Modern Digital Communication Techniques
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Lecture - 50
Performance of Digital Modulation Techniques (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. So, far what we have done is very interesting and we are standing at a juncture; where we could review what we have done, it is a very interesting juncture that we are standing at. Because now we could evaluate the difference schemes that we have studied, as well as bench mark them against certain performance so that it helps us in doing an exact performance comparison.

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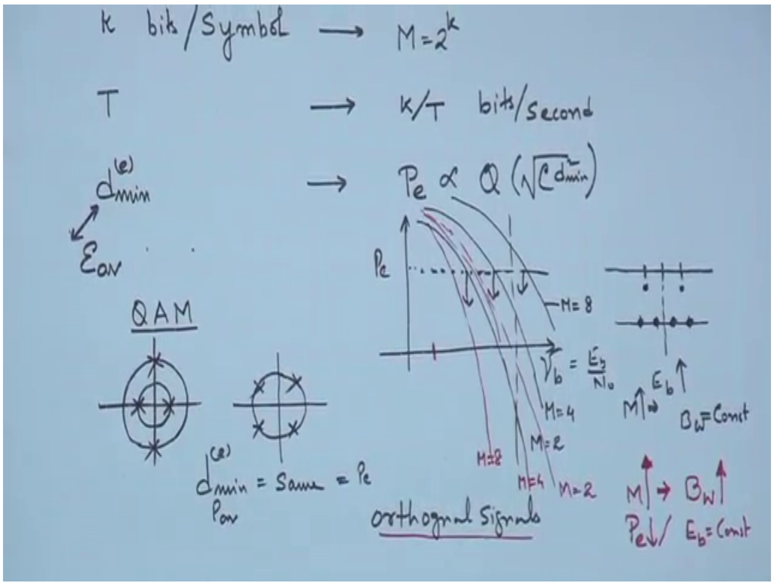

So, looking back at some of the schemes that we have studied; we could say that we have looked at situations where k; which is the bits per symbol is one of the important factors which is used to choose the constellation size; which is M is equal to 2 to the power of k, so, as k increases we have seen that M increases.

We stated that if T is the symbol duration, so the number of bits per symbol duration that is sent is T upon k bits per second. On the other hand, we have also seen that if there is a minimum Euclidean distance between two constellation points; then we found that probability of error is proportional to the Q function of square root of some constant that is a C and d min squared.

So, what we see is that an d min is somehow related to; E average, which is the average energy. So, what we see that there are at least 3 independent factors which are available in order to choose a particular size of constellation. We have also seen that when we studied Q A M; there are several possibilities of choosing constellation, even though the constellation size would remain the same. For instance, this could be one choice of constellation that we saw and in contrast there could be another choice of constellation what we discussed was something like this.

What we figured out that even if in these two constellations, we maintain the minimum Euclidean distance as same; which means the same probability of error. And even if we have the same average power in these two constellations, still we stated that this constellation maybe preferred because of lower peak to average power ratio compared to this. So, there are several criteria based on which you would be able to choose a constellation.

And we had seen in general for constellations like PAM and Q A M as you have E b by N naught on X axis and you have the probability of error on Y axis and let this be one of the probability error curves. So, as we increase M; so, if M is equal to 2; as we increase M equals to 4; the probability of error curves shift to the right; which means that to maintain a certain probability of error; as we increase the constellation size; that means, as we increase the bits k, our bits per second increases, but the minimum required SNR or E b by N naught also keeps on increasing.

And in another way, you could say that if we keep E b by N naught constant; as we keep increasing the size of the constellation, the probability of error keeps on increasing. So, this is equal M equals to 4; this is let us say M is equal to 8 and 16 and so on and so forth. So, as we keep on increasing M; that means, as we keep on increasing k by keeping same e v by M naught, the probability of error increases.

And we explained this in the form that; if there is a certain constellation and we want to keep the same average energy; however, we want to increase the number of constellation points. So, then what we get is the constellation points come much closer; compared to the previous case. So, the constellation points coming closer means d min become smaller which means probability of error becoming larger, so that is what we have here.

Whereas when we studied orthogonal signals, what we found is that the probability of error curves behaved in a different way. So, if this was the probability of error for M equals to 2; for M equals to 4; it could have been here M equals to 8. So, we could roughly say M equals to 4; M equals to 8 and so on and so forth.

And we did discuss that what would happen, if we keep on increasing M. If we keep on increasing M, is there a bound that we reach on gamma bar; was one of the questions but when we reach that bound, what we could actually get is make probability of error go to almost 0. But in this case what we had as M increases, if you think of emery orthogonal signally; this implies that the bandwidth required also keeps on increasing. So, in one case if you have to reduce or keep probability of error constant, while we keep increasing M; we need not spend extra energy or extra power, we can simply keep on increasing the bandwidth.

Whereas in the other case; if we have to increase the constellation size; that means, increase the bit rate we have to spend more power. That means, if I have to keep the same error probability and I have to increase the M value, I have to keep maintaining a higher E b by N naught; this is E b by N naught and since N naught is constant, it is in terms of increasing higher E b.

So, in one case your requirement on E b should go up as M increases; in the other case the bandwidth requirement goes up. So, here bandwidth is constant and in this case you can say E b is constant. So, we have two different possibilities and of course, there are different variations on the error probability. So, at this stage it might be important to discuss that how do you compare the performance of the schemes in a fair basis.

So, to do that there has been very important contribution from a very eminent contributor; in the sense the father of information theory Claude Shannon, who proposed or who developed the expression for an important term known as channel capacity. So, we are going to discuss a bit of channel capacity in this particular lecture and the aim would be to get an insight into how the channel capacity comes into play and what is the consequence of channel capacity in terms of the performance of a digital communication system.

So; however, when we are studying a channel capacity; we need to ideally speaking start off with the basics of probability theory, asymptotic equi-partition and build up a lot of theory in order to understand the tenets of capacity, which is a practically a full course on information theory.

So, since we do not have the luxury to go into those details; we will give an overview of how do we arrive at the expression, what is the meaning of the expression and how we can use the expression in the study of digital communications. So, with this let us proceed into the study of channel capacity as it is famously known.

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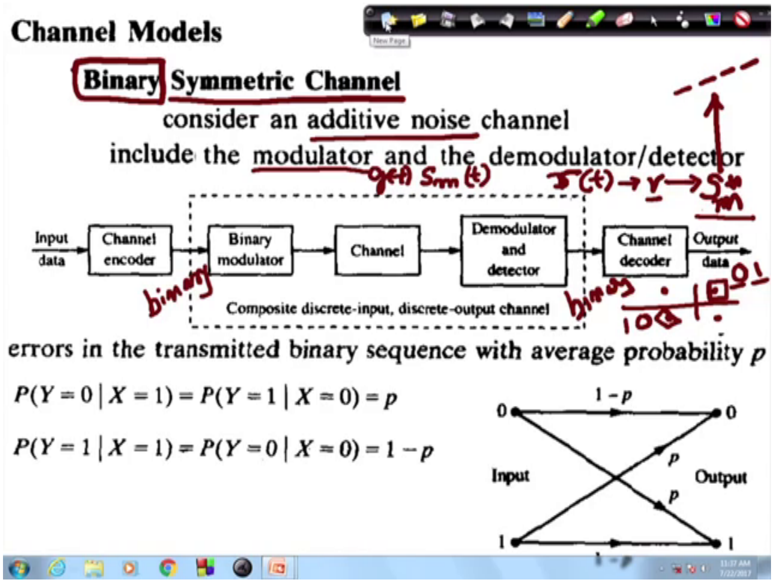

So, when we study channel capacity; generally there are we have to first study the different kinds of channel that may be available. So, one of the channels that is of our interest is the binary symmetric channel. The binary symmetric channel is one where of course, we consider additive void Gaussian noise; we will always consider additive noise and this kind of channel; the name already describes certain things; that means, it is telling these are binary channel and it also says, it is a symmetric channel.

So, before we go into the description let us see that how it is built; so, as it says it includes the modulator and the demodulator. So, if you remember in the modulator; we had the g of t and we had the S m of t. In the demodulator, again we would breakdown the received signal a; r of t, we would breakdown r t into its components; followed by that means, we would break it into components detector r from which we are going to get the choice of the wave form that could have been transmitted. This is what was available in the modulator and the demodulator.

So, now it says that we have to include this in the channel. So, I would like to remind you at this point; in the initial parts where we studied layering and breaking down into smaller components, we did say that you could view the channel at different points of the transmitter chain. That is you could view the channel as a medium which carries electromagnetic signal, if you are looking at the channel from the prospective of from the output of the antenna. And if you slowly go inside into the transmitter, we come to a point where we look at symbols; that means, the output of the symbol mapper goes into the channel. So, we have symbols going into the channel and symbols coming out of the channel so; that means, the up conversion is part of the channel model in that case.

But here we go even backwards and we say that the symbol mapper or the modulator is also part of the channel. So, the interface and layering that we discussed; if you remember that we were in a situation, where the bits go into the symbol mapper and at the receiver bits come out of the detector. Because the job of the detector is to select S m star; S m star means star indicates the particular result one of the S m.

So, once you detect the particular S m; you can easily identify the k bit sequence which mapped to it, because if this is your constellation and this is the detected symbol. So, definitely this symbol has a bit map which could be a 0, 1, so the output of the detector is 0,1. So that means, what we have is or let us say if this is detected at the receiver, s the output could be 1, 0.

So, what we have is input into the system is binary bits; so that means binary digits and output is again binary. So, we are at the binary interface right; so, when we are at the binary interface what we have is a bit sequence. So, we will use this; that means, 0’s and 1’s goes into the symbol mapper.

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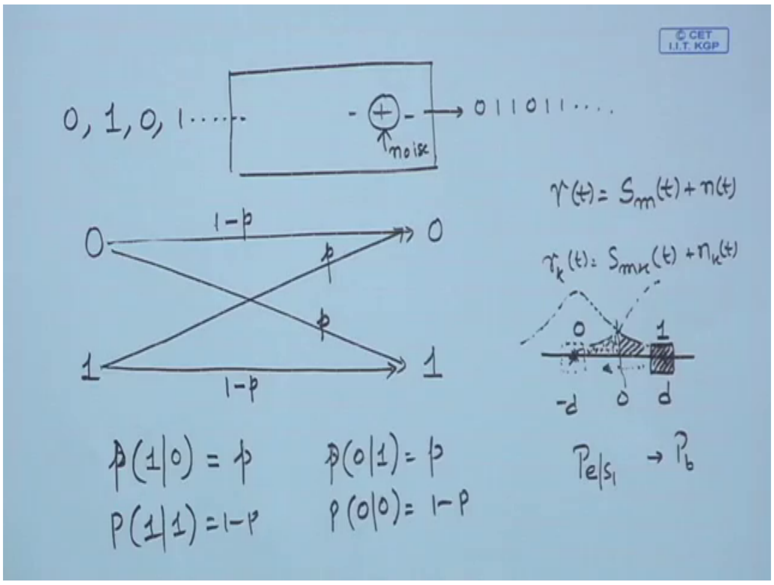

So, we construct the channel out of that; it goes through the channel passes through AWGN and demodulator detector and again it produces a sequence of 0's and 1's.

So, when you do so; what we have is a 0 as the input and the output could be a 0 or the output could be a 1. Because we have stated the detector must choose something; the detector cannot stay without choosing a particular solution. Similarly, when a 1 goes into the system; the 1 may be detected finally, as a 1 or it could be detected as a 0; so, this is the abstract from of the channel model.

So, at some point you may remember there is noise which is getting added to the signal. So, the received signal r of t is made up of S m; t plus noise, so if you think of components; then r k of t is equal to S; m k of t plus n k. So; that means, there is a signal component, there is a noise component and we have stated many of times. Let us consider the b p s k model; where we have d and minus d. Suppose, I map this to 1 and this to 0 and we have selected this for transmission.

So, if we have selected this for transmission; then if the noise is strong enough minus negative value and it shifts to this side, then the m a p or the ml detector; the ml detector would detect the received signal as possibly a minus d or a 0. So, in that case an error happens and this is what we have discussed earlier.

So, and we have calculated probability of error given S 1; so, for b p s k this turns out to be the probability of bit error; that means, the probability that when 1 is sent; it becomes a 0. So, probability that a 1 is sent; it becomes a 0 is p, similarly probability of 0 becoming a 1. So, how we calculated is by considering the conditional pdf and the probability of 0 becoming a 1 is the area under this curve and probability of 1 becoming a 0 is area under this curve.

So, probability of 0 becoming a 1 is p; so, now because of symmetricity; that means, 0 becoming 1 and 1 becoming 0, we have a symmetric channel. Because there is binary input, binary output we have binary symmetric channel. So, probability of 0 remaining a 0 is; 1 minus probability of error which is probability of being correct. Probability of 1; remaining a 1 is 1 minus probability of error; which is 1 minus p.

So, now we have the situation which maps a 0 to 0 and 0 to 1 and 1 to 0 and 1 to 1. So, these provide the transition probability values; that means, probability that I get a 1; when a 0 is transmitted is p and probability that I get a 0, when a 1 is transmitted is p. Probability that I get a 1, when 1 is transmitted is 1 minus P and probability that I get a 0 when 0 is transmitted is 1 minus P.

So, whatever we reflected here is shown in this particular setup. So, probability that I get a 0, when I sent a 1 is same as the probability; I get a 1, when I have sent a 0 and that is equal to P. Similarly, the probability of the output remaining same as the input; in both the cases is 1 minus the probability of making an error and that is probability of being correct which is 1 minus p; this is exactly that what we have written on the paper.

So, this is the basic model and this model we do use; in some of the calculations, so this is for mainly for you to know. So, just a side note over here; we are discussing this for sake of information so that our understanding is complete, but we will be mainly interested in the final expression as we have announced in the previous lecture.

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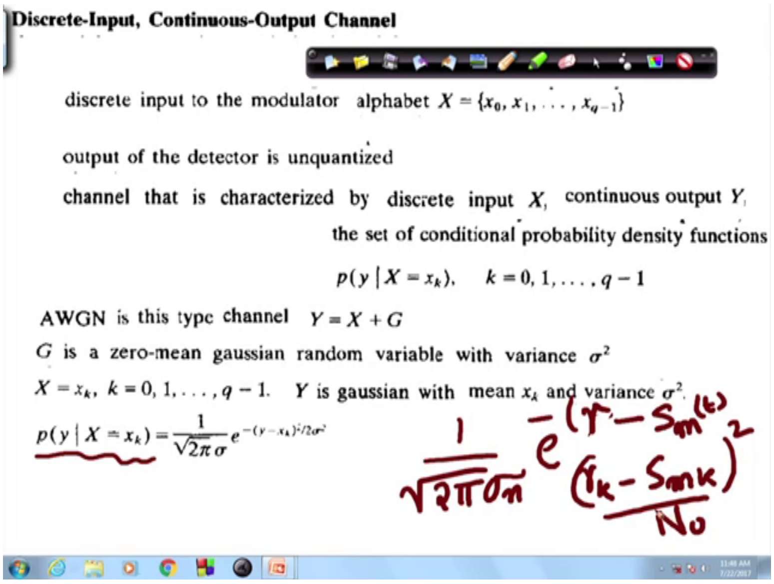

So, moving further the other important channel of interest is the discrete input continuous output channel. So, as the name tells you the input is discrete and clearly we have discrete input because our input takes values, let us say minus d or plus d; in case of binary pulse amplitude modulation. In case of (Refer Time: 20:00) pulse amplitude modulation, we will have two additional amplitudes. So, these are discrete inputs; in case QAM is discrete input; in case of PSK; it is discrete input, so we do have discrete input. And continuous output simply because if we send let us say d which is S m t; which is a value of d; what we receive is plus noise, what we receive is noise.

So, when a discrete value gets added to noise which is the continuous values signal, what we have the situation as discrete input, but continuous output. So, this kind of channel is this and it is very easily characterized by the modulator alphabet x 0; that means, the discrete symbols. And the output of the detector is unquantized, so that is what we have; the channel is characterized by the discrete input X and continuous output Y. So, it is simply stating that you have these variables X and Y; one at the input, one at the output and you have the set of conditional probability density functions which define the channel.

So, simply we have already discussed that thing here; we have already discussed it in the model that we have drawn here, this is one of the situations. So, if I am taking r k; I am already taking a discrete input continuous output, but if I am taking the discrete output; that means, the decision output I go to a discrete input; discrete output.

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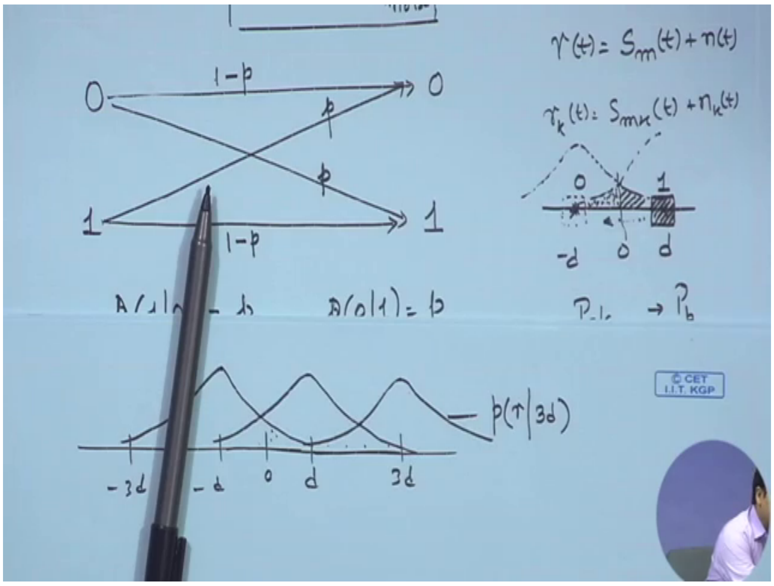

So, in case of emery; you may remember the situation was, so we have 0, d, 3 d, minus d, minus 3 d. So, the conditional PDF is; if 3 d is sent; probability of receiving anywhere in this is given by this particular curve. If d is send probability of receiving the signal along this is given by this PDF, probability of; if you say minus d, this y axis is the probability density function axis and then if you have to convert this to a discrete output, you have to calculate the probability of error. So in that case if minus d is send you will be calculating this area.

However what we have is probability of received signal given 3 d as this end, so; that means, we are here it is given by the set of conditional PDF. So, it is given by the set of conditional PDF; so these characterise. Now you can clearly understand that these conditional PDF's carry information about the channel; that means, they are characterizing the channel.

So, the noise impact is characterized into this transition probabilities; so, if we would have gone to the previous slide, here we find that these transition probabilities; these are the transition probabilities.

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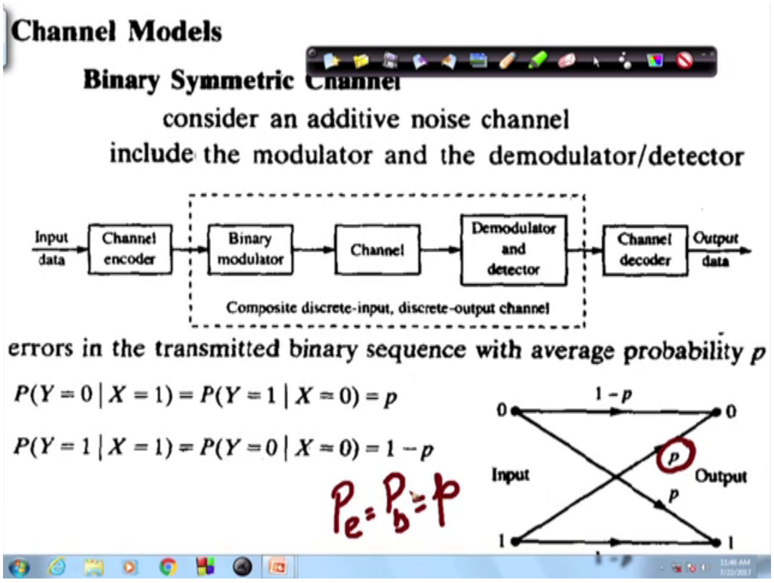

And these are clearly due to the noise effect, which you can easily see that because of noise you have a certain probability of error. So, probability of error in this case; it is probability of bit error which is equal to p. So, here the noise is getting translate to a transition probability.

So, similarly in this case as well the noise is effecting the conditional probabilities and this and in an AWGN channel, you have the simple model Y equals to X; the transmitted signal, no distortion due to channel plus there is noise getting added and G is 0 mean Gaussian random variable with variance of sigma square and for all situations; we have taken this to be N naught by 2.

So, that is what we have seen before and the conditional PDF we are used to this expression. So, there is nothing new in this expression; 1 by root 2 pi sigma n; e to the power of minus Y minus; Y is the received signal. So, we had done earlier is we had used r minus S m t; rather we had used r k minus S m k, squared upon N naught; this is what we had used.

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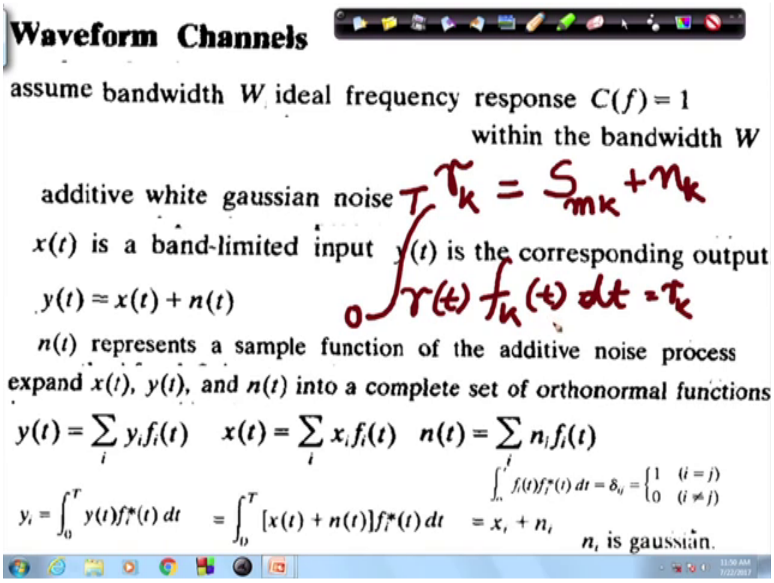

So, this is the same thing that you see over here; you do not see anything different at this point. So, the next kind of channel what we can think of is the waveform channel; so, with the waveform channel, we have to consider a channel bandwidth W and an ideal frequency response. So, in the earlier case also we had the ideal frequency response; so, it says that it has an ideal frequency response; C f equals to 1 within this bandwidth W. So that means, you have a response like this; so this is W and this (Refer Time: 25:55) is 1 and x; t is band-limited and y t is the corresponding output.

So, this is the band limited input because you have a certain bandwidth W and y t is the corresponding output that is a corresponding output. So, again you can see in additive white gaussian noise it is given by; so here x t takes continuous values, y t takes continuous values. So, continuous input continuous output and hence it is known as the waveform channel. And of course, n t represents a sample function of additive white gaussian noise process.

So, we expand x t, y t and n t into its complete set of orthonormal functions; which we have already done before, we have been doing it throughout. So, y t is equal to expansion in terms of a f i t's, y i t's are the components; same with x i t's and you could write; so, it is very small over here. Here we write that the functions are orthogonal to each other; so finally what we write is that y k or if we use are earlier notation; we have r k that is the kth component is equal to S m and its kth component plus noise and its kth component.

So, how you get it? You simply get it by projecting r on f k t; integrate from 0 to t, which is equal to 1 upon w; d t is equal to r k, that is how you get it. And you remember that f k and fm are orthogonal to each other.

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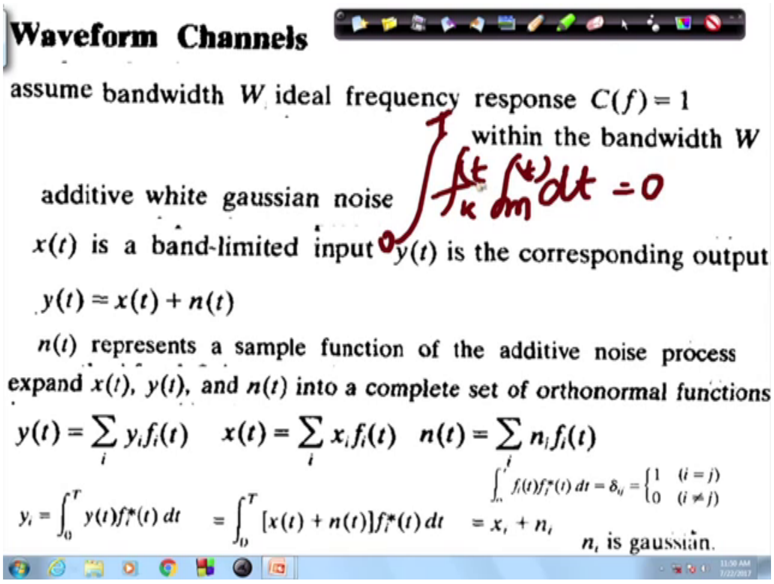

So, what we also have is f k; f m 0 to t, d t is equal to this of course, this is the function of time, this is the function of time.

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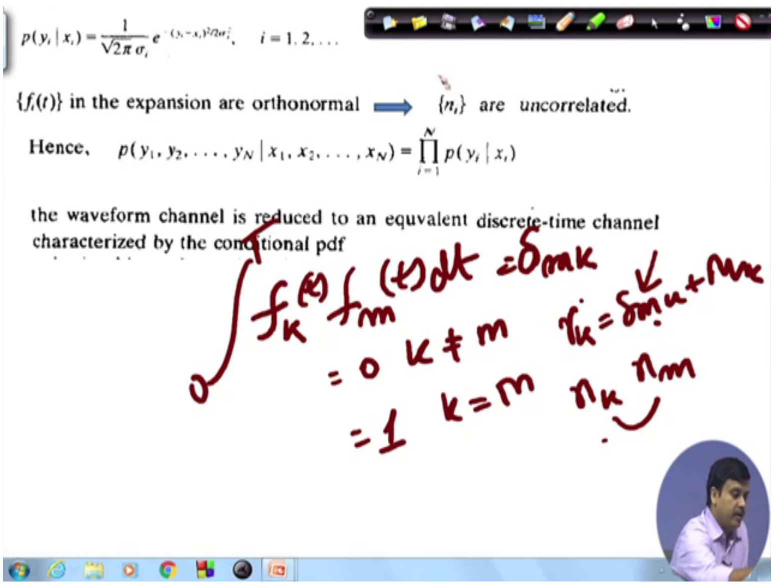

So, these are something which we have been using, so this is classified as a waveform channel. The main difference is input is continuous, so now in such conditions what we had seen is that these f k's been orthogonal because these f k's are orthogonal, what we found is that k naught equal to m. So, rather it is important to write it is delta m k; so, equals to 0 for k not equal to for k equals to m; this is the whole thing.

So, what we get is that the signals r k which is equal to s m k plus n k are independent because n k and n m; what we had calculated earlier is that they are uncorrelated. If they are uncorrelated, we know they are gaussian and therefore, they are independent. So, r k given S m k is a gaussian random variable with a mean S m k and again the same applies these are independent; these are uncorrelated, so again they are gaussian uncorrelated means independent.

So, at the receiver if these are the components, so the joint distribution of the received components; given the transmitted signal is equal to the product of the marginal conditional distributions, so simply because of orthogonality criteria that we have. So, these are some of the things that we use, so since we know this expression already; 1 over root 2 pi sigma n; e to the power of minus r k minus S m k squared by N naught.

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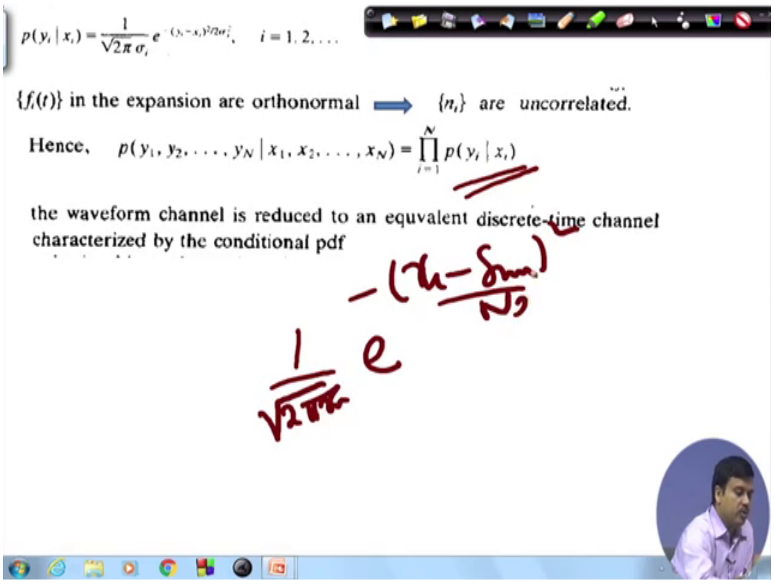

So, this is what we use, so now what do you will get is equal to use all these corresponding expression; you are going to take get a product of them and hence you will be able to compute the necessary things.

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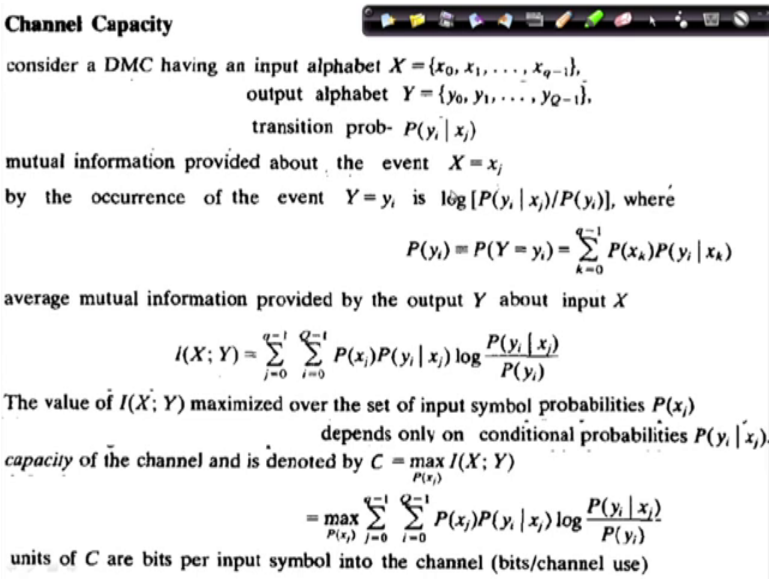

So, we stop this particular lecture here and we using the channel modules that we have developed, we will take a look at the channel capacity from which we will finally, take a look at the performance comparison of the different communication systems.

Thank you.