Modern Digital Communication Techniques

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Lecture - 38
With Memory Modulation (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. So, we have seen the different modulation techniques till now. And in the previous lecture we have mentioned that we have more or less covered some of the important modulating structures or the formats or generic representation by which you should be able to handle most of the modulations that we encounter and of course, there could be something beyond that.

But whatever framework we have started off with should equip with you with sufficient capability to able to handle the modulation schemes in the format or in the generics form that we have been working with. And while doing these things in the last few lectures we have specifically stated that these signals which we are doing which we are calling as memory less are having some problems, which are overcome with respect, when we are introducing memory.

And the way we explain this was we took the example of QPSK. Then we said that in QPSK, you would generally have the phases changing at every single interval where if both the bits; that means, there are 2 bits in QPSK like ordinary phase shift keying, that if both the bits change simultaneously and you have no control on that because it is a from incoming source.

Since one of the bits is going to the i channel the other bits bit going to the q channel it could highly happen that overall there is a one 80 degree phase shift. Now this drastic phase shift happening within a very short interval that is a switching period from one symbol to another, which is very, very short could give raise to the means 0 crossing and they could be a spectral re-growth and all kinds of problems.

So, one would typically like to reduce these kind of phase changes. So, we introduced the offset QPSK where we said that, let one of the bits change per symbol interval. And in that case, when you change one of the bits at most you can change by pi by 2. And not pi and this would reduce the delta phi; that means the amount of phase shift that you want to encounter and hence the frequency growth should be less.

And So, it will it will at least it will give you resilience against 0 crossing and envelope feeding and all kinds of things. And then we also saw that there is another modulation technique which is the binary FSK which could also be represented in the form of an offset QPSK, where the pulse shaping is kind of half sinusoid. And there also there is offset between the 2 i and q channels the i is used to modulate the cosine and the q is used to modulate the sin. And by virtue of maintaining the sinusoidal pulse shape, compared to rectangle pulse shape in the earlier 2, we would find phase continuity. We had also hand drawn the wave form structure, which is highly instructive to generate the wave form using some programming language and you could easily see the wave forms that is that you would encounter in such a system.

So, what we thought is probably it is wise to look at the power spectral densities or the spectral characteristics of signals before we conclude our discussion on modulation techniques and before we move on to the receiver structures. So, we begin our discussion today by looking at the spectral characteristics of digitally modulated signals. So, this is what is our main interest today.

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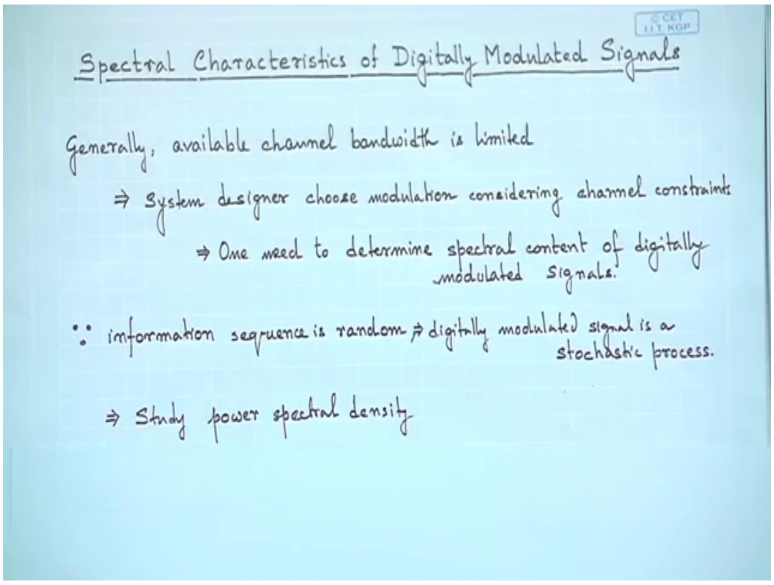

So, when we talk about channels we have been seeing is since the beginning that channels are all kinds of systems through which the signal propagates. And generally this channels are band pass channels. So, it allows a certain set of frequencies to go through and it does not allow certain set of frequencies to go through. So, usually channels are frequency selective. So, we therefore, have that generally channels are limited in band width right.

So that means, they do not have a band width which is spanning from minus infinity to infinity. And that would leave the system designer to choose the modulation technique considering channel constraints. So, so we are basically getting into design aspects slowly. So, till now we have been writing the expression of the transmitted signal we have been discussing in a bit of qualitative form about the spectral characteristics this being more or less.

What we aim to do today is to take a few steps forward in trying to more or less concretized this particular aspect. And why we want to do this the reason is that you would like a designer communication system and that is what we started off in the beginning as the goal of the course. So, if you design a communication system the moment we talk about design we talk about choosing parameter values, because parameter values are the ones which are the outcome of a design procedure.

So, when the channels are limited in bandwidth in that case the designer; that means, you are suppose to consider modulation schemes, which would allow the signal to go through the channel by conforming to the constraints. So, for example, we will see sometime later that QPSK has a certain spectrum compared to the spectrum of MSK and depending upon situation you would like to a choose particular modulation scheme over the other.

So, one has to determine the spectral content of the digitally modulated signal. Because once you know the spectral content of the modulated signal, then you would know that whether the spectral content would satisfy the channel constraint. So, what we will do today is try to look at the spectral content of digitally modulated signals. About the channel will not do much because will simply state that if we can calculate this will choose that modulation technique which would satisfy the constraint of the channel.

So, of course, that can be taken with examples. So now, the next important things to be noted here is the information sequence that we have; that means, a bit stream ones and zeros and which gets translated to INS; that means, the amplitude values is random I mean this we have been discussing for quite some time and this is not new for us, that the source if it is not random then there is no point in communicating that is what we have been stating since beginning. And the randomness that we have been stating would mean that it is not known a priory about what is going to be sent, although, you have your fixed options.

So, the example that we can take the basic, the most simple example that we can take is the tossing of a coin. So, we know that the outcome will be a head or a tale. So, there is no doubt about, that it will be either a head or tail. But what we do not know is what is the particular outcome in a particular tossing of a coin, right. So that means, the bits that are going to come we know for sure it will be either taking the value of 0 or taking the value of 1, but we do not know which particular value they going to take at this instant of time, and that is the information content that is being carried by that.

So, that is what we have that the information sequence is random. And hence since this random sequence is used to modulate a signal and we also have this information sequence digital. So, again just to word of reminder that we said initially that if at all they can be communication from the source to the destination or from the transmitter to receiver, you can do it through binary interface. So, that is what is the premise which is started off with.

So, back to that same thing so basically using whatever we have stated before, that we want digitally modulated signal the modulation has been an outcome of this information sequence which is random. So that means, this digitally modulated signal is a stochastic process. So that means, it is a it is a random variable which is a function of time and what we are interested in is a spectral characteristics.

So, if you have to study the spectral characteristics, you have to look at the power spectral density, right. Otherwise you would have looked at the Fourier transform the signal and you got it. Now, since we have this stochastic process to study spectral characteristics you generally look at the power spectral density, right; and again, reminder so that things become easier as we do in this particular discussion.

We did study the spectral properties of a narrow band stochastic process in one of the earlier lectures. And when we did that the example that we took was for noise and if you would recall we wrote the signal model which was very similar to that of a signal; however, we said that it is random that is the that is the difference the previous one was deterministic in case of signal where is for stochastic process, the expression looks the same, but it is not deterministic it is random that is what we stated and then you said that we want to study the spectral characteristics.

So, what we did if you remember closely we did study the auto correlation function of the stochastic process and then a Fourier transform of that, e led the power spectral density. So, that is the same procedure of what we are going to take.

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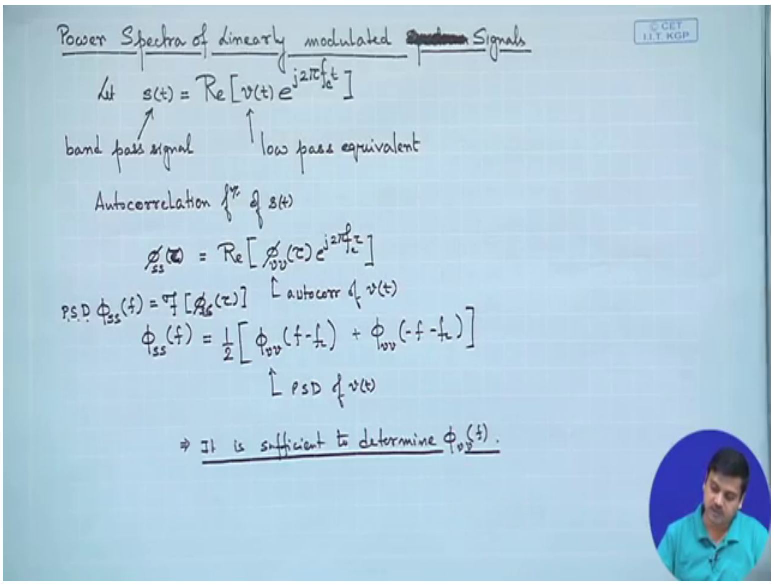

So, let us take the power spectral power spectral of the linearly modulated signal; so initially will begin with the linearly modulated signal. And our typical representation of a linearly modulated signal would be, a s of t is equal to real part of some v t e to the power of j 2 pi f c t. The closest expression which you would have was we used v t to represent the low pass equivalent for memory FSK in a few lecture back and just to match with whatever we can we have learnt before, v t was Am times g t for pulse amplitude modulation and v t was ya, I would not write here it is Am g t and in case of QPSK oh sorry.

In case of PSK you had v t to be equal to e to the power of j theta m, and where theta m was pi multiplied by small m minus 1 divided by capital M. And for Am for pulse amplitude we had to small m minus 1 minus capital M the whole thing multiplied by d. So, times g t of course, that was v t in these 2 cases in case of FSK we had v t as e to the power of j 2 pi m multiplied by delta f. And in case of continuous phase we had the integration of dt, and dt was I n g t minus n t.

So, that how we have these kinds of expressions; and if you look at this expression is definitely real part of v t if this is already real no problem times cos 2 pi f c t. So, that is the standard form which we are quite used to. So, as we said that we are interested in the auto correlation function. So, will take this auto correlation function of s of t because, this is the stochastic process that what we have said, because v t is an outcome of the information sequence and hence if this is stochastic this also stochastic process.

So, the relationship of the auto correlation function of the pass band to that of the base band was also established in our discussion when you studied the narrow band stochastic process. So, we had derived in details the relationship between the pass band and the low pass equivalent representation of the auto correlation function and we did identify that the auto correlation function expression looks very similar to the relationship of the low pass equivalent representation of the signal and it is corresponding pass band.

So, for the signals whatever relationships we have for the signal to the to the pass band from low pass to band pass, we have similar relationship for the auto correlation for the narrow band stochastic process. And the equivalent presentation of that as well as the band pass of that. So, if we you are interested in the power spectral density will represent it as phi ss of f.

Ss because auto correlation of s and s. So, power spectral density because of auto correlation. So, it is s and tau gets translated to f because from the delay domain we are going to the frequency domain and this is basically the Fourier transform of this. So, that is the relationship I have already told you before. So, this one going by this expression you also done a similar thing before where real, if you have real part of something you have the phi vv which is the power spectral density of the low pass equivalent. And this is the other term in the negative frequency generally there is this complex, if there is a complex.

So, for certain cases we did found we did find that this, this we do not need the complex representation. So, of course, what we have in this expression? We had before only thing is we are using the notations which are related to the signals. So, since the signals are now random signals. So, we have the pass band spectral properties connected to this power spectral density of the low pass equivalent process.

So, what we can take from this is that in order to study the spectral properties of the signal all we need to do is to study the spectral properties of the low pass equivalent signal of the low pass equivalent signal. So, if we can study the power spectral density of v then we could easily translate that to the power spectral density of s where, v and s are the low pass equivalent and the corresponding band pass signal.

So, that is what is stated in this line it is sufficient to determine phi vv of f. And these are not new to us we have been using this since the time we have discussed the power spectral density of narrow band stochastic process.

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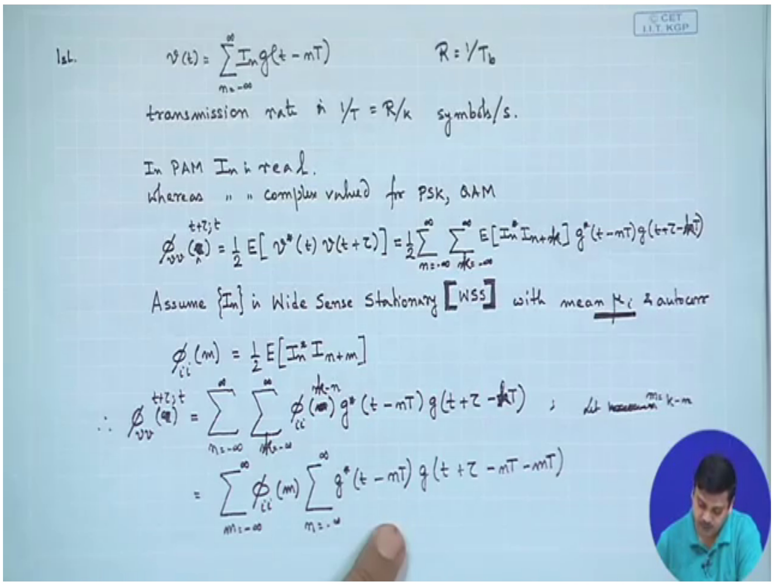

So, we look at v t and So, just because we have discussed this. So therefore, we need to find phi vv in order to find this we will start off with the expression of v. Because in this expression, we do not have the expression of v it is expression of s in terms of v right.

So, moving further let us consider a v which is in this form. So, as it appears, this form this is kind of a pulse amplitude modulation you can think of it in this way. And the information rate is of course, 1 upon t, where t is the symbol duration which is also the notation used here. And this is R by k, where R is equal to 1 upon T b; that means, this is equal to one by k multiplied by T b So that means, t is equal to k multiplied by T b.

So, all these notations are which are we are used to this is, this is not new to us. So, that is why I Am proceeding in form of already pride it in expressions. So, in case of PAM I n is real which we already seen where is in case of PSK and QAM it can be complex valued. And well you could easily separate them into the i channel and q channel and you could write it as I n g something, and plus j times i of something else with g of t minus 2 nT and things like that. And we already seen such an expression when we did study MSK. So, this is also not new to you.

So, if we have to take the auto correlation of v we will directly write phi vv, we have already defined this. So, we are not going to redefine these expressions. So, you should be now familiar with the way we are writing these expressions. So, we have phi vv of t plus tau and t, right. And which is half expectation of v of t conjugate times v of t plus tau, this is from standard definition.

So, when we have expectation operation we do need the pdf the joint, pdf in order to compute this. So, let us see how we proceed with that. So, this expression we should now expand. So, v of t is what we have over here a conjugate would lead to a conjugate over there. G is a real valued signal that we have already said since the beginning. And v of t plus tau should have v of t plus tau as another signal and in that second signal we should have different index; so i k and this expression.

So, when will write it down, when we write it down in terms of this kind of expression what we have is half summation this summation n equals minus infinity to infinity. I n g t minus nT. And there is a second summation k equals to minus infinity to infinity I n plus k. So, we have something shifted with respect to this because it is shifted with respect to this. And we have g t plus tau minus kt following this expression right.

So, this follows by using this expression in this you will get there. Now since we have the e operation e will not operate on g because g e is the deterministic. Signal g is the pulse shape unless we are choosing this randomly. So, e would operate on this information sequence. So, I n is the information sequence. So, what we see is the auto correlation is finally, depended upon the auto correlation of the information sequence.

So, it may be to assume I n that is the information sequence to be wide sense stationary. So, by wide sense stationary what we mean is that, the mean is not varying with time and the correlation is dependent only on the time lag and not on the function of time. When means the correlation and the mean will remain the same at whatever instant of time you are trying to calculate these values.

So, we also saying that there is a mean which is mu of i and the auto correlation would be given by phi ii m. So, we are using phi for auto correlation, and this is naturally the expression. So, always trying to remind here is that will be replacing this expression over here right. So, we have still not calculated it we have still left it in terms of notation calculations will follow. So now, if you would expand this expression we going to get phi vv t plus tau t there is the same expression here.

We have n, we have k and here we have simply replaced phi ii k minus n as per the relationships over here. So, if you work out these relationships you are going to get this expression here these 2 terms remain as it is. And then you could replace let m is equal to k minus n which is the difference between these 2 indices and you would you could write the expression as phi ii of m, m equals to minus infinity to infinity and this is the index of n, because and that is how you going to get the expression right.

So, this would be a typical expansion of phi vv of t plus tau and t. So, this appears to be a bit, I mean comer sum, but if you replace the terms you would come across the expression which looks in this form, right. We are simply expanding the particular expressions. So, if you look at this expression down below of course, we have this expression to evaluate.

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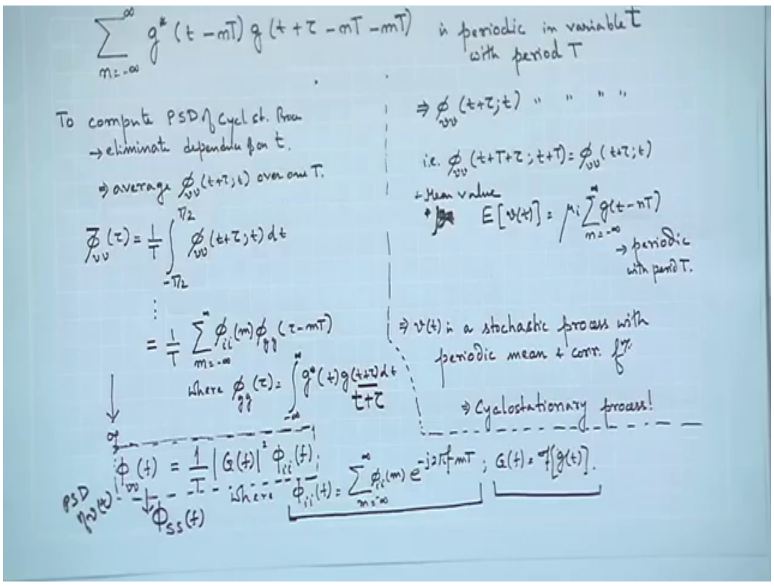

So, we will stop for a while and we will continue to evaluate first this expression. Once we have done with this expression then we may take a look at what is the influence of phi ii of m. So, as a first step let us look at the second part in this summation n equals to minus infinity to infinity and this particular sequence. G t is the pulse shape. Pulse shape means it is it would be the rectangular pulse shape which keeps occurring with time.

It could be the raised cosine pulse shape we had given you the raised cosine pulse shape. So, this keeps on repeating at intervals of symbol duration which is capital T. So, and hence, I mean from infinitive gases you could clearly see that it is periodic or you could also prove it based on the expression that you have of g t. So, you have seen different gts you seen half, half sinusoids, you seen raised cosine, you have seen rectangular.

So, if you would try to put the value of T equals to small t equals to small t plus some k times capital T you would find that all these g of t s are equal to each other. And then you could easily see that they are periodic. So, if they are periodic; that means, you would also claim that this which is the left hand side is also periodic with period t because it is a function of this, and that is periodic therefore this itself is periodic.

So, that mean periodic what it effectively means is that phi vv t plus tau if you add t with that and if you had t with this, this is independent of capital T; that means, after t it gets back to it is original value, right. And of course, the mean value would be if you would see v of t was equal to I n g t now expectation of I n was mu, I we had already pointed out and you have this. And again this thing is periodic with t. So, that clearly shows that this thing is periodic, this is the mean is also periodic with t right.

So, what we say is that v t is a stochastic process with a periodic mean and a correlation function, right. Which is also period. So, we have seen the correlation function is periodic the mean is also periodic now. So, what we have is a cyclostationary process right. So, since we have cyclostationary process what we need to do is, in order to calculate the power spectral density we have to eliminate the dependence on t and therefore, you would like to average this expression over one time interval right.

So, we continue to do that. So, the average value of the auto correlation function would be integrate from minus t by 2 plus t by 2 that is one capital T and this expression, right; that is whatever is expression that comes out to be. So, if you follow a few steps and if you would expand whatever expression we had there we would have, it is 1 upon t and phi ii of m, would remain phi ii of m and you are going to get phi gg which is the auto correlation of the pulse g tau minus m times capital T.

So, this is no longer a function of t because t has been integrated out from this expression it is depended on tau, it is depended on tau. And of course, phi gg that is the auto correlation of the pulse is, given by this particular expression. Which is phi gg of tau is minus infinity to infinity g conjugate t g of t plus tau. So, this expression is t plus tau dt, right. And if you are interested in calculating the spectrum, what we have to do is you take the Fourier transform of the auto correlation of the average auto correlation function averaged over one time interval t and you going to get 1 over t g of f.

So, where g of f is the Fourier transform of this, right. And phi ii of f which is the Fourier transform of the phi ii of m. So, phi ii f you would write it as the expression as given here and g of f is the Fourier transform of g t. So, this would be the typical structure of the spectrum of the low pass equivalent stochastic process or the PSD of the low pass equivalent signal. Now the signal we call it as a stochastic process because it is being modulated by the information bearing signal. Now what we get from this particular expression that we have over here is something very important to note is that the PSD that is the power spectral density of v of t that is what we have. Is dependent on the power spectral density or the frequency characteristics of the pulse shape and the frequency characteristics of the information bearing signal right.

So, we have 2 different parts to it. And what we can think from this is if we have to control the characteristics of this because this influences the PSD of phi of s; that means, from this you could go to phi ss of f through the relationships which we had shown before. You can clearly see that you have a choice to work with the pulse shape which would result in certain spectral characteristics as well as you have a choice to work with the information bearing signal, try to do something with that, So that you get a desired characteristics.

So, we would like to conclude this particular lecture here and like to summarize that where we arrived at the end of this particular discussion is the relationship which connects the power spectral density of the low pass equivalent signal to that of the pulse shape and the information bearing signal which in turn means that review have to control the characteristics of the outgoing signal, I can either control the pulse shape or the spectral characteristics of the pulse, or I can do something with the information bearing signal.

Thank you.