Modern Digital Communication Techniques
Prof. Suvra Sekhar Das
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Lecture - 36
With Memory Modulation (contd.)

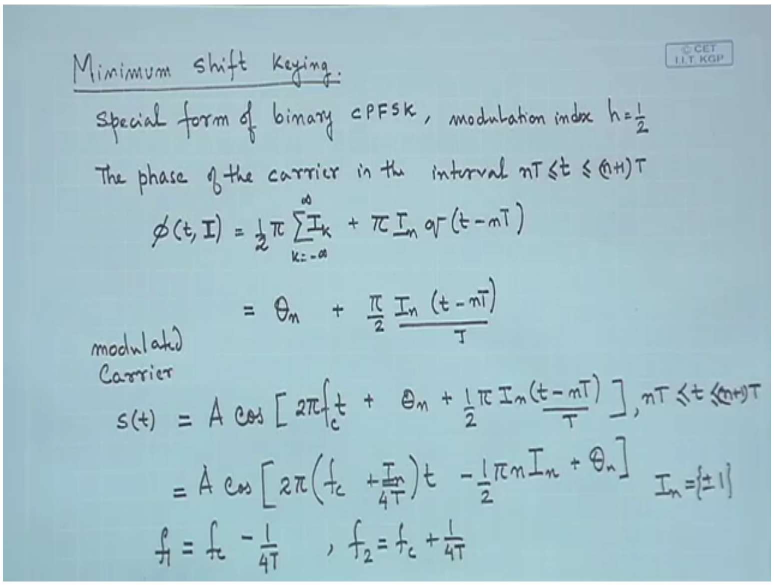
Welcome to the lectures on Modern Digital Communication Techniques. So, just to summarize: till now we have covered modulations with without memory, and then we have moved into modulations with memory. So, we have considered 2 kinds of modulations one is without memory and one is with memory. And when we did with memory modulations we have established that why would you call it with memory modulations just to quickly or briefly recapture what we have d one. We started with the FSK where we said that you could select any one of the frequencies, and then we said you could do it by just selecting one of the oscillators, but that would cause an abrupt switch from one frequency to another and this would give rise to a huge expectral side low.

So, to reduce that the other way you could do it is you could think of doing a frequency modulation using a digital wave form input to the frequency modulator. So for that, what we did is we took the data sequence and converted it into a PAM signal and that PAM signal was used to frequency modulate the single oscillator. And then we found the expression looked that expression had the typical expression of a frequency modulator which has integration in the frequency term. So, when we integrate over all possible previous symbols; that clearly mean that you want to remember you want to accumulate and therefore, you want to remember the previous phases. And therefore, we have with memory.

Then we did work around with the expression of the phase, and when we restructured in and try to represented in a new form without changing anything just trying to put the terms in grouping them accordingly what we found is that the expression of the phase looked like the expression of a continuous phase modulated signal which is a more generic form and there we had defined a few terms and we had the modulation index. And we had the accumulated memory term in the CPM we said that each symbol could have a different modulation index whereas, when we kept h is constant we had a cp FSK signal.

So, we continue on our discussion on the same on the similar lines and today what we have to discuss right.

(Refer Slide Time: 02:50)



Now, would be one of the special forms of cp FSK which is the minimum shift keying. So, minimum shift keying it is a special form of binary continuous phase frequency shift keying where you have the modulation index h set equals to half. So, this is the baseline that we have with us. And we will proceed with this and the phase we have been always discussing the phase of the signal phase of the carrier is what we are interested in the interval t less than or equal to n plus 1 t and n T. That means, t in this interval would be equal to phi of t and all accumulation would be half pi k equals to minus infinity to infinity I k.

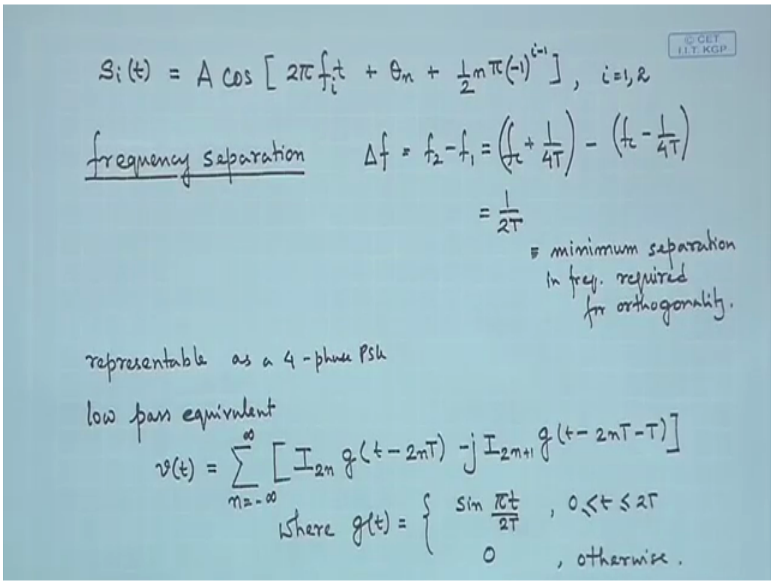
these are the PAM signals plus pi I n q of t minus nt and we have defined all these terms in the earlier lecture and we could write this as theta n which is accumulation of the memory we of course, put h equals to half that is why you do not see f h anywhere and h is equal to 2 fd. If you would remember so that means 2 fd is equal to half; that means f 2 fdt. So, fdt you could easily get it as 1 upon 4. So, that would work out to this expression. And then you have the remaining term as pi upon 2 I n T minus n T upon t, because you would remember qt was t by 2 t in this interval. So, this you could easily convert to t minus n T upon 2 t. So, this 2 t comes from q of T.

So, once we have this then you could write the carrier modulated or the modulated carrier that is S of t that is the band pass signal which you have been always writing as S of t as a cos 2 pi fct plus. Now we have this whole phase term plus theta n plus half of pi I n T minus n T upon t that is the expression directly from here. And of course, in the interval of t lying between n T to n plus 1 t and this you could rewrite as a cause. So, we can collect the terms we have an fct term we have int terms. So, you have a pi over here. So, we would rather take 2 pi and then fci would take from here and the remaining term from this.

So, since I have taken 2. So, I should have 1 upon 4 T of course, is here pi of course, has gone out I n should remain. So, you could write I n and this t and this t which is common you get over here. The next term that you would write is this particular term along with this. That means, minus because of this minus sign half pi t and t would cancel out n I n plus now you have the theta term left. So, you have theta, and of course valid for this interval and in this case we would take I n to take the values of plus minus 1 because we are choosing between 2 levels we are talking about binary CPFSK.

So, moving into this expression what we can see is that this term we could view it rather as there are 2 different carriers. So, we could write that f 1 is equal to fc plus. So, this I n since it can take plus and minus values you could take it as 1 by 4 T and you could also take f 2 another value with a plus sign fc plus 1 upon 4 T. So, if you look at it it appears as if it can take 2 different values of frequency f 1 and f 2.

(Refer Slide Time: 08:37)



So, moving forward with this expression that we have just generated what we could write is that we have 2 signals we could write as I of t is equal to a cos 2 pi f I could take these values t plus we have theta n from here which we keep as theta n plus we have 1 upon 2 n from here pi and we have minus 1 to the power of I minus 1 for I equals to 1 and I equals to 2. So, if I equals to 1 you have a cos 2 pi f 1 t. That means, there is a minus over here and n pi minus.

So, when I equals to 1 you take the minus value and you have the minus value over here this is also minus when I equals to 2 you have 2 pi f 2 t f 2 t is this value which is because of I n to be plus. So, it is 2 minus 1. So, you could reverse it actually you could say that f. So, this could become a minus and hence this one would match with the expression that you have over here.

. So, moving further what we have is we. So, we have this 2 frequency values and these 2 symbols that are with us and then we could see an interesting feature of this is that the frequency separation. So, the frequency separation that we have is delta f which is equal to f 2 minus f 1 which you could easily calculate as fc plus 1 upon 4 T which is f 2 minus fc minus 1 upon 4 T. So, fc and fc would cancel out you would have 1 upon 4 T plus 1 upon 4 T which is equal to 1 upon 2 t. So, this would remind you that this is the minimum separation in frequency required for orthogonality.

Now if you would make the separation smaller than this you would not be able to make it orthogonal. So, what we see is that by choosing the value of h which is half and by choosing in as plus and minus 1 you could generate a binary cp FSK signal, because we have used the same phase terms only in the phase terms we have replaced the values of h with half and we have taken in to be plus minus 1. Accordingly, we have got the expression that we just looked at and by calculating or by working with the expressions we could write it in terms of 2 different frequencies as if there are 2 different signals S 1 and S 2.

So, this would remind us that we are supposed to select the waveform based on the input bit stream that is the job of the modulator. So, we have to select 2 different waveforms and these 2 waveforms that we have over here are with 2 different frequencies one is fc plus 1 upon 4 T the other is fc minus 1 upon 4 T. And of course, there is some memory term associated with that and then when we look at the frequency separation what we find it is 1 upon 2 t. So, this is the minimum separation that we have. And therefore, this kind of a signal is also known as MSK signal that is minimum shift keying signal.

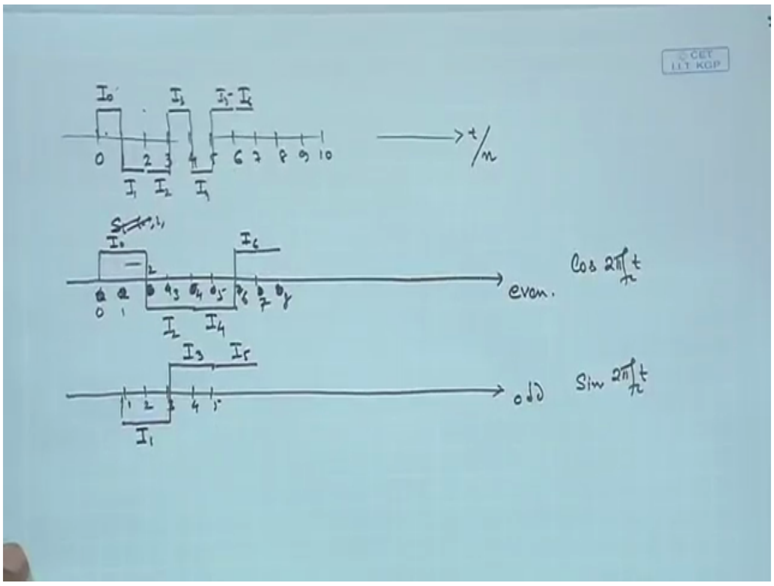
So, if we look at the expression that we have just had over here and is the S T and this is the si of t that we have. So, we could also represent this particular signal that is with us in a simpler form. So, we are going to interested and we are going to write that particular expression and now what we are going to do is write it in the form which is representable as 4 phase PSK that would make things easier.

So, the low pass equivalent the low pass equivalent that is what we are always interested in vt you could also write that expression that we had given there in a form which is sum of n equals to minus infinity to infinity I 2 n will explain what does this mean g of t minus 2 and t minus j i 2 n plus 1 g of t minus 2 nt minus t where gt is equal to sin of pi t upon 2 t for t in the interval 0 to t and it is 0 otherwise.

So, this is the typical expression you could write it in this form now we have I 2 n and I 2 n plus 1 and we have some g associated with that. So, would like to explain briefly what is the consequence or what is the meaning of writing it in this form. So, what we have with us in the previous expression we had I n if you would see, but now we have I to n and I to n plus 1.

So, the reason of writing it in this particular way is that we are taking even sequence of symbols here and we are taking odd sequence of symbols over here.

(Refer Slide Time: 15:19)



So; that means, if this is my time line and these are the values of n that you would get. So, let us say this is t oblique n that I put 0 1 2 3 4 5 and so on right. So, then we are going to get sum value of I n because of and we are having 2 different values.

So, let us say that we have some values going like this right. So, this would be I 0 this would be I one this would be I 2, I 3, I 4, I 5 and I 6. So, what it says is that if I had to send this signal I would send this through selecting sums waveform in this interval and selecting another waveform in this interval selecting another waveform in this interval. And each wave each interval I would select one of the S i is I is equal to 1 or 2 the other thing that we could think of doing is I could break it into 2 streams in one stream I would put the evens and I would stretch it to double the duration this is I 0.

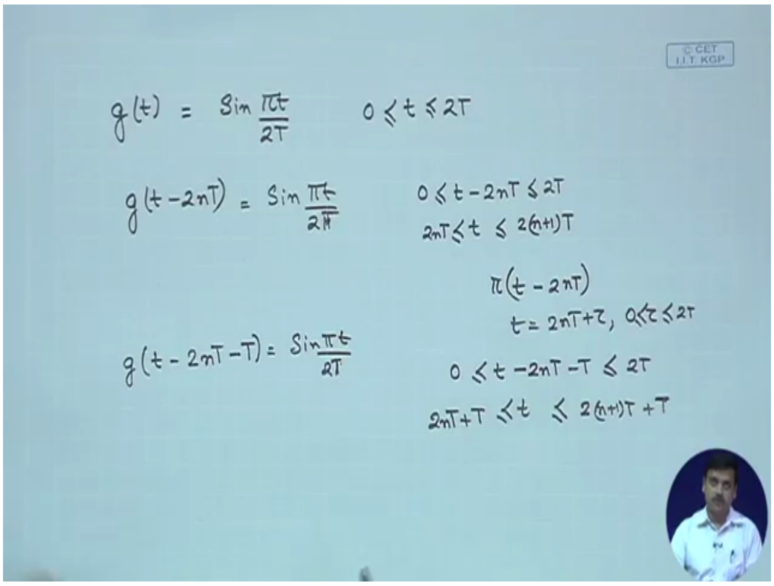
And then at I 2 I am going to get at the n equals to 2 I am going to get I 2 I would stretch it for double the duration I 2. Then again at 2 I would get this I would this I is 0 I would go there I would take I 2. Again I am going to get I 4 and I am going to stretch it for this to duration I 4 I going to stretch it here then I am going to get I 6 and I am going to stretch it there. So, this point is 1 2 3 4 I 2 this is I 4 this is I 6.

So, let me see if I have made a tiny mistakes somewhere yes. So, this is 0 this is 1, this is 2, 3, there is some numbering offset that I got into right. So, this is the right numbering and I would have another stream. So, this I would call the even sequence and this I would call the odd sequence. So, here what I am going to do is I 1 is starting at the index one I am going to take I one and its stretching till this point. So, this is 1 2 3 4 5 and at 3 I am getting I 3. So, I choose I 3 and it continues for I 5 at I 5 at 5 I get I 5.

So, here I get 5. So, here what you see is this is I 1 this is I 3 this is I 5 and I would modulate a cosine with this and I would modulate a sign with this right. So, if I am going to do this in that case each of the symbol duration would be becoming twice this period of t this also is twice, but in a 2 t interval I am sending 2 I values overall one along the I channel 1 along the q channel.

So, overall my bit rate has not been hampered between these 2 and when I write it in this form this modulates the cosine carrier this modulates the sin carrier. So, that is what we have. So, if we would now look at what is gt minus 2 t things would be becoming bit clearer now.

(Refer Slide Time: 19:37)

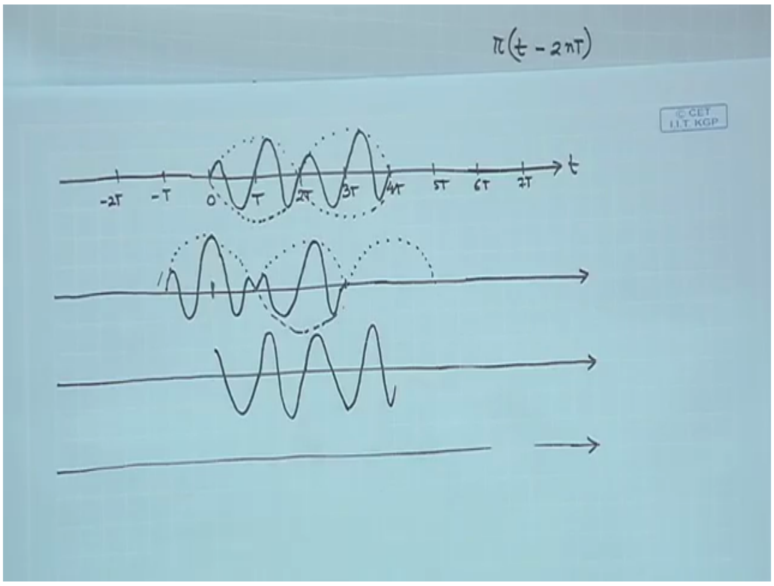


So, if we would look at the expression of gt; gt was mentioned as sin of pi t upon 2 t for the interval t lying between 0 and 2 t . So, now, we have the expression gt minus 2 and t and gt minus 2 and t minus t. So, we would like to look at those expressions so; that means, if we would say g of t minus 2 and t. So, this is equal to this function for the time t minus 2 nt less than or equal to 2 t 0. So, you could get t lying between 2 and t and 2 n plus 1 into t in this interval this function is equal to this the other form you could have d one is you could have substituted t as t minus 2 and t over here and you would know that it is valid for this interval and this would again turn out to be the same thing.

For instance, I could say t minus 2 n t. However, I would put t is equals to 2 and t plus tau tau lying in the interval to t 0. So, in that case the moment I put over here 2 n T 2 n T cancels out I have pi of tau and that is exactly what we have and it is valid for this interval of time if we would see the other one g of t minus 2 and t minus t that is the expression that we have here for the I component for the for the q component. So, here it was the I component cosine modulator this was the sign modulator so, that is what we have over here and if we examine this term. So, for t minus 2 n T minus t in the interval this; what we get is t greater than this and t greater than 2 n plus 1 t plus t.

So, in a similar way if you would like to try putting this whole value over here you would find this turning out to assign pi of tau in the range of tau here. So, we could write this is also equal to sin pi t by 2 t for t in this interval right or in this case tau in the range of 0 to 2 t right. So, these are the 2 things that we have and using these 2 we could now see how would the waveform look like.

(Refer Slide Time: 22:34)



So, if we would have time axis over here and we could mark let us say 0 this time I should draw it a little bit bigger and so on. So, I need to draw quite a few timelines. So, that it helps us this is another timeline is another timeline there is another timeline. So, what we would find is from this expression that we have is g of t is equal to sin pi t for this interval.

That means, if I would put n equals to 0; that means, t from 0 to 2 t right. So, I am going to get for 0 to 2 t sin pi t right if I put n equals to 1. So, I will get 2 t 2 3 2 this is 1 plus 1 2 4 t. That means, from here to here I am going to get a sign right. So, pi of small t upon capital 2 T. So, when [small t equals to 2 t you get sin of pi which is 0 here again and t equals to small t equals to capital T you have sin of pi by 2 which is equal to on1e which is reflected here the other time interval that we have depicted here.

So, if we would put n equals to 0 then we are going to get t to 3 t. So, n equals to 0 means t to 2 multiplied by t it is 2 t plus t that is 3 t. That means, from t to 3 t and again in a similar fashion you would find when t equals 2 t in this expression it gets some maximum value. So, what we see is that the; I phase or the in phase signal gets modulated by gt. So, now, what we have a slightly different situation we have the situation where gt is not equal to 1 upon 2 t earlier when we did FSK we said gt is equal to 1 upon 2 t, but in this situation we are trying to write it in a different form. And we are seeing that this form of expression could have the meaning that is what we are writing over here.

So, here gt given by this term yields a a picture which would be like for the item we are going to get a pulse shape which is sinusoidal in nature starting from 0 to t 2 t 2 4 T and so on. And the other one you are going to get from t. So, there is an offset of t and which is reflected by this term as well and if you would combine the modulations what we could get is that in one of the signals we could draw that you are. So, these signal you getting from here. So, this is a sinusoid that you are getting; that means, there is a carrier which gets modulated right there is a carrier that gets modulated.

So, this is getting shaped over here in this form it is getting shaped over in here in this form and of course, it will get shaped in this form as well and the second one is also going to modulate the carrier. So, what we are going to get over here is. So, you can also clearly see that at 0 there is a cosine form and here there is a sin form. So, there is a co-sinusoid and there is a sinusoid which gets modulated and if you would use this together it is some kind of a picture which is hand drawn looks like this if you generate the signal it would appear better. So, if you would merge these 2 signals the waveform that you would generate would be something typically of this order.

So, what you are seeing is that in some portion there is a higher frequency component and in some portion there is a lower frequency component right. So, as if you are getting 2 different frequencies. However, since you are doing an fm there is a continuity of phase and this has primarily been, because you are using as if there are 2 different they are using a pulse shape which is non-rectangular and each of the non-rectangular pulse shape is modulating a carrier in the phase and in the quadrature phase.And they are taking the input bit stream dividing them into the in phase and quadrature phase in order to maintain the bit rate you are stretching the even bit stream to 2 t duration you are stretching the odd to 2 t duration. So, that overall bit rate remains the same.

However, you are modulating this with the cosine and you are modulating this with a sin and together you would get a waveform which looks a bit complicated to draw by hand, but you could generate this if you are using some programming language and you would feed these functions into it you could generate the modulated waveform for MSK.

So, in summary we have got a waveform which selects between 2 frequencies and we have seen the waveform how it looks like where continuous phase condition has been maintained and primarily there are 2 different frequencies which are used to indicate what is the original information bearing signal being 0 and 1. And it could also be represented as a 4 phase PSK where you could write it in a form as if you are modulating a cosine and a sin with 2 different bit stream, but here tactically it has been handled in a way that the 2 different bit stream are as if at half the bit rate each. So, that together they get back to the same bit rate.

So, we conclude our discussion on MSK. In this particular lecture here we continue to see a few more similar things in the next lecture.

Thank you.