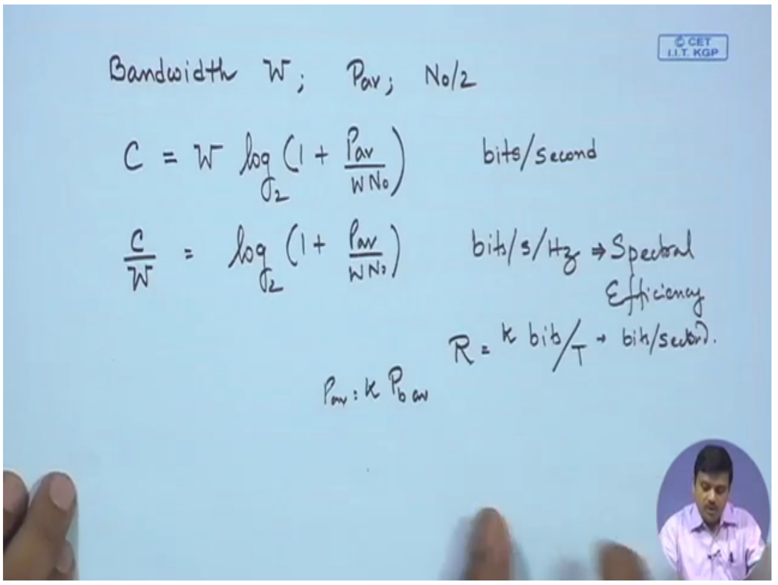

So, what we have is the expression of capacity for a band limited channel. So, if we have the bandwidth W and we know that the signal power the average signal power is P a v and we have the noise power spectral density given by N naught by 2 equivalent baseband is N naught.

So, what we could do in turn is we write the expression of channel capacity C is given by W log base 2 one plus P a v one W naught right this is what we have got. So, from this expression what we can see is that first thing the expression of capacity in this form it is in bits per second you could also say bits per channel use and channel use T is one upon w. So, bits per channel use you could consider in this way and generally we are interested in the term C upon W which is log base 2 1 plus P a v by W N naught which is bits per second per hertz. So, bits per second per hertz is an important entity also known as spectral efficiency.

So, we had seen K bits earlier and when divided by T, we had bits per second and of course, divided by W that is the bandwidth would give bits per second per hertz. So, we would have whole divide by W as bits per second per hertz and we can compare the performance of the different digital modulation techniques or given a W you could compare bits per second, right. So, W and T are related with as inverse of each other.

So, we would be able to compare the capacity in terms of bits per second to the term K upon T which is also known as R bits per second that is what we have. So, our interest is whenever you are given an average transmit power and you are given a bandwidth that is band limited channel we should be able to compute; how many bits per second that we are sending.

So, P average you could also modify to K times P bit that is per bit. So, P average is P average power that is available per symbol you can say. So, you could relate within this form.

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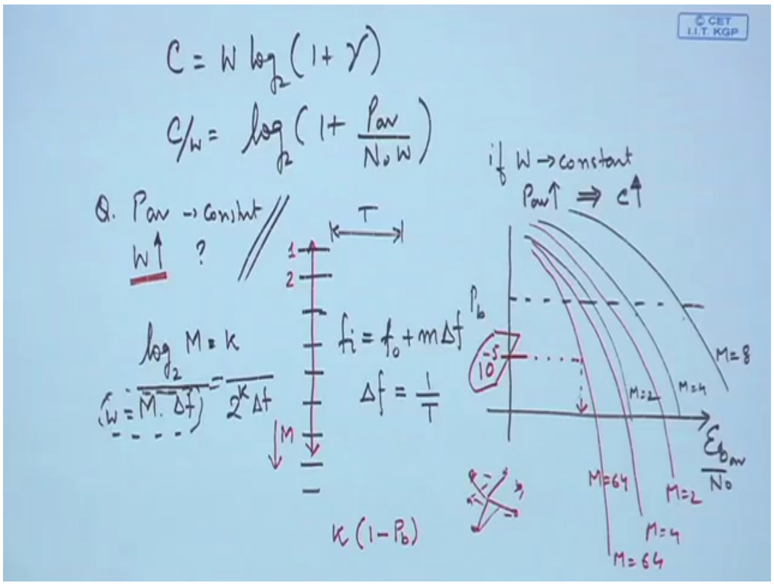

So, we will primarily look at the expression C equals to W log base 2 1 plus will indicate gamma as the ratio whenever needed we will translate it and C upon W is equal to log base 2 1 plus P a v by N naught W.

Some of the important things we briefly highlighted in the previous lecture is that if W is kept constant and P a v; that means, the transmit power is increased this means you could keep on indefinitely increasing C and 1 way of looking at the result is will focus only in this particular curve figure will not look at what is looking elsewhere.

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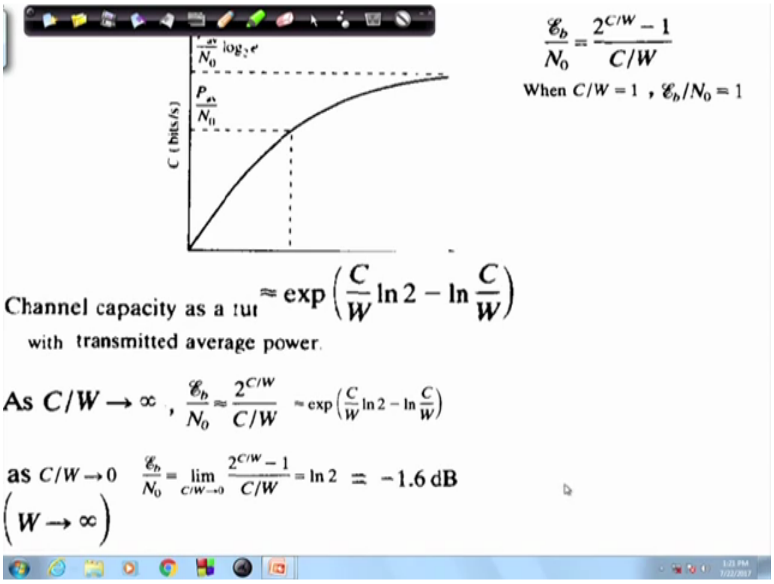

So, we will look at whatever is present at here. So, as you keep on increasing P a v, you can keep on indefinitely increasing C upon W or C if we keep W as constant. So, if you keep W as constant. So, denominator you can neglect and N naught is constant anyway. So, you can neglect that. So, you can be discovered as increasing W increasing P a v, it goes up it is also relevant from expression that we have in front of a C R.

You would also remember in terms of error probability expressions for different m. So, as we keep increasing M 8 and. So, on and. So, forth this probability of error for a given probability of error that is what we said as we keep increasing P a v; that means, the average energy that is equivalent to in going on the right on this side, we can accommodate higher and higher number of bits. So, that is in consistence with this. So, we should be able to compare C with K upon T this is one part of it.

And the next important thing is the question that we had raised earlier that if we let P a v to be constant and we would like to increase W indefinitely, then what happens? We give the result briefly in the previous discussion. So, this reminds us of the situation that we have different frequencies the M-ary FSK in which a particular frequency F I is equal to sum F 0 base frequency plus M delta F and if the symbol duration is for a duration of T to maintain orthogonality, we have to ensure that delta F is equal to 1 by T and there we had seen that as we increase M the curve shifts on the reverse direction.

So; that means, as we increase if we keep P a v constant as we increase K the E b requirement reduces, right. So, as we increase m, in other words if this is 1 2 let us say M as we keep increasing M our requirement of E b by M naught decreases and as we increase m, we definitely find that the bandwidth occupied also increases.

So, therefore, by seeing this expression which is which gives an ultimate limit on the maximum bits per second that can be send with almost negligible amount of error in the communication system we are interested in seeing if this particular expression could give us some results of our interest.

So, if we keep increasing W which is equivalent to increasing M in or corresponding to what we meant by increasing M in this particular situation while keeping this constant so; that means, if I would keep the P a v constant. So, that is what we had our average because our signal; the distance of the constellations from each other were all the same that is what we discussed. If you would remember with drew this kind of an imaginative figure indicating that all this signals are equidistant from each other. So, what would be the impact on P a v? So, previous discussion we kept W constant and we increased P a v in this discussion we would like to keep P a v constant and increase W.

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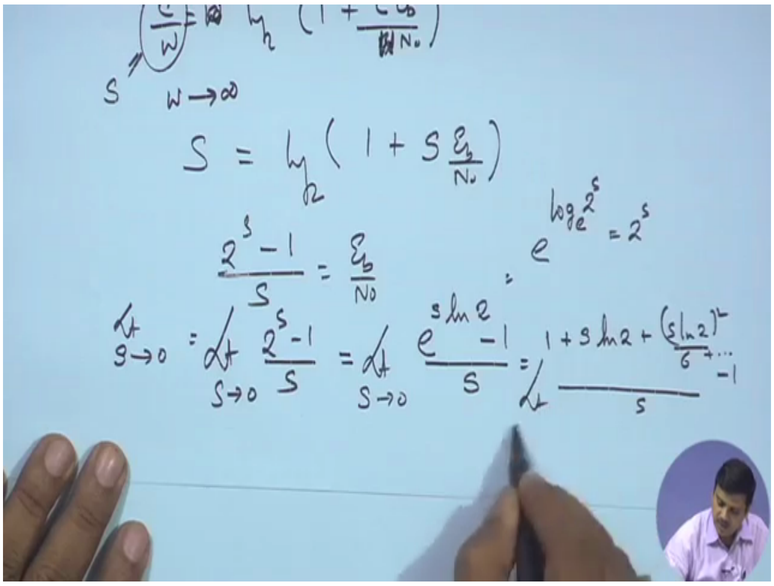

So, let us look at the expression of C is equal to W log base 2 1 upon P upon. So, I am writing P for in short notation as this. So, we could say that we could also write this in terms of one plus C times E b bits per second times bits per energy per symbol, we are going to get the average b N naught log base 2 of W. So, we can write C by W.

So, what we could do now is if you would look at C by W, we already said C by W is the spectral efficiency that is bits per second bits per second per hertz so interested in the spectral efficiency.

So, what we have is we are interested in making W go to infinity and what is the question on E b upon N naught, right. So, let us denote this by S. So, what we would like to say is if we want the spectral efficiency S to go to 0 what would be the impact, right.

So, let us look at the expression. So, we would write S is equal to log base 2 1 plus S E b by N naught and then from this by change of sides you could write this as 2 to the power of S minus 1 upon S is equal to E b by N naught. So, we could have this expression that is 2 to the power of S minus one upon S is this.

At this point, we would like to study what is the limit if S tends to 0; that means, if we let W tends to infinity we are going to get S tending to 0. So, this would mean we would like to put limit S tends to 0 2 to the power of S upon S; that means, we are going to find the limiting condition on E b by N naught given S tends to 0; that means, W tends to infinity.

So, this we could expand as limit S tends to 0 E to the power of S l n of 2 because l n is log base E S over here means 2 to the power of S. So, this effect will means E to the power of log base E or 2 to the power of S which is simply 2 to the power of S. So, 2 to the power of S; we are replacing by this term minus 1 upon S. So, then we could expand S E to the power of S l n 2 and that would be one plus S l n 2 plus S l n 2 square by 6 and so on; there is a minus 1 term. So, we have a minus 1 upon S and of course, limit S tends to 0.

So, going further on the same expression, so, if we would cancel this minus 1 out and we would take out this S will be left with E b by N naught; that means, the right hand side of what we have over here is equal to S l n 2 plus S.

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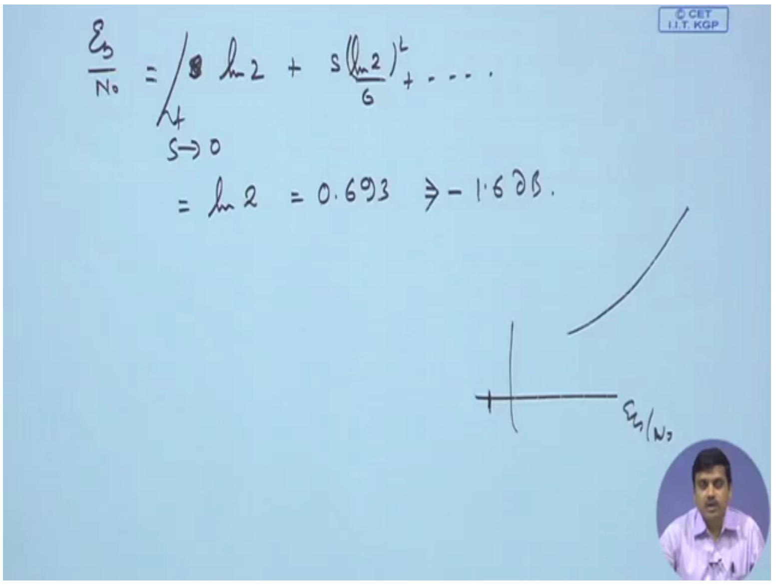

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So, we also have canceled out S and the other S remains S l n 2 squared upon 6 and so on and so forth with limit S tending to 0. So, if limit S tending to 0 this would turn out to be natural logarithm of 2 which is 0.693 which is equal to minus 1.6 T b which translates to minus 1.6 T b.

So, what we have seen here again that if we try to increase W indefinitely by keeping P a v constant, we get a few things. one is the spectral efficiency tends to 0 simply because we have let W tends to infinity. So, this ratio must tend to 0. So, trying to correlate that with what we have studied before. So, as we increase M and it tends to infinity, the number of bits that is being sent is log base 2 of M which is K and we divide it by b bandwidth that is M times delta F, right. So, on the right hand side, we will be having 2 to the power of K delta F.

So, as we keep increasing K this ratio becomes smaller and smaller. So, if this ratio becomes smaller and smaller as K tends to infinity; this ratio tends to 0 and that is exactly what we have over here as W tends to infinity S tends to 0 which is achieved by making K very very large. So, the total bandwidth available becomes very very large. So, it is consistent on both the sides.

So, when we did the union bound; from there also, we realized that the minimum E b by M naught that would be required is minus 1.6 T b, from this analysis also, we see that the minimum E b by M naught that can be reached when W tends to infinity is again minus

1.6 T b. So, that is the lower limit on E b by N naught that one is required to maintain a reliable communication in this kind of a communication system that is a digital communication system.

So, since we have seen the 2 situations, one is when we would increase the power indefinitely the capacity keeps on increasing and if we keep decreasing if we keep increasing the bandwidth, then we see that there is a lower limit on E b by N naught somewhere around minus 1.6 T b is the limit.

So, we have seen 2 different dimensions of the expression that we have used. Now we are equipped to study the performance of digital communication systems with reference to what we have discussed so far.

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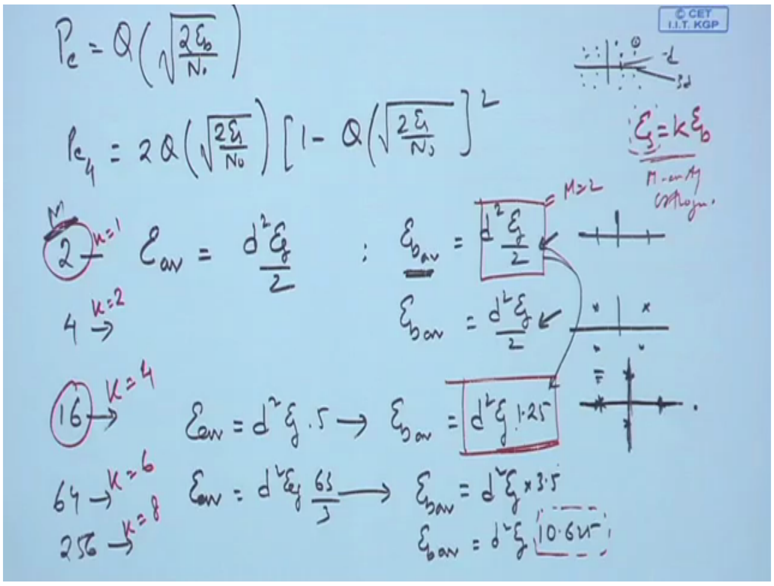

So, let us take a look at some of the interesting things before we proceed into the exact comparison. So, what we have with us is the probability of error expressions probability of error expression, let us say for binary P S K is Q of square root of T 2 E b upon N naught and probability of error for 4 level constellation will be 2 times Q root over 2 E S by N naught into one minus Q of square root of 2 E S by N naught square and so on and so forth.

So, what we will find is that the E a v the average energy of the constellation for 2 constellation point can be written in terms of T squared E g upon 2 where d is the scale factor and E g is the energy of the pulse and E b; that means, the energy per bit on an average is related to d squared E g by two. So, this is what we have used before. So, this is not new.

So, when you move to 4 level constellation; you will again find E a v is equal to d squared E g by 2. So, interestingly this is for 2 level this is for 4 level; that means, here we have BPSK here, we have QPSK; this has an equivalent form in terms of constellations here. So, you can imagine this to be BPSK or binary pulse amplitude modulation in the I channel and Q channel and hence the performance of BPSK and Q P S K would be similar.

So, if we have to go for 16 level constellation; that means, 3 bits; sorry; 4 bits in that case you are going to get E b a v to be d squared E g times 1.25; this is based on the average signal energy that you would require. So, E a v; we can compute to be d squared E g times 5 this will what it will be.

And when it is 64 constellation 64 QAM, you are going to get E a v, we did the calculation of E a v for M-ary QAM would be d squared E g with 63 upon 3. So, of course, you can do this calculation yourself E b average turns out to be d squared E g multiplied by 3.5 and if we have to go for 256 QAM; E b a v turns out to be d squared E g with factor of 10.625. So, these calculations will be based on the method that we did; that means, if you have constellation points all over. So, we have to calculate the energy of this energy of this energy of this and energy of this and take the average of it. So, and this would be d this could be 3 d and so on and so forth following the same methods.

So, if we have to keep the same minimum distance; that means, if we keep d to be constant what we find is as we increase the m; that means, the size of the constellation the E b energy per bit is related to the pulse energy and d in the form over here.

So, if I have to maintain the same error probability what we find is that E b average keeps on increasing. So, between BPSK and Q P S K there is binary PAM and QAM K, we find that energy per bit requirement has not been different and the simple justification is as if there is binary PAM on 2 axis.

However if you go to 16 QAM, what we find is that the factor is 2.5 times if I compare the energy per bit the required so; that means, if I am comparing this which is for M is equal to 2 and the energy per bit required which is for M is equal to 16 or K is equal to four. So, here K is equal to 2 here K is equal to 1; what we find is that this requires 2.5 times energy per bit to maintain the same error probability.

If we go to 64 QAM; that means, K is equal to 6 what you will find is that this requires seven times more energy per bit compare to a binary PAM to maintain the same error probability for 256; that means, K is equal to 8, we find it requires nearly 21 times energy per bit.

So, what we find is that as we keep increasing the number of bits per symbol the average bit energy keeps on increasing at non-linear fashion this huge amount of excess energy per bit required. So, this is an important message that we that we need to take; that means, in order to increase the bit rate per bit the amount of energy that has to be pumped in is significantly higher for high values of K compared to low values of K and as we go higher and higher, values of K; the amount of energy per bit required will be significantly large and the growth is not a linear growth it is a non-linear growth that we get from these expressions.

Now, we deglaze it is not necessary over here to repeat the similar things for M-ary orthogonal signals because we have already shown that if you keep the energy per symbol as the same and you have the relationship E S equals to K times E b. So, in case of M-ary orthogonal, if I keep the same constant E S I keep increasing K E b goes down. So, in that case the energy required for bit goes down we have been saying this thing.

So, now remembering these relationships that we have over here and the relationship we have for M-ary orthogonal and considering the expressions that we have arrived at for capacity for 2 situations when S tends to 0; that means, W tends to infinity and as E a v tends to infinity 2 different conditions, we would like to put all of them together in one expression in one particular picture and compare the performance.

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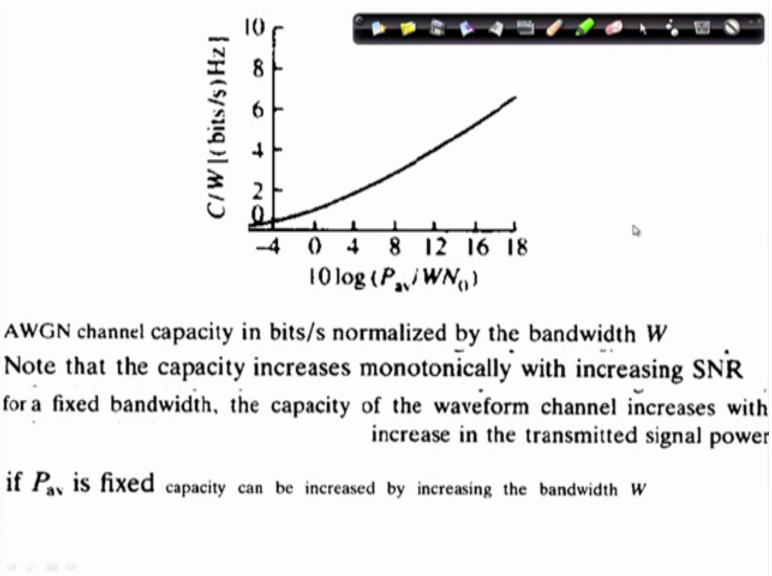

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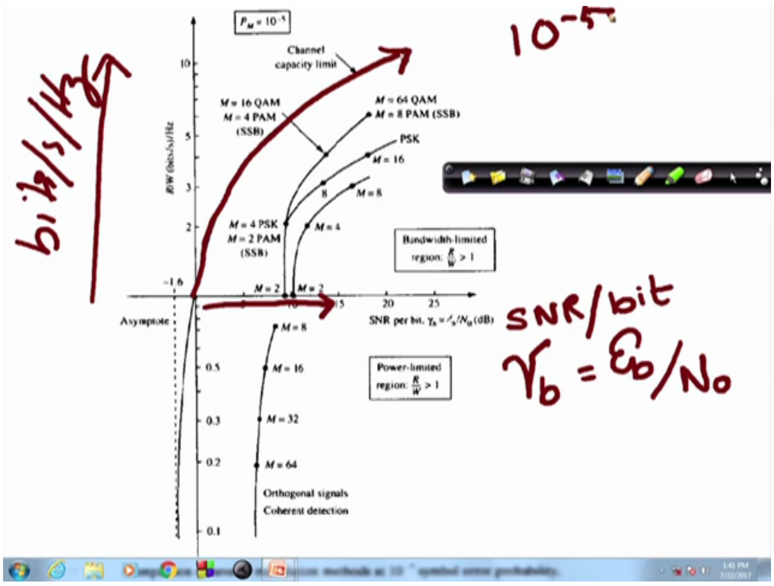

So, now what we have with us is particular famous representation which is usually available in many references. So, if we look at this particular picture we have the x axis as SNR per bit also we have been noting it as gamma b which is E b by N naught right and on the y axis, we have bits per second per hertz bits per second per hertz. So, what we see over here is that as we keep increasing the E b by N naught the channel capacity or the spectral efficiency keeps on increasing following this line and this is well established by the expressions what we had seen before.

If we now compare the spectral efficiency of digitally modulated signals for error probability of 10 to the power of minus 5. So, what we mean to say is that we have this error probability curve and suppose we choose some point which is 10 to the power of minus 5, right at 10 to the power of minus 5 whatever is the E b by N naught, we choose that because if we calculate that there are K bits per symbol then the number of bits that are received successfully is on an average 1 minus P b. So, if P b is 10 to the power of minus 5; what we have is nearly K bits are received successfully. So, K 10 to the power of minus 5 is a is a good reference for considering error probability.

Of course just a side note for other situation like wireless links and that 2 short links especially like mobile communication links these numbers this encoded performance numbers can be taken to be 10 to the power of minus 3 as a bench mark, but generally 10 to the power of minus 5 is a good reference point considering all kinds of communication system.

So, now going back to this particular curve what we see here is the constellation; the point here is corresponding to N equals to 4 PSK and M equals to 2 PAM with SSB single side band because PAM is a double side band signal. So, there is unnecessary use of bandwidths; if you use single side bandwidth you are optimally using the spectrum. So, E by N naught requirement is the same and for 4 PSK, you can think of it in many ways as we said before. So, as if there is 2 binary PAM on the I axis and Q axis. So, it is as if DSB and 2 DSBs. So, if there 2 DSBs on the same bandwidth of curse on orthogonal carriers what we have is effectively the spectral locations of SSB that is why they are here together, right.

So, then as you keep increasing M for P S K you would follow the curve as noted here right this point is M is equal to 8 this point is M is equal to 16. So, as we keep increasing the number of bits; that means, constellation we would require higher and higher energy per bit to maintain the same probability of error

If we would go for PAM or QAM. So, what we have over here is 8 PAM and 64 QAM. So, 8 PAM means 3 bits per symbol 64 QAM means 6 bits per symbol, but 6 bits mean 3 PAM on the I plus 3 PAM on the Q. So, if we use QAM it is double side band signal and if we use PAM with SSB; single side band, it is going to give the same spectral efficiency. So, that is why they are at the same point.

So, the PAM kind of constellations would follow the curve here which is indicated there and as we see if we consider orthogonal signals this particular point is the 1 for M is equal to 8 level orthogonal signals; that means, K equals to 3. So, what we find is that this energy per bit requirement decreases and the bound can be minus 1.6 T b on this axis.

So, what we find is the M of the communication system design engineer is generally to find a modulation scheme which has the minimum distance from the capacity curve which is given by this; that means, at a certain E b by N naught; this particular point is been maximum number of bits per second that one can achieve with negligible amount of error whereas, we see that the PAM kind of curves follow this. So, there is a capacity gap between the maximum limit and what can be achieved by PAM, right.

If you want to see the capacity gap with PSK, we have to refer to this particular curve and if you have to refer to that for M-ary orthogonal signals of course, how this curve extends there is what we have to see

The other way of viewing this curve would be that given a particular spectral efficiency what is the excess amount of E b by N naught required to achieve that and clearly if we increase our Q S criteria; that means, if we make things more stringent error probability instead of 10 to the power of minus 5, we make it 10 to the power of minus 10 all these curves that we have drawn here with this colors; these colors, they would shift and will require higher P b by N naught and if we reduce the error probability threshold; that means, say we want to compare a 10 to the power of minus 3 will be here.

So, generally 10 to the power of minus 5 is a good number and ideally speaking during such comparisons one should use error correction codes in order to compensate for the assumptions that have been made with the capacity expression.

So, with this I would like to conclude this particular discussion and like to keep you engaged with the comparison of performance of digital communication modulation techniques with that of the channel capacity.

Thank you.