Modern Digital Communication Techniques
Prof. Suvra Sekhar Das
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Lecture - 39
With Memory Modulation (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. In the previous lecture we have been discussing about these the spectral characteristics of the digitally modulated signal. And what we have seen in the first step is that the spectral characteristics of the band pass signal can be calculated from the spectral characteristics of the equivalent low pass signal. And when we wrote down the expression of the low pass signal we found that it is a function of the information varying signal, which is a standard thing which we had been using since the beginning. And since, the information bearing signal is random we stated that the signal the baseband or the low pass equivalent signal is represents a stochastic process.

Now since, this is a stochastic process and we are interested in the spectral characteristics we said that we need to look at the power spectral density- that is the frequency characteristics of the signal. So, if you have to do the power spectral density we found that the power spectral density of the pass band signal could be calculated from that a bit low pass of the PSD of the low pass equivalent. And to calculate the power spectral density you need to know the autocorrelation function, because these are duals of each other as Fourier transform and inverse Fourier transform.

So, since the power spectral densities of the low pass equivalent are connected to that of the pass band signal so is the autocorrelation function relationship. So, there is also a direct relationship between them. And if you know the carrier frequency which translates from the low pass to the pass band you could easily get the complete information.

So then we took the I n that is the information bearing signal to be real; sorry, I mean we said that in case of PAM it is real in case of others its complex and you would write it using the j function. And we assumed that this information bearing sequence is a wide sense stationary process. That is fair enough because for the duration of time that we are under consideration is not infinitely large it is typically quite small.

So, over the small duration that we are considering; that means, the source that we are under consideration we are saying that the mean of the signal is not changing with time and the autocorrelation is dependent only on the lag between the 2 signals and not a function of time. So with these few assumptions we moved ahead to calculate the autocorrelation function.

And then we wrote the expression of the power spectral density of the low pass equivalent and what we found is that the these power spectral density of the low pass equivalent signal could be expressed as product of the power spectral density of the pulse that is g of t and that of the power spectral density of the information bearing signal. So, now, if we have to change or control the power spectral density of the of the outgoing signal or what we see is that we could play around with the pulse shape, because changing the pulse shape would of course change the spectral characteristics we had seen in one of the previous lectures. And it also gives us the opportunity that you could do something with the modulating signal so that it affects the spectral characteristics.

So, we had seen some of the basic forms like in case of MSK we used a different pulse shape. that means, a half sinusoid and we had also seen that you could change the incoming signal in a way that they are kind of staggered that could also help you in some form and there could be other forms where you could do an exhorting of the output with that of the previous we had seen before. So, there are various ways of doing it. And this gives us an opportunity to control the spectrum of the outgoing signal.

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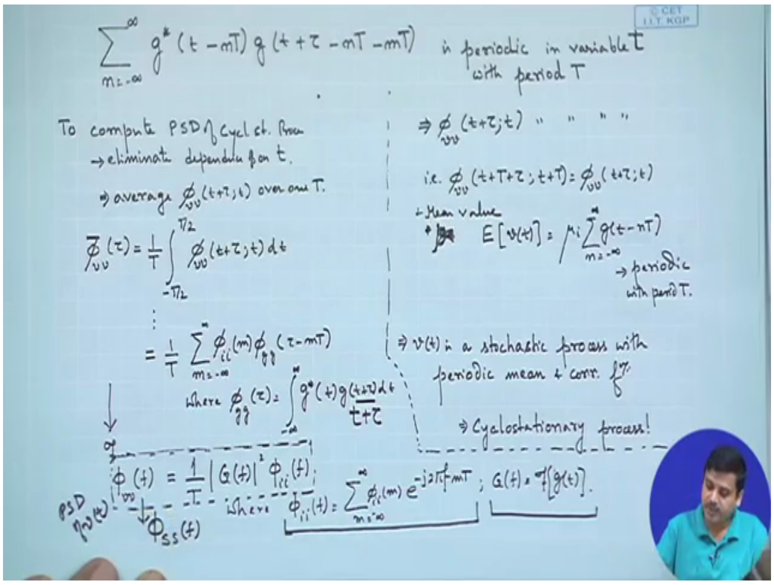

So moving ahead we had come to this particular expression in the previous lecture where we identified the spectrum of the pulse along with the spectrum of the information bearing signal which affects the pulse the power spectral density of the baseband signal.

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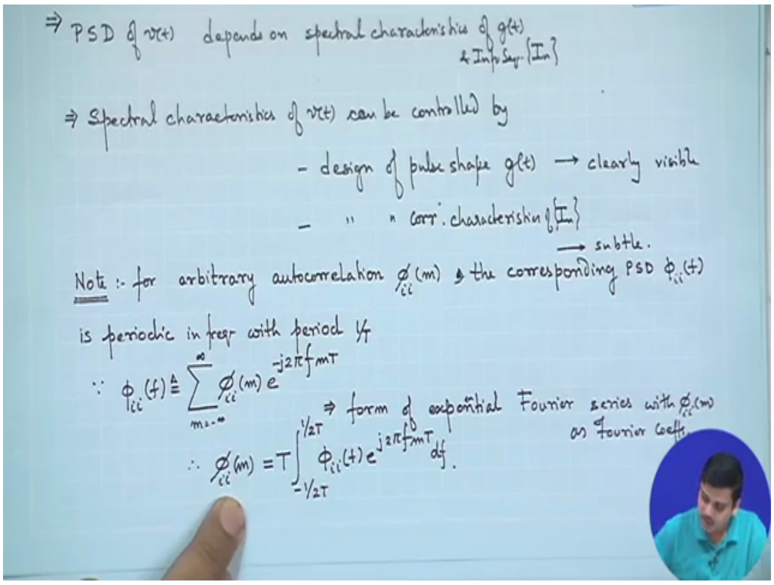

So, whatever we told you is a kind of briefly stated over here; that means, the PSD of the vt depends on spectral characteristics of gt and in so, therefore, you have some options over here. So, spectral characteristics of vt could be controlled by the pulse shape g this clearly visible and how does this effect is not. So, clear because we have not looked at the phi ii of m.

So when we get a look at that then thing will be a little bit more clear. So, so we could say that the for arbitrary autocorrelation function the corresponding PSD phi ii of f is periodic frequency because of this expressions that we had already there. So, we had said that phi ii of f see because this is from the discrete sequence because this is obtained because of e of ii of n. So, which we have over here?

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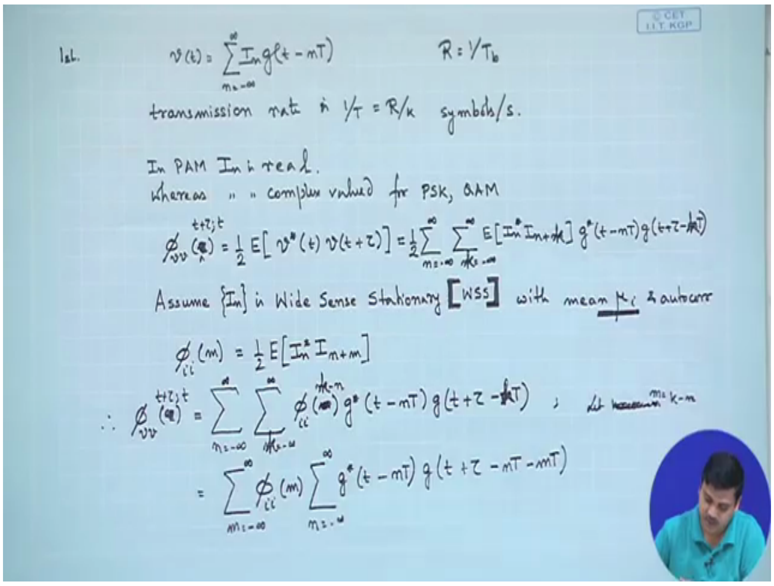

So, you have phi ii f these are discrete sequences right. So, since because of that you could write it as a Fourier series right you could easily write it as a Fourier series. So, that is what we have over here.

Now this Fourier series it is periodic with period 1 upon t and what you could see over here that it is in the exponential form of Fourier series this particular relationship and therefore, you could say that these are the Fourier coefficients. So, these are the Fourier coefficients so; that means, there is this Fourier relationship between phi ii f m and phi ii of f which is also natural you could say phi ii of m could be obtained using the inverse Fourier transform of phi ii of f and this would be the relationship that you are going to get. So, given this we can say that.

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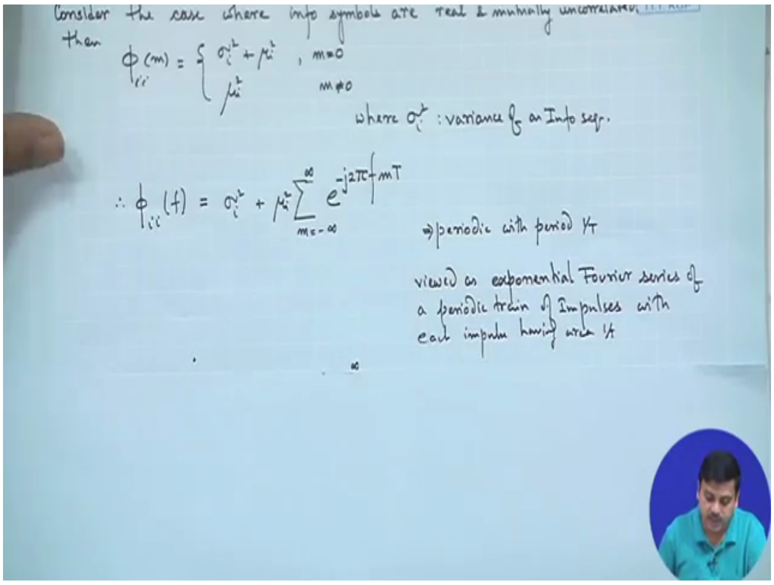

Let us consider the case where the information bearing signals are real and mutually uncorrelated, but this is in terms of assumption its not a very critical assumption because if you are taking PAM then these are real and if you are taking memoryless source then you can say that well its uncorrelated. That means, the ins are uncorrelated right we are not talking about the modulating form we are saying that the source is generating symbols which are not dependent on the previous 1.

so which is also a pretty good assumption and in that case we could say that this phi ii of m right which is the autocorrelation of I am right which is expectation of in and in plus m right. So, that would be equal to sigma squared I plus mu squared I because mu is the is the mean of in and this is the variance of in for m equals to 0; that means, when they are the same. So, if there is a mean and there is a variance you are going to get this as the phi ii of m and for m not equal to 0. That means, for any other value if they are uncorrelated right then you are going to get this as mu I squared because sigma ij would be equal to 0 right oh sorry sigma I of m. That means, of a certain lag would become 0 un-correlatedness would give only leave you with the mean.

So, of course, we have said that this is the variance of the information sequence and therefore, you could write phi ii of f in a form where there is this sigma squared I plus mu I squared times this term going by the previous expression if you look at this previous expression over here right from this and using this expression you could write it in this form for m equals to 0 and this is for all other terms, right. So, that is the relationship that you have and again you will find this is periodic with period 1 upon t, because of this expression e to the power of j 2 pi fmt and it can also be viewed as an exponential Fourier series as and this particular thing right can be viewed as a periodic train of impulses each having an area 1 upon t.

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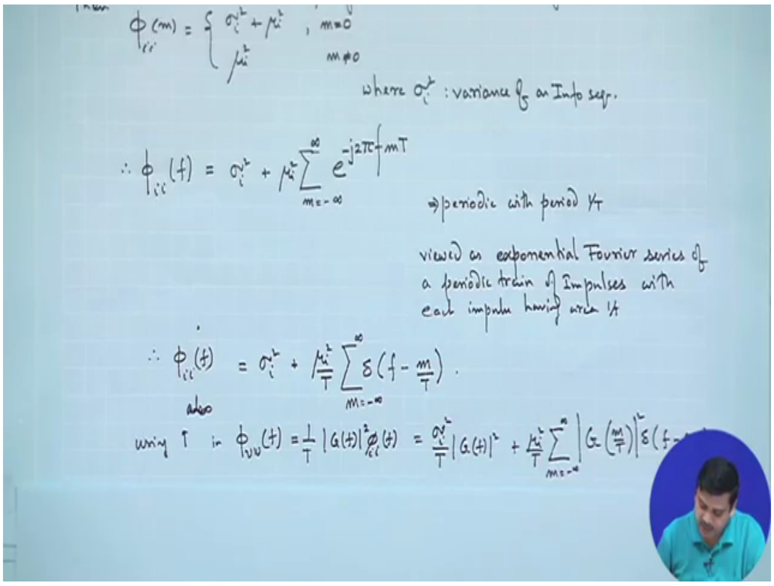

So, you could also view it in a similar form and then you could also write it as this kind of a structure, right.

So there is with this concurrence you could expand it in this form right where the ii phi ii of f would have a similar form and using this particular expression you could write phi vv of f which we had arrived at in the previous expression. So, here is what we have phi vv of f. So, we have these 2 terms we now have an expression for phi I i of f while making certain assumptions about I right which we just did. So, then you could write it as using this expression as 1 upon t g f squared phi ii of f this is what we had before this particular expression. So, sigma squared I gf and you have this particular term, right.

So what we see is that there is a gf and there is also a train of repetitions for t. So, that that is what we get a structure which is not. So, I mean straightforward, but we cannot help it that the expression looks in takes a form which is this. So, now, if you look at phi vv of f whatever form it takes what you can say is that there are 2 terms in this thing.

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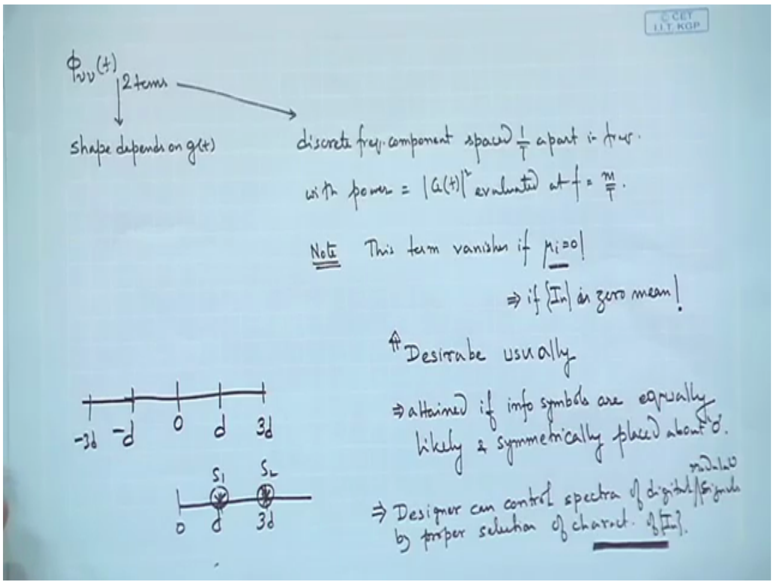

One is the dependence on shape of gt and the other is the discrete frequency component spaced 1 upon t apart which is clear from this there is a delta f m upon t which is the discrete frequency component and there is this gf which is also present and this one you would evaluate at f equals to m m upon t.

So these are the 2 different things that you have in this particular expression and if your mu that is the mean is 0 right then this particular term goes away mu is 0. So, discreet frequency component means you are going to have like spikes of frequencies at f equals to m upon t and clearly if mu is 0 this whole term would go to 0. So, if that term is 0 you are left with only this term; right where you are you have the variance only and this is usually desirable because you would not like to have the spikes of frequencies in repeated intervals.

Now if you want to make mu 0 if you have to make mu 0 then all you have to do is if the information sequences are equally likely and symmetrical; symmetrical about 0 so; that means, this is 0 and you have let us say d and let us say you have minus d and there is equal likelihood of d and minus d the mean would be 0 if you have 3 d and if you have minus 3 d then again the mean is 0; however, just for the sake of example if we have d and 3 d as 2 constellation points this is s 1 this is s 2 this is not going to yield in a situation where mu I 0. So, you to place them symmetrically and there has to be equal likelihood.

likelihood.

So; that means, the designer can control the spectrum of the digitally modulated signal by properly selecting the characteristics of in so; that means, you have option of choosing some characteristics of in thereby you could control the spectrum which is apparent in this particular expression right. So, that is how we connect the; these this spectral the spectrum of the signal moving forward. So, let us take the example of a rectangular pulse shape.

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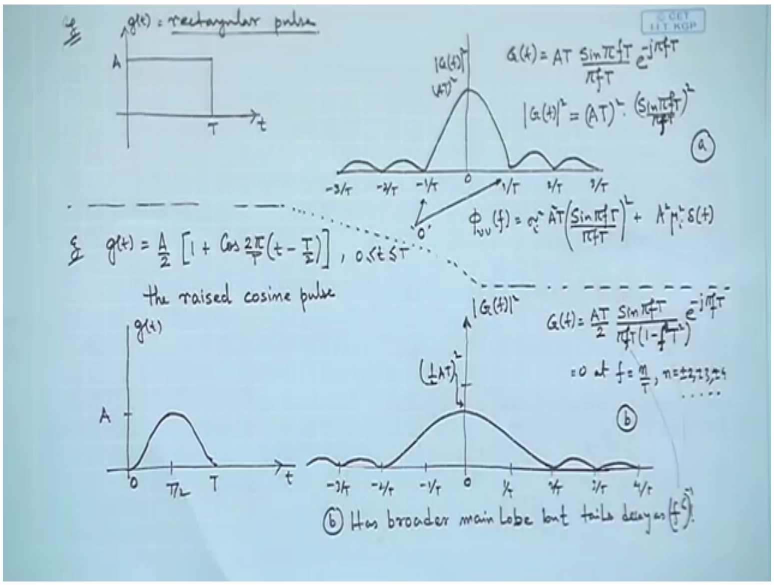

So, if we take a rectangular pulse shape of gt and we need to find the the spectral characteristics the first thing that we have to do is to calculate the Fourier transform of gt because gt is a fixed one it is not a stochastic one and gf is found by simply taking the Fourier transform of gt unlike that of I n for I n you need to take the autocorrelation function and go to the power spectral density.

So, if you take the Fourier transform g of f would have an expression which looks like this has indicated there at sin pi ft upon pi ft. So, these basically sink e to the power of j of phi pi ft. So, we have seen this kind of expressions before and the power spectral these the power; power spectrum is mod of gf squared where you have at squared and mod of this is going to be one sink squared right it is a sink squared. So, a rectangular pulse shape has a power spectrum which looks like this with 0es at 1 by t 2 by t 3 by t and so on and so forth. So, they will be 0 crossing at these points right.

So, now, instead of doing this if you would take a raised cosine we had given a raised cosine pulse pulse earlier the raised cosine pulse would look like this it rises slowly to the maximum value and then it falls from the period 0 to t right in this case your gf would take a structure which appears like this. So, in the denominator you had pi ft earlier now you have pi ft 1 minus f squared and t square and this is 0 at f equals to nt. So, when f is equal to nt it will become 0 for n plus minus 2 compared to this. So, there is as at the 0 value there is a peak because of the sink both the things this goes to 0 at 1 upon t this goes to its first 0 at 2 by t and minus 2 by t.

However, the difference between these 2 that one the important difference between these 2 is that if you look at the denominator this decays as f squared or 1 upon f squared because there is a single f here in the power there is 1 upon f 1 upon f squared. So, there is an f here whereas, here it decays the mod gf squared decays as f to the power of six simply, because there is an f square term over here there is another f f term over here. So, what it means these side lobes decay at a much faster rate than these side lobes. So, it goes to the first 0 very quickly, but these keep on coming with quite good amplitude whereas, here it spreads out, but the side lobes are much much smaller.

So what hint we can take from this will probably see something more is that if you have a requirement where you are you need to have the null within a certain small bandwidth you can go for a rectangular pulse whereas, if your requirement is that you are adjacent channel interference is low; that means, you want to create less interference in the neighboring channels then you could use a raised cosine pulse where your spectrum the main lobe would be wider, but the spectrum would fall very sharply after a while.

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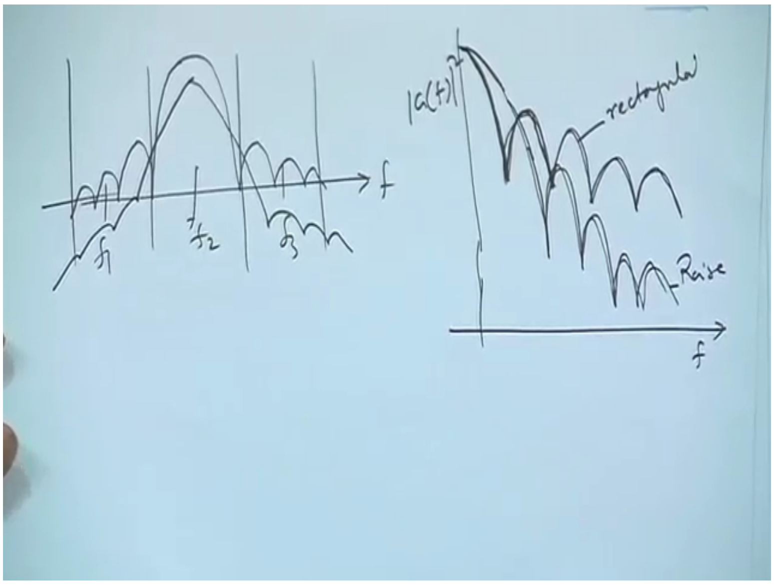

So, I mean if you would compare this against each other probably in one case you will get spectrum falling like this and in the other case you might get going like this. So, this would be the case for the raised cosine. So, this is let us say gf squared and this is f and this would be the rectangular.

so although this comes to 0 earlier, but still there is lot of side lobe which is present in this and which decays much much slowly compared to this these are rough hand drawn picture. So, things going to exact; so, this decreases the advantage of this is that if we have frequency axis and there are channels like fdm channels this is f one this is f 2 f 3 and. So, on and there is a spectrum right. So, you would have this kind of situation in one case the other case you are going to have going down and down and down and down like this right.

So then one has to decide how to design the communication system what should I consider as my system bandwidth and how should I choose the pulse shape and not only that in the raised cosine you also have certain design parameters by which you could control this thing. So, what clearly it means is that you have an opportunity to control the spectral occupancy of the signal based on the pulse shape as evident from this particular example right.

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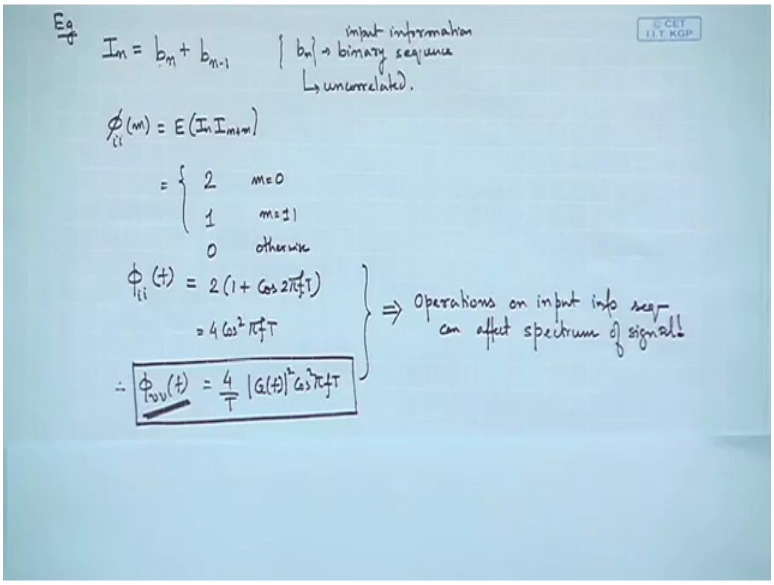

We move on further and take a look at another example and we say that let your information bearing signal that is in that we are interested in is generated in the form that in equals to bn plus bn minus 1 where bn is the binary sequence right.

So bn is uncorrelated, but in is not uncorrelated. So, bn could take a value of 0 and 1. So, in could take a value of 0 one or 2 it would take a value of 0 when both of them are 0 it would take a value of one when either of them are one and the other one is 0 it would take a value of 2 when both of them are taking a value of 1 so; that means, the phi ii of m the autocorrelation between this; this sequence would be a value of 2 for m equals to 0 m equals to 0 means in bm and bm.

So in that case you are going to get autocorrelation value of 2 and the for m equals to plus minus 1; that means, when there is a difference of one you can get a value of one otherwise it is 0. So, that is how the autocorrelation function would turn out to be and if you take the Fourier transform of the autocorrelation function you are going to get power spectral density. So, you have a power spectral density which is 2 times 1 plus cos 2 pi fct which is clearly different from that of the power spectral density of if you were considered bn and therefore, this you could write it as four cos squared pi fct because it is 1 plus cos 2 theta which is equal to 2 times cos squared theta.

And then of course, you could have phi vv of gf in this case sorry phi vv of f which is the power spectral density of the low pass equivalent signal as four upon t this t comes from the original reference where it you had mod gf squared upon t and this four comes from here and cos squared 2 pi cos squared pi fct. So, this is the overall structure whereby you not only have converted this to a cos squared function because of this operation which includes memory; that means, 2 bits are there; there is an autocorrelation present over here as well as there is gf.

So what we could see is that you can control the spectral content of the signal by modifying the input sequence that goes into the modulator as well as by selecting the pulse shape. So, before concluding we will quickly take a look at some of the spectrum that has been pre calculated and we will just see how things are. So, what will do is we will move to the power spectral density of a cp FSK signal, because otherwise we would have had to do all similar calculations for cp FSK modulations and that is pretty time consuming and which we cannot afford to do in this particular course, but I would encourage you to go through the derivations available in typical textbooks and mainly the one that we are following as of now.

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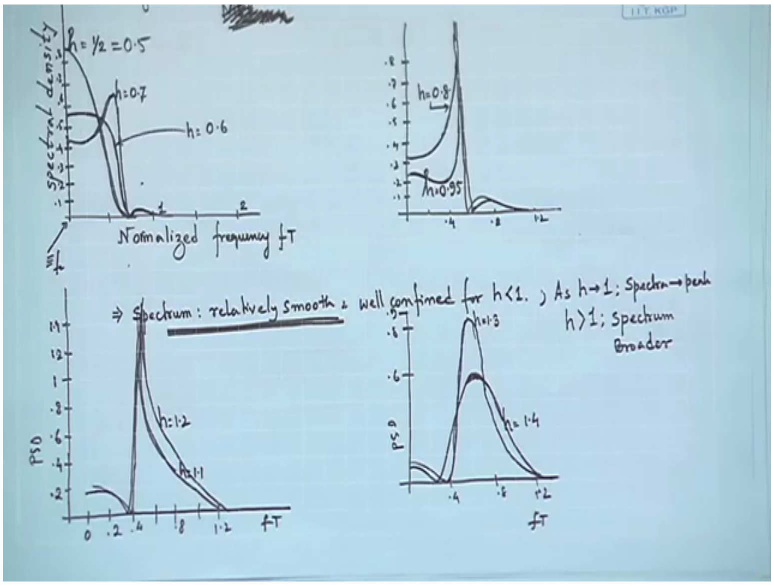

So, the power spectral density of a cp FSK signal I have simply drawn a few pictures available from references and we would like to see what it yields. So, before I go into this I would like to remind you that in cp FSK signal binary cp FSK signal you would remember we said that the modulation index h is equal to 1 upon 2 and you have the in the modulating signal has 2 values plus 1 or minus 1; so, using that. So now, sorry I stated something else in case of MSK we said in case of MSK you had h equals to half. So, please not take my previous statement in case of MSK you had half.

So in case of binary cp FSK; that means, in this case we have h as the modulation index and we would choose this particular spectrum as you see is a normalized spectrum with ft because you would have you had you had 2 fdt as the value of h. So, and this is the spectral density. So, this is the power spectral density and this is normalized power spectral density this is normalized frequency ft. So, if we take different values of h what is the power spectral density that we get?

So what we see is that if you would choose h equals to half your curve power spectral density would look like this if you; that means, half means 0.5 if you take it as 0.6, it would become like this if you take it at 0.7 it would turn out to be like this as you increase h from 0.7 to 0.8 it becomes peaky at 0.95, it becomes even more sharp and as we move on it is the same here you have h equals to 1.1 as this 1 an h equals to 1.2 and here we have higher values this particular one is for h equals to 1.3 and this one is for h equals to 1.4.

So, what we see from all these power spectral density curves for a binary CPSK cp FSK is that the spectrum is relatively smooth and it is well confined for h less than one. So, h less than one are all of these values here it is more than 1 right. So, if you keep h less than one you are more or less quite confined in spectrum, whereas, as you increase h it becomes broader right it stretches this one increases there you see the stretch over here it stretches there and there and so on and so forth right and as h tends to 1. So, when it is close to 1 if you look at 0.951 0.1; what you see is that there is a peak that is coming right there is a power spectrum. So, there is some particular carriers which are coming and one thing of course, a bit out of context, but if there is peak and there is a peak interference at some point it could jeopardize your signal and if h is greater than one you are going to have a broader thing.

So, this is some hint towards our discussion what we did for MSK in case of MSK we have h equals to half and that would mean this particular spectrum that we follow right this particular spectrum that we follow. So, it is pretty well contained over here.

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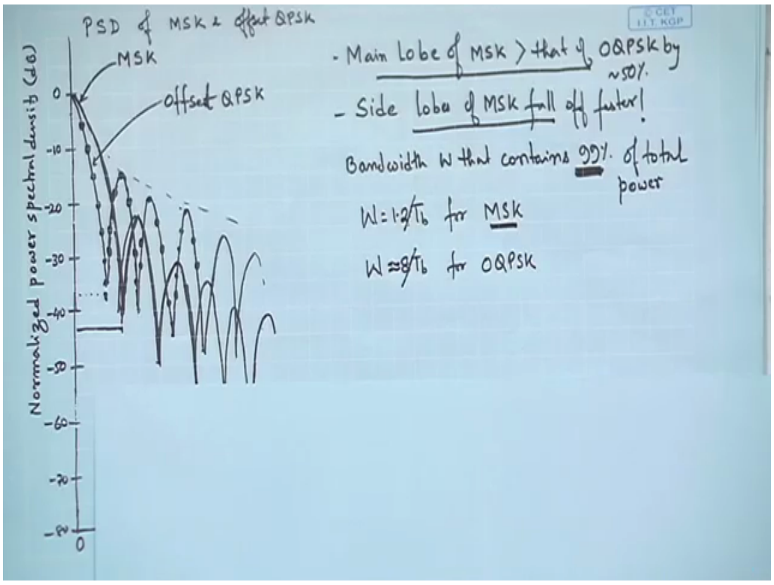

So, now we take a look at the relative comparison of the power spectral density of MSK and offset QPSK. So, this is again a rough diagram. So, this is the normalized the power spectral density in db and this is the normalized frequency axis fc times tb minus fctb. So, what I had drawn before in the in the rough sense I have drawn it over here as well.

so this particular curve that you see which I am making it a bit more thicker here is basically the curve for MSK and the curve for MSK clearly tells us that it is a bit broader compared to offset QPSK. So, which curve is here? So, offset QPSK goes to 0 earlier that was the null is earlier compared to this, but the side lobes you clearly see falls much much faster than. So, this would go somewhere there and it would continue to go like this right. So, this one falls like this the other one falls much much faster.

So, then the side lobes are much better. So, what you have is the main lobe of MSK is much larger than that of QPSK offset QPSK and the side lobes fall off faster and generally it is known that the 99 percent of the total power is contained within 1.2 times the signaling rate for MSK signal whereas, it is nearly eight times the signaling rate for offset QPSK. So, if you have to contain ninety nine percent of the energy within a small band of signals you would go for MSK whereas, if you have to constraint the null within a smaller bandwidth you would go for QPSK and in this case in case of MSK you are choosing one signal at a time and in this case you are choosing 2 bits at a time.

So there are certain differences. So finally, what we make to what we want to make the statement is that given a constraint on the channel especially in terms of the amount of bandwidth available or the amount of side information leakage that you can allow you should find opportunities to choose the modulation scheme which satisfies the constraints.

And this you can do by studying the spectral characteristics of the signal in the by the mechanism that we have demonstrated in this. And the previous lecture I would like to also remind you at this point is they are still quite a few more things to do especially when we study the error probabilities which would help us in taking a concrete decision and doing a performance evaluation of different modulation schemes which would help you in finally selecting an appropriate modulation scheme for a particular kind of channel.

Thank you.