Modern Digital Communication Techniques
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Lecture - 40
Optimum Receivers for AWGN

Welcome to the lectures on Modern Digital Communication Techniques. So, far we have covered the transmission section. And now we are almost ready to get into the next part that is the receiver. However, we will revisit some important things of the transmitter at an appropriate time whenever we feel that it is pertinent enough.

Now, some early we have started off with the source and we have converted it into a bit sequence then from the bit sequence we have mapped the bit sequence to waveforms and then we finally, said that depending upon the channel one should choose the waveform in an appropriate manner and one of the ways to do it is to look at the spectrum of the signal and that would help in matching the waveform to the channel.

So, now, at this point we said that we are more or less discussed some of the important aspects of signal transmission and throughout we have been stressing the point that we are looking at the low pass equivalent signal. And even for the stochastic process we have been looking at the low pass equivalent process as well as we for both of these cases we could study their spectral characteristics for the for the stochastic process through the spectral density which is the Fourier transform photo correlation function whereas, for the deterministic signals we have used the Fourier transform.

So, with all these things more or less we have covered some of the different aspects required to express modulated digitally modulated signal. Now, we can imagine as if we are in a state where the signal is out from the transmitter. Now of course, to send this digitally modulated signal one must up convert that is bring the signal to the pass band for that the general activity is you multiply e to the power of j to 2 pi fct and take the real part of it that is good, when we write in terms of expression in reality one should multiply the output of an oscillator pass it through an amplifier. And then band pass filters and high power amplifiers to antennas and into the channel. And then when it goes to the channel ideally speaking one should study the channels the different aspects of what happens to the signal through the channel and then what does the signal look like when it is at the receiver and then one should start working with the receiver.

So, when we start studying about the signal being received there are different kinds of channel that one usually encounters so amongst.

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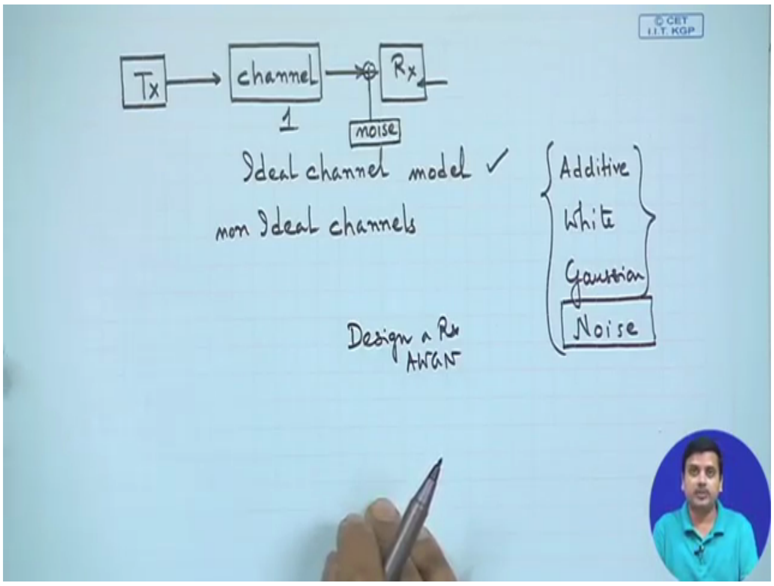

So, let us let us do like this we say that there is the transmitter from the transmitter signal goes into the channel this kind of a figure we had drawn before and from the channel it goes to the receiver right. So, when we talk about the channel there are different kinds of channel that can be used. So, it could be an ideal channel. Of course, we are talking of ideal channel model and there could be non ideal channels right and then of course, there is a receiver. So, will begin with the ideal channel model and in the ideal channel model what you get is that when the signal goes to the channel the signal is remaining as it is that is undistorted signal.

So, if you have the undistorted signal then there is hardly anything to do at the receiver you just demodulate by multiplying e to the power of minus j 2 pi fct and then you pass it through a filter and you get the signal back and you can almost do with any kind of transmission power there is no problem with this kind of a system; however, typically what we say is at the receiver we generally model something like addition of noise which is due to all components at the receiver. So, generally we say noise addition happens at the receiver. So, all effective noise of the communication system in form of additive noise gets added here because of the amplifiers and all passive components that are present in the system.

So, one of the simplest channels that one can look at is the additive white Gaussian noise channel. So, it is additive in nature it is spectrally white its pdf is Gaussian and of course, it represents noise. So, all these characteristics are characteristics of noise and we have discussed this when we looked at the stochastic the representation of band pass stochastic process.

So, when we continue our discussion here we will assume that there is additive white Gaussian noise and the channel is such whose gain is 1; that means the signal is left undistorted only there is addition of noise in the receiver.

So, for this kind of situation which is quite idealistic we would like to design a receiver under that condition. So, one would question that what is the point of discussing such a system where the channel is non-distorting there is no effect of the channel. And there is only noise well this is the simplest form of a communication link that we can ever think of. And you can think of an idealized wire or a wire which is with very good properties where there is hardly any capacitance that could be present.

And there is only resistive component which is present ohmic resistance that is present and when the signal is received there is only thermal noise which distorts the signal there is nothing else. Of course, will assume that the bandwidth of the signal is much smaller compared to the pass band of the channel a similar situation can also happen in wireless links. For example, in satellite communications where there is line of sight in case of microwave link where there is line of sight and there is clear line of sight view.

So, under those conditions you would study it as AWGN channel and of course, the additional losses such as the attenuation due to path loss and all can be captured without much of a problem. So, these attenuations would be taken into consideration as loss of power. So, at the receiver there will be reduction of power due to path loss and the rest of it is addition of noise.

So, there is simple attenuation and noise. So, you can study such systems using properties of AWGN channel also when you have distorting channels it is possible to remove the effect of the channel and then use some kind of a whitening filter after which you would be able to get a system, which appears like a AWGN channel and before and further before getting into any complicated system any complicated receiver design. This is the simplest form of system which still has a lot of practical connections.

That means, you could still design communication systems using this and which is a fundamental baseline receiver. So, so this kind of a channel is very fundamental and very important that is why it is necessary to begin the study of receivers using such a kind of channel model where only one kind of a problem is present in the link.

So, we would like to design a receiver for AWGN channel.

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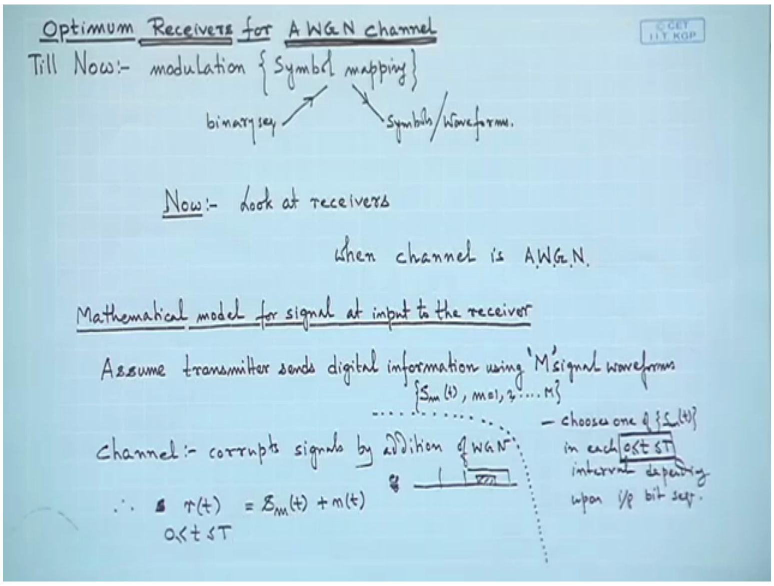

So, that is our objective right. So, with this we would consider our objective as designing of an AWGN for designing of an optimum receiver for AWGN channel. So, till now we have considered modulation. So, typically the receiver would mean that you have to demodulate. So, we will start looking into the details of it. So, when we talked about modulation we talked about symbol mapping. So, when we did a symbol mapping we had a binary sequence getting mapped to certain waveforms or symbols and then sent out into the system.

Now when you look at the receiver and when the channel is AWGN we have described what we mean by AWGN channel. So, first what we need to look at is the mathematical model for signal at the input of the receiver. So, for this will assume that the transmitter sends digital information using m signal waveforms. That means, there are m number of signal waveforms and these are S mt m equals to 1 to capital M and our job at the symbol mapper is to choose one of S mt S right.

And these will be chosen in the signaling interval 0 to capital T right and at the receiver. So, sorry these S mt would be chosen based on the input sequence. So, that is what we have described to this arrow diagram and the channel we have already described is additive white Gaussian noise where of course, the bandwidth of the filter at the receiver is much wider than that of the noise and noise respectively white and we have described this in an earlier discussion.

So, received signal r of t could be written as S m of t that is the transmitted waveform plus noise which is additive in nature and nt represents noise and this r of t is in the interval of 0 to capital T for the next interval you again have a similar expression. So, we proceed with this kind of model.

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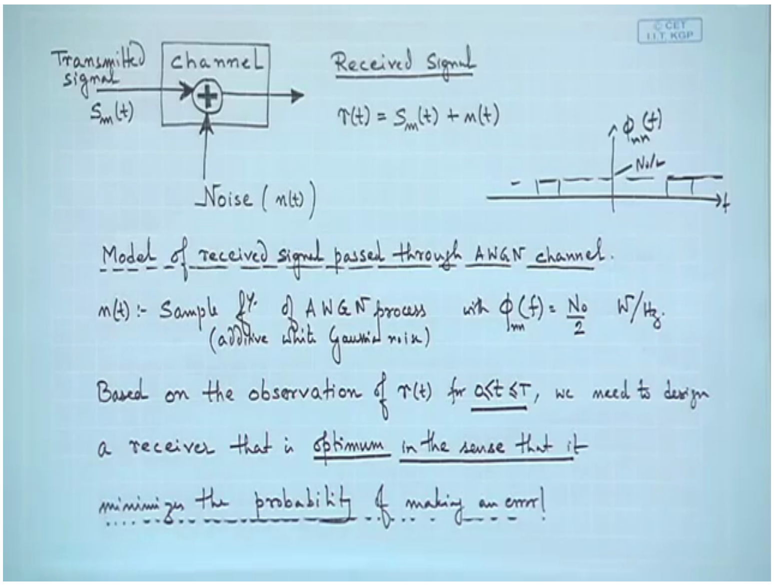

And we have the diagrammatic representation as we discussed earlier there is a transmitted signal S mt is transmitted the channel is all, but addition there is no modification to the signal and noise which is N t and the received signal is of this form which we had described earlier.

So, now, eloquently look into the model of the received signal when it passes through the AWGN channel as described here and there we have to describe the noise process. So, noise process N t is a sample function of the additive white Gaussian noise process with the power spectral density denoted by phi sub N n 2 ends indicating we are taking from the autocorrelation function its Fourier transform of f is equal to N naught by 2 this is what we had seen earlier.

So, this is phi N n of f and this level is N naught by 2 this is f right this is the picture that we had used before now based on the observation of r t in the interval 0 to capital T. So, we have our observation interval up to 0 to capital t we need to design a receiver that is optimum in the sense that it minimizes the probability of making an error. So, recall we have started our discussion on designing optimum receivers for AWGN channel.

So, when we say we want to build an optimum receiver for AWGN channel we have first described what we mean by AWGN channel and we will be talking about the receivers. So, before we talk about receivers we will talk about what is meant by the term optimum right we say that it is optimum in the sense. So, whenever we define optimality you have to design you will define a particular criteria based on which optimality should be taken. So, that it minimizes the probability of making an error. So, why we make such a system such a statement the reason we make such a statement would be clear if we look at the signal that we are talking about. So, we can use this representation.

So, we have sent S m receiver receives S m of t plus there is noise the job of the receiver is to identify which of the sm was sent because if the receiver is able to identify which of sm was sent; that means, if it is able to identify the waveform it can do the reverse mapping and it can get the binary sequence. So, that is the process of reverse modulation. So, here modulation goes this way reverse modulation should go this way right. So, when you go this way you have to identify which waveform because each waveform is arrived at by mapping a unique sequence.

So, if I am able to identify a waveform I will be able to identify the corresponding unique sequence and hence the statement about making an error because all that I can do at the receiver is either find the appropriate sequence. That means the appropriate waveform that has been sent or finds some other waveform that has been sent.

Because try to recall what we have discussed the job of the transmitter or why we are doing communication the reason is there are a set of possibilities we know the possibilities. So, the transmitter and the receiver they both know all possibilities. So, if it is a sequence of 0s and 1s getting transmitted the receiver knows that the receiving signal should be a 0 or a 1 it has to make a decision whether it is a 0 or it is a 1. So, it has to choose between the 2 it cannot create a third entity which is neither 0 nor 1, right.

So; that means, when it has to choose between the 2 it will choose either 0 or it will choose 1; That means, if it chooses if the input bit was 1 and it chose one it is correct otherwise it is error right. So, for the receiver it makes sense to define optimality criteria in the sense in terms of making an error in making decision.

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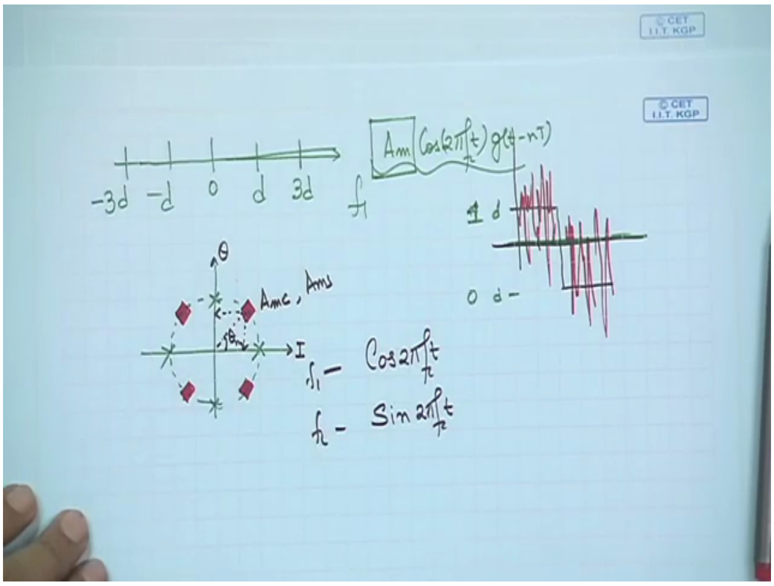

So; that means, you want to minimize the chance of making an error right and why we will make an error is what we had briefly discussed at some point where we said that I will quickly talk about this that suppose you have 2 levels. And there is noise that is present which gets added to these now whenever signal crosses this threshold if it is on the up upward side you would identify this as d means Let us say plus 1 and minus t would mean A 0. That means, A 1 or a 0.

So, because of noise if instead of detecting it as at this level I detect it to be negative value. So, instead of 1 I would make a decision of 0 and vice versa. So, that is why it makes sense to describe optimality in the in the sense that there is some kind of an error that may happen at the receiver right. So, that is why you define optimality in the sense of either making either identifying the right symbol or waveform or the bit or making a mistake in that.

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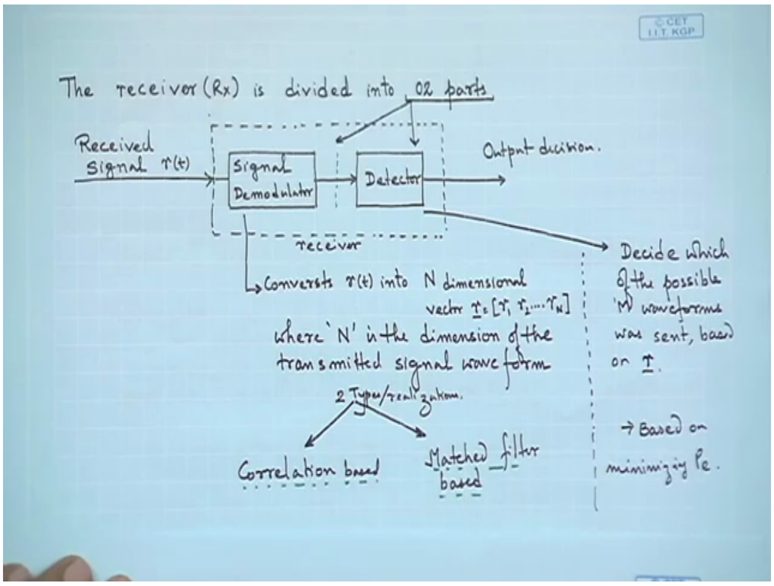

So, a receiver is divided into 2 parts just like the modulator or the transmitter is divided in 2 parts one is the symbol mapper the other is the up converter here also we divide into 2 parts, but it is not in correspondence to the not in the exact correspondence, but there is a lot of similarity in the way we do.

So, in one part what we do is deconstruct or decompose the received signal into its components and the second part would be using these components to detect which particular S mt was sent. So, what we have is the received signal r of t comes in and it goes through the signal demodulator right.

And then it sends the output to the detector the job of the demodulator is to convert the received signal rt into the N dimensional vector are composed of r 1 r 2 up to rn components the job of the detector is to decide which of the possible m waveforms was sent. So, you partition the job into 2 parts and things will be clear soon and of course, when we say that it decomposes into N components. So, N is the dimension of the transmitted waveform of the transmitted signal waveform we will describe what we mean by the dimension.

So, in this case the demodulation there again are 2 possible realizations one of the realization is known as the correlation based realization the other one is known as the matched filter based realization; so before we discuss further onto this id like to mention about the dimension of the signal that we have just described here. So, if we have a PAM signal this is 0 you have d and you have minus d you may also have a 3 d and you may have a minus 3 d . So, in this case you have a 1 dimensional signal space right and if you look at the Pass band signal it is a m cos 2 pi fct and there is a gt minus nt.

. So, you recall that we defined in this case there is only one basis function and that is in terms of cos 2 pi fct times gt. So, g is giving the time orthogonality and this is giving the function. So, when we are modulating we are only changing the amplitude of this particular signal. So, when we studied the next constellation we studied QPSK. So, either this could be one constellation or this is pi by 4 shifted QPSK.

So, clearly there are 4 phases; however, each signal has a component on the I channel and another component on the q channel and this location could be represented by amplitude on the cosine and amplitude on the side. However, for QPSK we had one phase parameter theta m which would control this, but we could represent this as cos 2 pi fct as one dimension and f one of t and sin 2 pi fct as the other dimension.

So, one of the basis functions was in the cosine the other basis function was the sin and they are at quadrature with each other.

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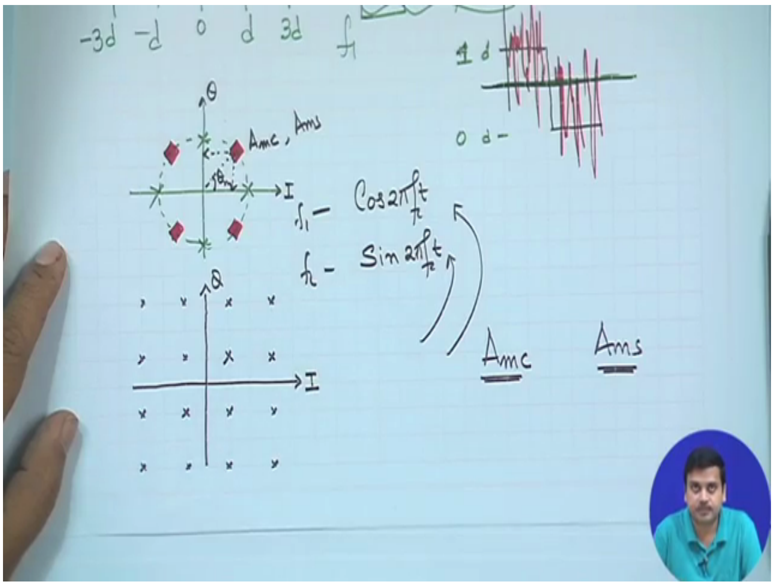

And this is also in the signal space diagram we have a quadrature. So, when we moved into the QAM constellation in the QAM constellation also we had the; I and the Q and the same basis functions were available. So, in this case we had the AMC as the amplitude for the basis function along this dimension, and we had AMS as the amplitude for the basis function along the q dimension or that of the sin. And we said that these were the components of the original signal on the 2 basis functions from this point we moved further and we looked at N dimensional signaling.

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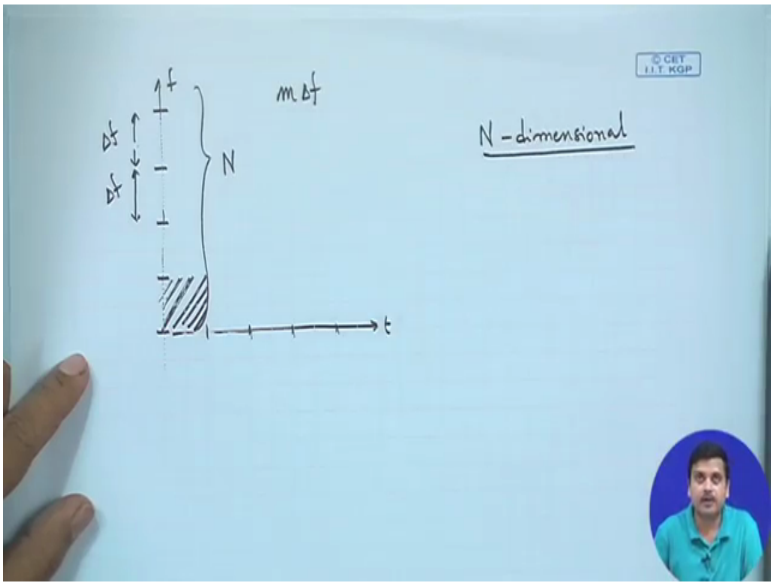

So, in the N dimensional signaling right we had situation of selecting values of f where each of the channels were of bandwidth delta f. And one would select m times delta f in communicating which of these frequencies has been selected.

So, if there are N such possible selections available and each of them were orthogonal to each other we had M-ARY FSK. So, there was like N dimensional signaling that we said we also extended this to N dimensional in the time domain. And then together we said there could be N one multiplied by N 2 dimensions and in each of these windows you could further use all kinds of all kinds of modulation techniques.

So, in this case it was N dimensional right where as in other case it was 2 dimensional or one dimension. So, when we talk about decomposing the signal into its basis components into its components what we mean is that we would take r of t and project it on to the basis functions right.

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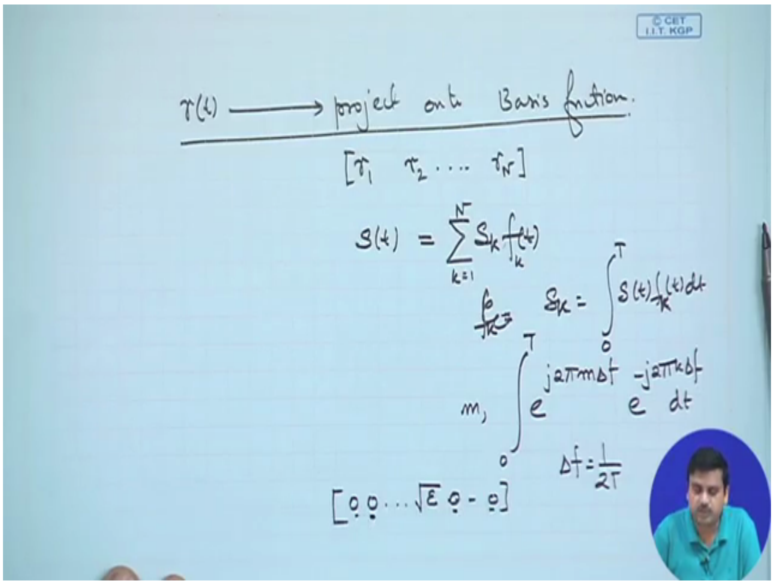

So, if you project it into basis functions you would get its component r one on the first basis function r 2 up to rn if there are N dimensions in case of PAM there is only one dimension and the component we are going to get one single component of it if it is quadrature modulation you are going to get 2 such components one along the i channel one along the q channel.

If it is N dimension you are going to get along all this N orthogonal dimensions now at this point we should also remember that given a particular signal S of t we said that S of t could be expressed in terms of its coefficients k equals to 1 to N where S k is the component of the signal on the particular basis function orthonormal function and f k could be found by f k is of course, there sorry sk you could find by integrating S of t with f k of t dt from 0 to t this would give you the component of the signal. So, basically you have to project the signal onto the basis function you find this component.

So, if there are N dimensions you have to do that. So, when we discussed M-ARY FSK we did say we are calculating the cross correlation function. So, when we are calculating the cross correlation function we did integration of the received signal multiplied by any one of the other signals right and the complex conjugate of it and we wanted to establish that the received signal is orthogonal to all other signals than that particular component on which it has been sent.

So, if you had chosen m; that means, e to the power of j 2 pi m delta f and if we are correlating with e to the power of j 2 pi k delta f in the interval 0 to d 2 dt and if delta f is selected as 1 by 2 t and we are doing a PAM in that case we would found that we had found that if m is equal to k this would turn out to be one else m not equal to k this would turn out to be 0. That means, in that case our components would be maybe 0 0 somewhere there would be some component which is the energy of the signal.

So, in ideal case you would have this in a non ideal case you would get some component values in other places. So, you are basically breaking down the input signal into several components once you have broken down into components the job of the detector would be to use all these components and find out which value of S was sent; that means, you have S m of t.

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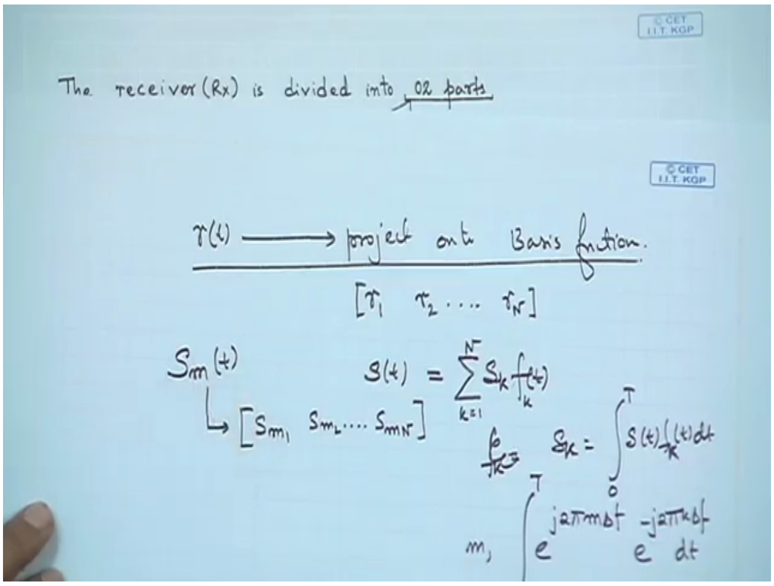

So, S m of t can be broken into S m 1 S m 2 up to S m N different components. So, using this you can reconstruct S mt following an expression which looks like this; so now, since you are broken it into components. Now using these components the detector would find out which of the signals because each of these signals will have its components. So, if I have the components then I can identify with signal.

So, this is the basic philosophy based on which a receiver is usually designed. And in the next lecture we are going to discuss the correlated demodulator of this particular structure following which we will discuss the matched filter realization and once we have studied these 2 methods. And we have been able to decompose the received signal into its components, then we will move forward in using these components in the lecture down that after that into designing the detector for the receiver in an AWGN channel.

And once we have studied AWGN channel we have been able to find out the error probability expressions the performance, then we can move into studying the effect of non ideal channels thereafter.

Thank you.