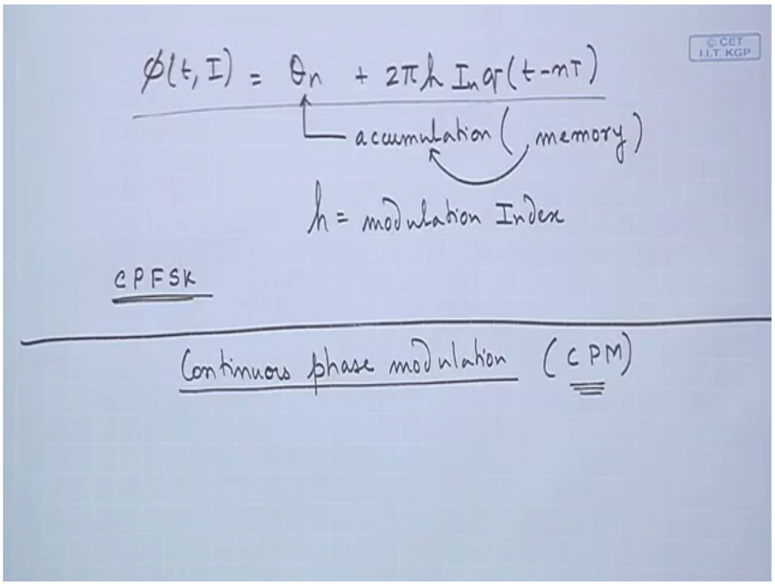
Modern Digital Communication Techniques
Prof. Suvra Sekhar Das
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Lecture - 35
With Memory Modulation (Contd.)

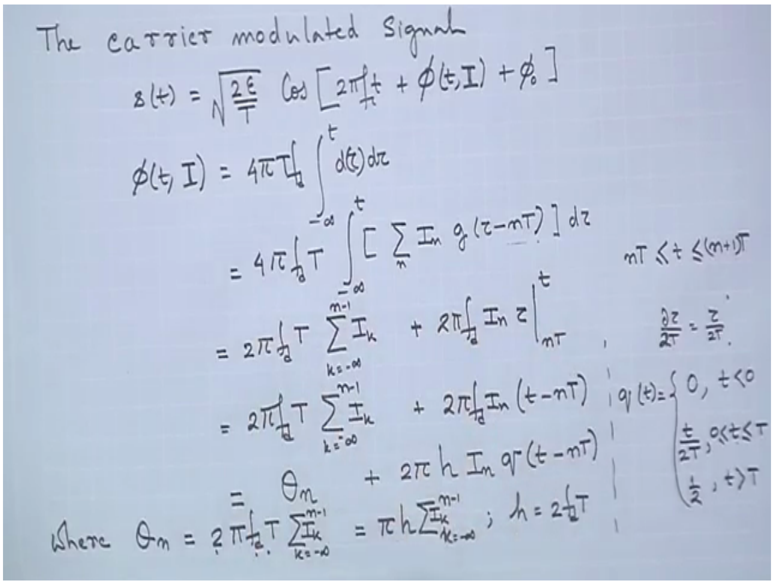
Welcome to the lectures on Modern Digital Communication Techniques. So, we have been going through the expression for the frequency shift keying, where we want to put in continuous phase and we have actually started discussing the continuous phase.

(Refer Slide Time: 00:36)



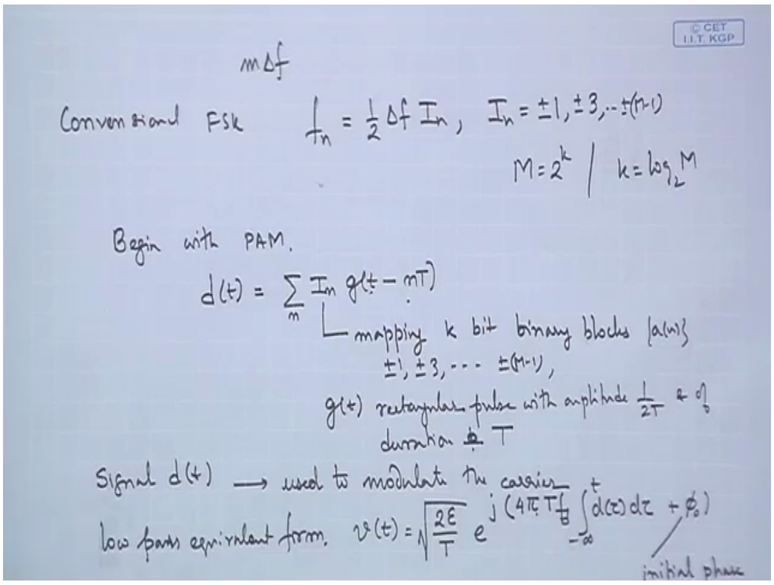
So, we have discussed continuous phase frequency shift keying where we had identified the phase in terms of a relationship where there is the accumulation and the instantaneous signal has a having some kind of modulation constant.

(Refer Slide Time: 00:54)



Where the signal is written in this form in the pass band and in the equivalent low pass we have it in this form.

(Refer Slide Time: 01:01)



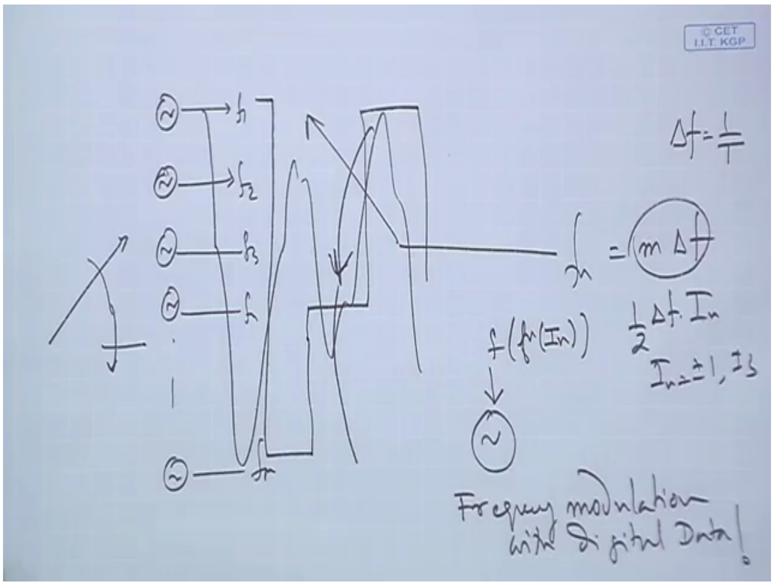
So, all what we are trying to do is instead of directly choosing the frequencies fn in this form which we had done before where you might remember we had written e to the power of j 2 pi fct plus m delta f right where m is the value that we choose m we had earlier written 1 2 3 4 and so on. And it could also be plus minus 1 plus minus 3 and so on and so forth. And here we have a half indicating that whenever it is having a 2 d kind of a thing it gets normalized by this factor.

So, these all expressions are consistent and instead of having fn directly with this we instead have integration in the term to include the phase continuity and all we have done is written it in a particular form where we have got the phase in this particular expression. The next important thing that we should move into is known as continuous phase modulation. So, it is a more general class of expressions than cp FSK and in fact, cp FSK is a special case of cp m you could say you could call this as continuous phase modulation.

So, this is a more general class and cp FSK what we have studied is a special class now you could ask why did you take this up before doing CPM the reason we did it because you are acquainted with FSK and we simply changed the way you select the frequency instead doing m multiplied by delta f we said that you now use the continuous phase property and what you do is you use the data sequence which is the pulse amplitude modulated sequence and you use the philosophy of fm.

So, what we probably have skipped in the earlier discussion is that instead of having several oscillators that send out different frequencies.

(Refer Slide Time: 03:18)



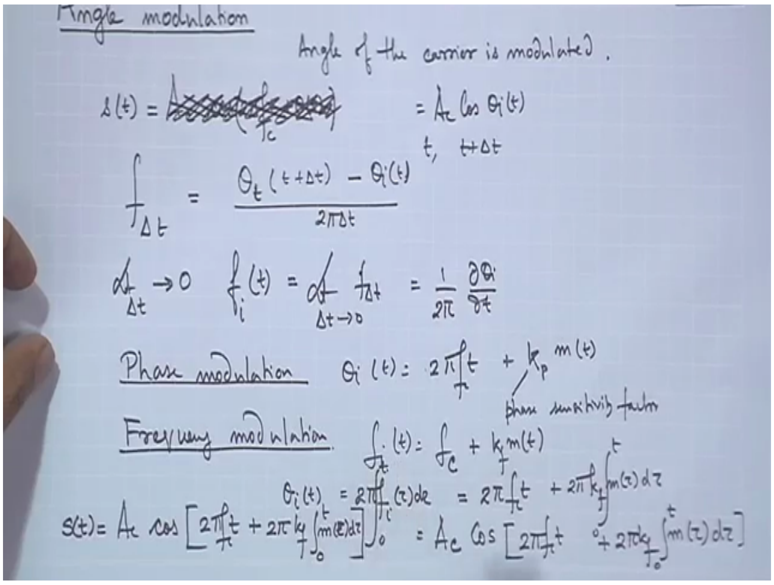
And you have to select between these or you could say that you are selecting between this these frequencies and you are sending the signal out and choosing fn which was m times delta f or we also said it as half delta f times I n where I n equals to plus minus 1 plus minus 3 and so on and so forth right what you do is you have one single oscillator where you modulate the frequency of this particular oscillator as some function of the original message sequence.

So, in order to do that so basically you had to do some frequency modulation with digital data so, that would remind us of one of our very early discussions when we were looking at comparing analog communications and digital communications. We said that in analog communications you modulate the amplitude the phase or the frequency directly as a function of the modulating signal m of t the message signal. Whereas, here when you have to go immediately to digital without knowing much of digital transmission techniques you could say that let my message signal take discrete values.

I had said this in the beginning if the message signal takes discrete values. That means, one level or another level or another level. And I use this to modulate my carrier be it the amplitude the frequency of the phase the signal I will get out of it would be looking similar to a digitally modulated signal only thing I have to ensure is when I am maintaining an amplitude level I have to maintain that for a certain duration of time which is t.

Now, this is that is exactly what we have done in the previous continuous phase frequency shift keying we have chosen one oscillator and we have done frequency modulation of that oscillator. However, the input sequence was a digital sequence right. So, we should remember that and that is what we have actually expressed over here.

(Refer Slide Time: 05:48)

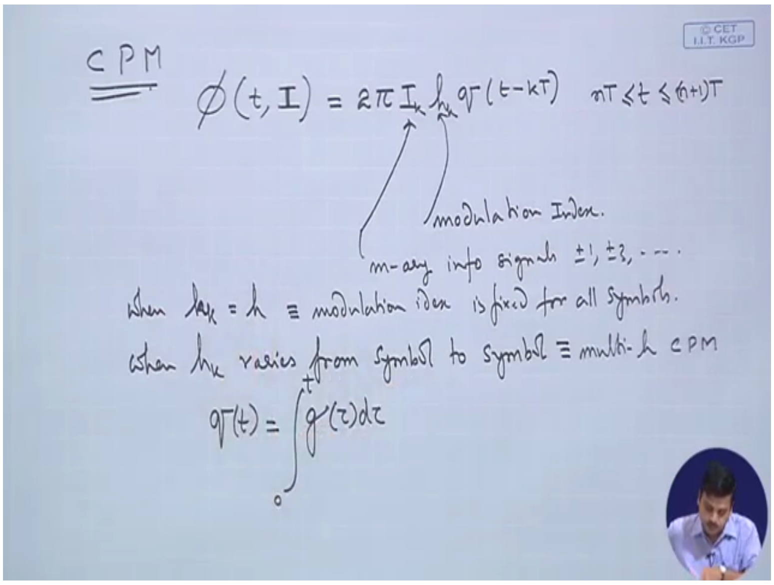


So, you are doing continuous frequency modulation and whatever expression we had over here is exactly what we have been using in the expression that we have here in this expression or you would like to go for the carrier the carrier signal which is here that is that is here where this is the phase which is in form of integration.

So, I would strongly urge you that you look at frequency modulation and compare the FSK with that of a continuous phase FSK. So, that these things are clear this might appear a bit confusing, but this is nothing but frequency modulation with digital sequence and when we did study this we came to an expression where we had a form like this and since we have a form like this we say there is accumulation and then we explain the relationship and now what we say is that this form is helpful, because what we would recognize this form is as a special case of continuous phase modulation.

So, when I write the expression of phase for a continuous phase modulated signal things will be a bit clearer or things would be justified that why we are interested in writing in this particular form.

(Refer Slide Time: 07:17)



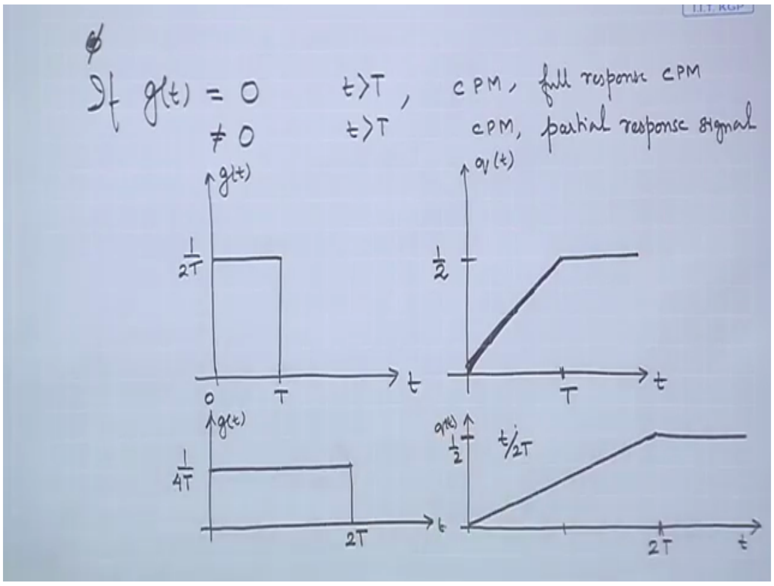
So, when we look at continuous phase modulated signal in this case the phase which is a function of time and the input data sequence set. So, when I had a bold I mean it is a function of not only the instantaneous signal, but it is also a function of several incoming sequences which is all the past history. So, that is why we have this I n over here.

The general class is 2 pi I k h k q of t minus k t this is the phase of the signal valid for the signal interval n plus 1 t 2 n t so; that means, in this instant of time which is between n plus t nt 2 n plus 1 t this is the instantaneous phase and this h k this is the modulation index. So, I would keep this expression here. So, that you can connect this and I k is the M-ARY information bearing signals which is plus minus 1 plus minus 3 and so on and so forth. And when h k is equal to h what we have is the modulation index now we are talking about the modulation index is fixed for all symbols. So, this is what we have at least over here we have it fixed for all symbols and when h k varies from symbol to symbol what we have you could say is a multi h continuous phase modulated signal.

So, what you see is that you could easily connect over here and you could break this particular signal up into the parts which have k up to n minus 1 and then you have the part n. So, this part would be valid for k equals to n and this theta which is accumulation of the past phase would be for up to n minus 1. And of course, we have q of t is equal to integrate from 0 to t g tau d tau. So, if you would put all of this in place you are going to get this expression as we had done before for continuous phase modulation continuous phase frequency shift keying for the case when h k equals to h. So, this is a mode generic form and this is a special case of CPM.

So, since we wanted to connect the fs cpf sk 2 CPM we have introduced this particular structure of the signal right.

(Refer Slide Time: 10:58)



So, you could say that this particular phase that we have phi of t sorry we have already discussed this phi t and all I need to tell you here is that in general I mean if we have this g of t g of t is the pulse shape. If you remember g of t is equal to 0 for t greater than t. So, this is generally the form that we are used to we would call a continuous phase modulated signal which is a full response CPM signal and if this is not equal to 0 for t greater than capital T we would call it a continuous phase modulated signal, but it is a partial response signal right and things would be a bit clearer if we look at some of the waveforms. So, suppose I have time t on the x axis and on the y axis I would draw gt and then I would say that for the time interval 0 to t gt looks like this that is 1 by 2 t this is exactly what we have used when we did cp FSK.

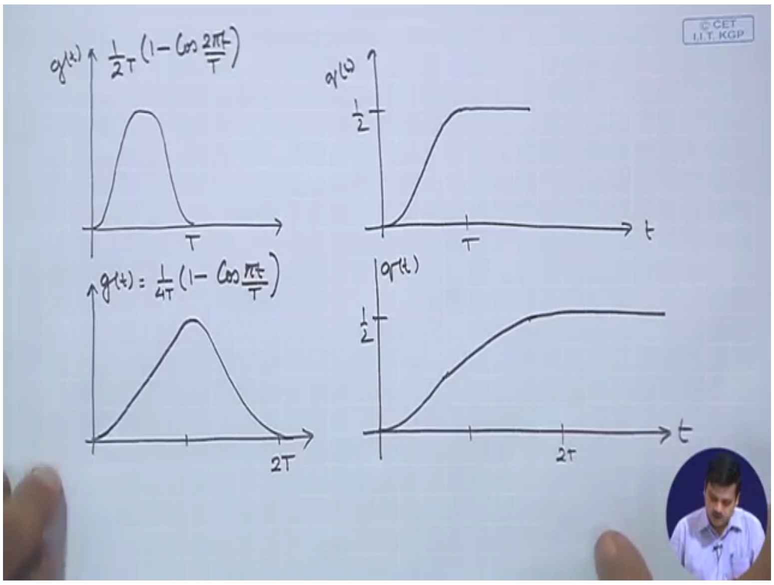
So, for this the q of t would look like up to t its value would be should start from 0. So, I get a bit thick to avoid the confusion one half and if you look at this it flows as t upon 2 t. So, when small t is equal to capital T you have the value 1 upon 2 which is over here and it remains as half rest of the time. So, this is exactly the situation that we have for continuous phase frequency shift keying and going by the particular definition that we have given just on top of this.

This kind of signaling is known as a full response signaling; that means, within the signal interval you have the full information that is present there could be some other waveforms also which would be discussed, but we could also have another waveform which could look like let this be t and you have till 2 t it is a rectangular pulse and the amplitude is 1 upon 40. In this case naturally you would extend the results that we have obtained earlier.

So, this is t this is of course, our g of t and this is of course, q of t in that case till the time. So, maybe there is a small difference in scale that is on the left hand side on the right hand side picture, but will at least have these 2 scales which are the same the 2 t you are going to have a value reaching half and then it continues. So, if you look at the difference between these 2 signals what we see is that here the information is present only up to the interval t, whereas, in this case information can be present up to an interval which is more than t up to 2 t. Now there are several reasons for doing this some of the reasons would become clear when we discuss on inter symbol interference and how to handle them, but; however, and also when we discuss about the bandwidth restrictions that are available.

So, as of now going by the definition we have over here if our gt is restricted to this interval we would call it a full response signaling qt would look like this and gt would be in this shape if when we call it partial response signaling. So, a signal is present across more than one symbol right and your qt would have a shape which looks like this.

(Refer Slide Time: 15:52)

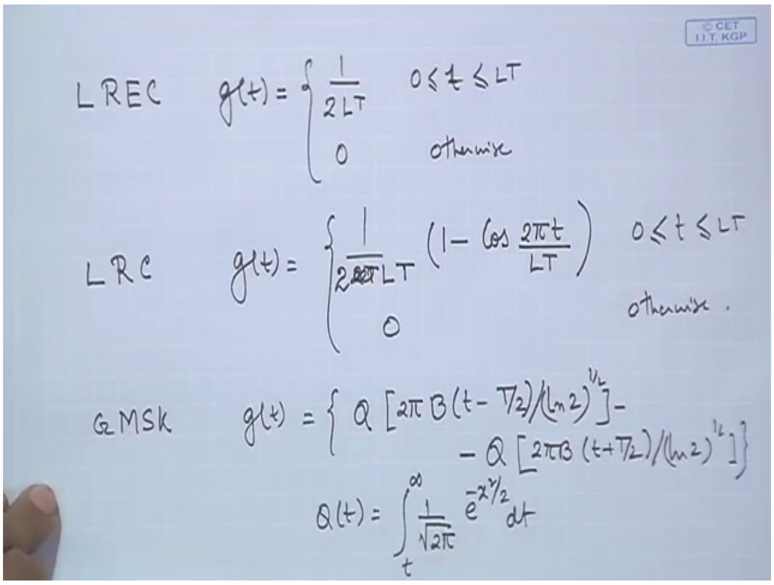


You could also have other waveforms and you could for example, you could have a waveform which would look like let us say this for the interval 0 to t and this I would call it gt and you would have it 1 by 2 t 1 minus cos 2 pi t upon t and the q of t for this case would look like a slowly rising smooth rising and then going ahead and here the value is half and this is t.

So, you can also recognize this is the full response signaling and this kind of a pulse shape is a raised cosine pulse shape will we will talk about this kind of pulse shapes of course, when we discuss inter symbol interference and you could also have a pulse shape which goes like this and it falls over here. So, this is 2 t and your gt could be 1 upon 4 t 1 minus cos pi t by t.

So, because you have a 2 t this 2 cancels out and your q of t would look like up to 2 t and the value would of course, be same as half as we did before and here the rise of qt because you doing the integration would go like this. So, this is a partial response this is a full response signaling right. So, all these waveforms that we have we have drawn our outcome of certain pulse shapes.

(Refer Slide Time: 17:56)

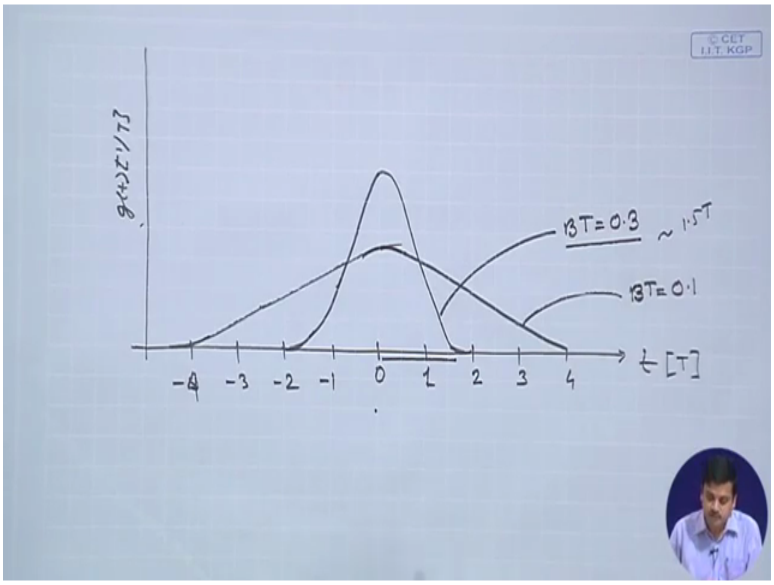


So, 1 of the pulse shape is known as the rectangular pulse shape with the parameter l and you would define g of t as 1 upon 2 l t for the interval of with a for the interval of t lying between 0 to l t and it is 0 otherwise right and the other one is known as raised cosine l indicating number of symbol intervals you would define gt as 1 upon l t 1 upon 2 l t times 1 minus of cos 2 pi t upon l t and for t less than l t less than or equal to 0 and it is 0 otherwise right. So, these are some of the pulse shapes.

So, instead of rectangular you can have a raised cosine pulse shape and what is the importance of a raised cosine pulse shape and what is the consequence of a rectangular pulse shape pulse shape you will be able to get things clearer when we discuss about the inter symbol interference at much later time from now.

So, there are some other pulse shapes also one of the other pulse shape is g m s k this looks a bit scary, but for the sake of completeness let us have it here you have it the q function of 2 pi with a b and t minus t by 2 upon l n of 2 raised to the power of half minus I would put the minus over here q function of 2 pi with a b b is a bandwidth parameter t plus t by 2 which clearly gives an offset natural logarithm of 2 raised to the power of half and where q of t is equal to t to infinity 1 upon root 2 pi e to the power of minus x square by 2 dt.

(Refer Slide Time: 20:33)



So, it is a much much complicated view shaped a pulse shape and one could get the g of t. So, this is let us say t in units of capital T and on this axis we have g of t and in units of 1 upon capital T. So, if I mark this as my central point what we are going to have is we could have a pulse shape. So, first we have to label this and we are labeling this as one.

So, remember this is t of t; that means t 2 t and 3 t and 4 t and so on minus 1 minus 2 this is simply going on the left hand side and if we have b t. So, we could draw one of the pulse shape as this kind of a pulse shape when b t the product of b and t is equal to 0.1, whereas when we have a pulse shape which is b t equals to 0.3 we are going to have pulse shape which would go like this and this particular pulse shape you could almost draw it in this; this form this one would be b t equals to 0.3.

So, in this particular expression the product of b and t is what we are looking at when we are drawing this in this particular picture. So, what we see is that as we are. So, if t is constant if t is constant as we are reducing the parameter b we are seeing that the pulse duration is increasing right this; this duration is increasing and as we are increasing this b parameter this duration is decreasing. So, there is a tradeoff between the bandwidth and the pulse duration and typically for this you could truncate the pulse up to nearly 1.5 t and that would be good enough.

So, there would be some approximations errors which are manageable. So, what this one tells you is this particular figure tells you is that clearly this kind of a pulse is I do not know how to place this, because of limited view this kind of a pulse is not a full response pulse it is a partial response pulse. And just for your information that these this kind of a pulse is very spectrally efficient and it maintains a lot of phase continuity and this is one pulse shape that is used in one of the very successful communication system that is GSM communication system where GSMK pulse shape is used for transmitting the signals because it restricts the signal from spreading a lot in the bandwidth.

So, these are some of the examples of with memory modulation. And so to summarize at this point I think it is important that we summarize the discussion because we have discussed something which is quite different than what you have studied before. So, I would d like to bring up the point that we did study FSK in the beginning and when we studied FSK we had fn that is selected frequency would be chosen in terms of multiples of some spacing.

And you could choose this delta f as 1 by t to maintain orthogonality or you could write it in this form where you have I n ranging from plus minus 1 plus minus 3 and therefore, this half it works out. So, all these are same. So, you could be switching between these oscillators. So, when you are switching between oscillators we said that you would be bringing in a lot of quick frequency changes thereby a lot of spectral outgrowth and them and we phase discontinuity in the signal.

So, to avoid that you would like to do a frequency modulation with the digital sequence so; that means, you would like to go from this to this and then to this and maybe this and so on so forth. So, you would be traveling along the different frequencies instead of jumping from this frequency to this frequency or instead of jumping between the frequencies. So, instead of doing that if you could smoothly go from one frequency to another then they will face continuity and to do that what we said is if you would recall your frequency modulation through this particular expression that we have. So, we use a similar method when we are doing this kind of modulation in case of FSK and what we end up with is the expression which looks like this over here.

So, we have this modulating signal which is similar over here and then we said that well you could do in terms of the carrier modulated signal where we have isolated the phase term from the carrier term we restructured the expression in such a way that we get form where we have structure which looks like this and at the bottom of this page you could see here theta n which is the accumulation of the previous terms. And therefore, we have stated that you have accumulated the previous information through this and which is the memory and since this is controlling the amount of information that is used for choosing the phase this is termed as the modulation index.

And then we finally, said that these class of modulation are a subclass of continuous phase modulation and the continuous phase modulation when we discussed we said that continuous phase modulation has a general expression which looks like this, where if you would choose k to be constant and you had a rectangular pulse shape that would become a cpf sk and this is a much more generic form which could be used to generate many other signals and then we defined that if your pulse shape is not restricted 2 t that is what you have over here.

So, a partial response signaling and then we moved on further and we give the example of or the numerical form of certain pulse shape and which we drew to explain how would the pulse shape look like and one particular example we took where there is a bandwidth factor associated. And finally we said that if you look at the graphs what you would find is if I keep t constant as I change the bandwidth factor there is a opposite influence on the pulse duration where one particular value of bt this 0.3 is used in practical communication systems, but what I am trying to point out through this is that these practical communication systems are an outcome of the kind of signals which you have just seen and unfortunately this is this expressions are not as straightforward as the memory less modulation, but the advantage that you get to this is there is less spreading of bandwidth and there is a lot of phase continuity that can be brought in through this modulation techniques.

So, we would like to stop this particular lecture here and in the next few lecture we would like to take up the special case which would be using in these particular structures were would like to see the case of binary continuous phase shift keying and will also look at what is the MSK; that means, the minimum shift keying and something known as the offset QAM or the staggered QAM which would be also quite interesting to see that how these kind of structures that we have or the expressions that we have developed would map to what kind of waveforms and what are the advantages of them in our particular study.

Thank you.