Modern Digital Communication Techniques

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Lecture - 34
With Memory Modulation (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. So, far we have covered modulation without memory initially, and that basically covers most of the things that we are usually doing in common place; that means, almost in our everyday encounter we are usually talking about memory less modulation.

But however, we have introduced one very minimal form of with memory communication or modulation technique or transmission of signal technique in the previous lecture where we have stated that one of the reasons for introducing this kind of memory through the XOR operation is because it would not let a continuous stream of once to be transmitted. And one of the advantages of such a scheme is that it is going to bring in several switching between the amplitudes. And that would be advantages because you go to get several transitions and those transitions are going to help the receiver in re-synchronizing with the transmitted signal.

So, with that a basic form of with memory modulation we will now move on to other forms of with memory modulations which would be quite interesting to know. There are of course, several applications of such kind of with memory app with memory modulations, and we will slowly get motivated with what could be the reasons.

Now before we get into these details I would like to quickly get back to the memory less modulation and just do a few revision in terms of how do we calculate the bit rate in a few seconds before we move on with our with memory modulation.

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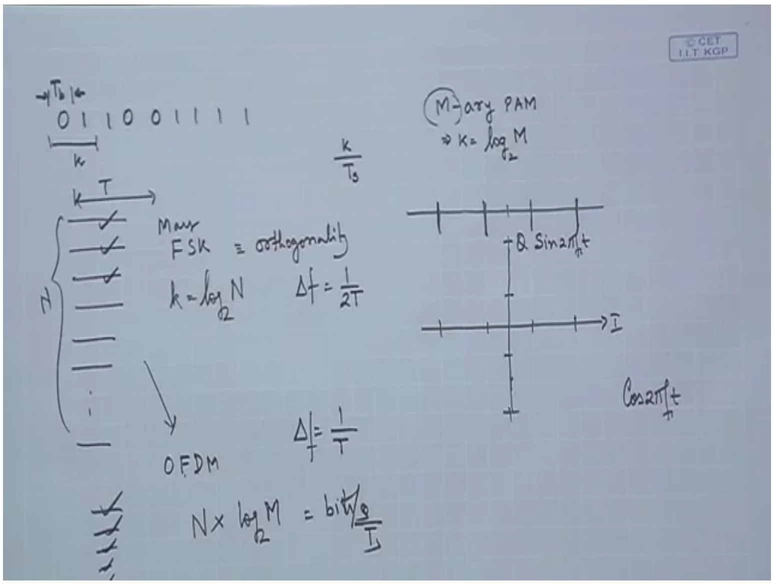

So, we have stated that let there be the bit sequence coming from the source which could be given in this pattern and definitely this is the bit interval. So, these are the bit interval and suppose you have an M- aray PAM. So, if you would have an M- aray PAM; that means, you are interested in k equals to log base 2 of M number of bits to be taken together in order to select 1 of the M possible waveforms. So, here if you are selecting let us say. So, many bits these are this is let us say k. So, effectively you are sending k bits per symbol interval right. So, this should be very very clear and the symbol rate or the bit rate will be k upon T s bits per second

So, we had we did mention in the in one of the previous discussions that if you are getting a constant bit stream and your k increases right so; that means, your ts increases, because ts is equal to k times td that is what we said because I have to wait for all these k bits to come and then that has to be transmitted for the duration till the next k bits come. So, as ts increases your bandwidth decreases and as bandwidth decreases your numbers of bits send per hertz would increase.

So, basically your spectral efficiency increases the other way to look at it is suppose I give you a bandwidth; that means, I am giving you 1 upon T is the bit is the symbol rate of sending the symbols 1 upon T symbols per second. And as we increase the number of bits basically as we increase let us say from 2 level PAM where we send 1 bit per symbol and then we go to 4 level PAM there we are sending 2 bits per symbol we have definitely increased the number of bits per symbol duration. So, we have increased the bit rate.

So, just to remind you that as we increase M we are increasing the bit rate of the transmitted signal when we go to QAM we are using the same bit duration and you could use the q axis to send additional bits because they are orthogonal to each other the I channel and the q channel. And we did mention that if you are using cos 2 pi fct as the carrier in the I channel you would use sin 2 pi fct in the q channel. And since they are orthogonal which each other they will not interfere and therefore, you could send twice the number of bits compared to a PAM.

However, we should remember that in case of PAM since ordinarily it looks like DSP modulation you could as well go for an SSB modulation that is single sideband modulation and the spectral efficiency that you would obtain by doing SSB would be the same as you would do QAM with DSB on both the I channel and q channel. And then we should look at just to remind you about the FSK in FSK we had N possible frequencies to choose from. So, if this was n. So, you would require k equals to log base 2 of N number of bits to select any one of these signals then we moved on from this to OFDM.

So, this was for FSK M-ARY FSK and of course, we did talk about orthogonality condition right and then we went to take this orthogonal FSK and instead of doing FSK we did frequency division multiplexing because then instead of selecting which sent across everything. That means, if these are the frequencies we would send across everything and the condition for orthogonality that we had obtained is delta f is equal to 1 by 2 T that is what we had obtained over there.

And if we are doing OFDM that me and you are doing QAM transmission your orthogonality condition would be 1 upon T delta f if you are doing QAM if you do PAM it will be 1 upon 2 T in this case you are having N different frequencies each one of them are carrying bits and each one is carrying let us say log base 2 of M which is the order of the constellation; so order of the constellation over here so, many bits per second right or bits per symbol duration ts. So these are some of the fundamental differences that we should keep in mind.

So, since we have got back to this FSK that is a bit interesting we would not get back to OFDM. In this course most probably because that requires many other special issues which we cannot cover in this course, so, will restrict our self to frequency shift keying kind of modulations at most. So, moving for the; if you would look at FSK. So, way you do FSK is you are selecting one of these frequencies. That means, there could be N different frequencies and of these N different frequencies you could be choosing any one. So that means, you could have N different oscillators and you would be switching from one oscillator to another in order to send or select the frequency to be transmitted.

So, this kind of switching causes a lot of spectral growth and we would see what can be done in order to reduce this spectral growth. So, before we get into those details would like to revise a little bit of phase or angle modulation in general which would form the basis of our discussion when we go to this kind of continuous phase frequency shift keying which is the next important thing in our agenda.

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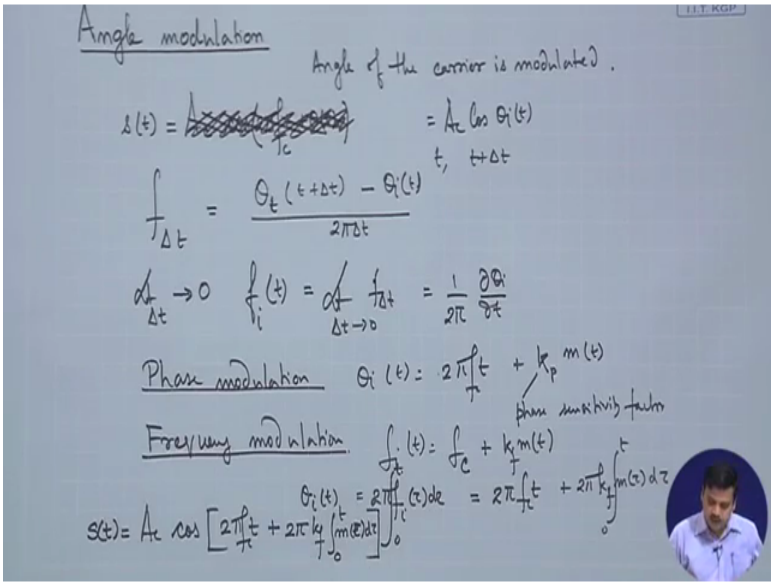

So, if we would look at angle modulation just to revisit what you studied earlier. So, that it reinforces or reminds our knowledge.

So, in case of angle modulation we know that the angle of the carrier is modulated this you have studied earlier. In this case what we have is the signal s of T you could simply write it as some amplitude which is constant and cos 2 pi fc or 2 pi fct plus theta of T this is the typical way as you would write or you could also say the instantaneous phase f ac cos theta I T you could also say this where I is the instantaneous phase of the particular signal and we could say that when this theta I T changes between the time interval T and

T plus delta T if in this interval this phase is this whole thing is the instantaneous phase then we could say that f delta t. That means, the frequency resultant because of the change in phase in this could be denoted as theta T in T plus delta T minus theta I of T upon 2 pi delta T because 2 pi delta T times f in that interval of time is basically the change in phase that occurs.

Now, if you let delta T tends to 0; that means, over a very very short interval of time what you get is fi of T or the instantaneous frequency which you could write it as limit delta T tends to 0 f delta T and which in short you could write it as 1 upon 2 pi d theta I dt. That means, the rate of change of this phase is an indication of the frequency that is being sent across.

Now this is also an indication that if you are changing if phase of the oscillator at a very fast rate then you have a huge amount of spectral growth. Of course, you have studied that in such a modulation since amplitudes its constant you have a lot of resilience from noise compared to a simple amplitude modulation along with this you have also studied frequency modulation well. Of course, before we go into frequency modulation let us summarize phase modulation.

So, we identified or generally we write in phase modulation the theta I T that we have reached over here as 2 pi fct that is the carrier component plus there is some phase sensitivity along with M t. So, this is the phase sensitivity factor right and this is your message signal this is what you have studied before now this change is primarily depend upon this particular rate of change of the signal.

So, this is the general form of angle modulation we would rather skip this we would rather write this as the general form of angle modulation. So, that we discriminate phase modulation from frequency modulation right in case of frequency modulation we would say that fi of t. That means, the instantaneous phase the instantaneous frequency is basically the frequency plus some kf times mt which is again the frequency sensitivity factor and. So, also known by other names in this case the theta I T the instantaneous phase that we have over there could be written as 2 pi fi 2 pi integral 0 to T fi of tau d tau.

So, in case of fi you would replace this term and you would get 2 pi fct because fc is constant d tau integrated from 0 to T would lead to T plus 2 pi kf integral 0 to T M tau d tau. So, this is the typical expression of the phase that you would have and your signal s of T you would write this as some constant amplitude times cosine of 2 pi fct plus 2 pi the frequency sensitivity factor 0 to T M tau d tau right. So, this is what you are going to get and in case of phase modulation this integration is not there. So, that is the typical expression of things that we use.

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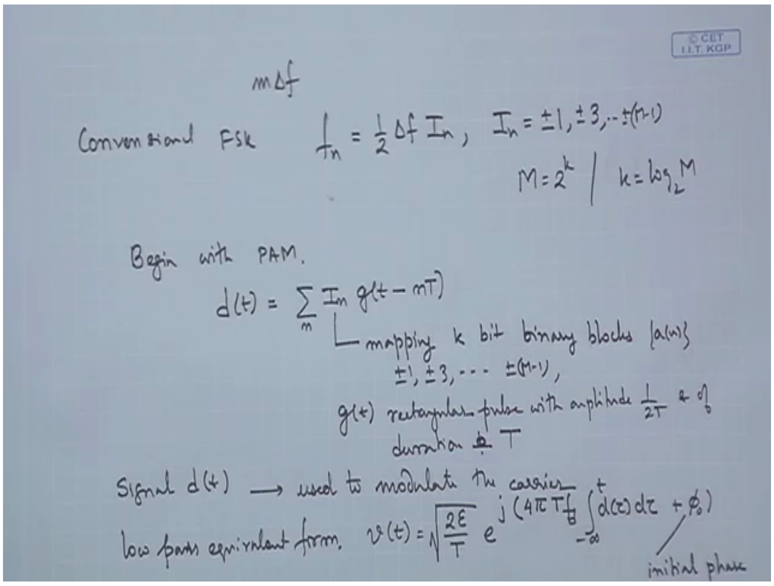

Now, in case of frequency shift keying what you had seen is that you are using M delta f to choose the frequency right. So, basically you are switching between frequencies at a very very fast rate the instant you have to change from one oscillator to another. So, we had in case of the conventional FSK what we have is the fn the in a frequency is equal to 1 half delta f I N where I N is the modulating signal you would have it at plus minus 1 or plus minus 3 or up to plus minus M minus 1. So, basically depending upon the value of I N you would jump from one frequency to another and that jump would remain for the symbol duration T right and this causes a lot of spectral re-growth and which is not very desirable.

So, we are in this case you basically let us say you have M equals to 2 to the power of k oscillators and there simply because k equals to log base 2 of M that is what we had said right. So, if is this is your possibilities then you would choose the different frequencies accordingly. So, instead we would have to find a mechanism by which this instantaneous jump from one oscillator to another oscillator is avoided. And we could maintain some kind of a phase continuity in the signal which would help us to reduce the sudden jump and phase discontinuity and thereby reducing the spectral outgrowth. So, to do this what you can begin is with the PAM signal. So, you begin with the PAM signal.

So, to say where you could write the data bearing signal is equal to sum of I i N which is this is amplitudes into g of T minus nt. So, this is what is your data and this is obtained by mapping mapping the k bit binary blocks from the input sequence a N let there be an input binary sequence a N into this such that the levels that you get are plus minus 1 plus minus 3 up to plus minus M minus 1 typically. So, this is how you get I N and gt we have indicated earlier.

So, in this particular case let us take this to be rectangular pulse with amplitude 1 upon 2 T and of duration one duration of T seconds. So, this is what we have as our basic expression to begin with this is the baseband expression. So, if you would do PAM you would simply multiply this with e to the power of j 2 pi fct take the real part. And then you have this times the cosine 2 pi fct that is the a N cos 2 pi fct where a N is basically this value and which is taken as 2 N minus 1 minus capital M multiplied by d. So, here we have taken you can assume that d to be equal to 1.

So, now the signal dt that you have. So, this is the signal dt this is used to modulate the carrier. So, again the same philosophy as you have studied earlier we are not changing much from what you have studied earlier in typical analog communication. So, you have the message signal which modulates the carrier, but here the difference is the message signal is not necessarily analog it could be anything.

And then we finally, have a source of bits coming from somewhere we choose this bits map them to some. Let us say PAM and then we considered this as the modulating signal which is not exactly the original message bearing signal it is some transformation of the message bearing signal. So, if you use to modulate the carrier then you could write the low pass equivalent form of the signal as v of t.

So, we are not writing the pass band immediately this is equal to 2 times root over 2 e upon T where e is the energy of the signal e to the power of j 4 pi T fd this is the peak frequency deviation or the sensitivity factor integrate from minus infinity to T d of tau d tau plus phi 0 where phi 0 is the initial phase of the of the signal. So, let this be some initial phase not much of a thing to worry and you might be concerned that why we are using this for pi and this t.

So, fd is similar to kf that what we have studied and here you have G. So, we have said that gt is 1 upon 2 T and it is rectangular. So, if it is 1 upon 2 T t goes out T goes out. So, you have 2 pi fd dt and that is similar to the expression that we have just worked out over here 2 pi fct.

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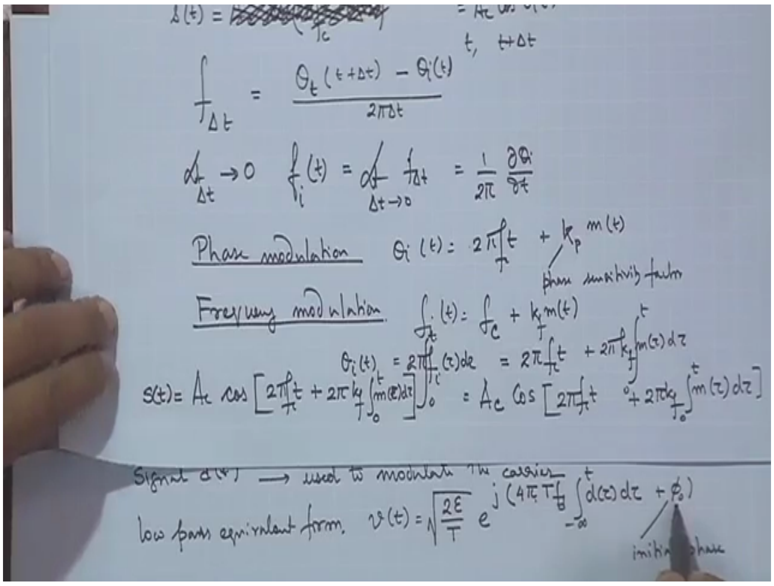

So, if we would write it clearer ac cos 2 pi fct. So, we are not talking about this part we are simply talking about the remaining part kf integral 0 to T M tau d tau. So, we are talking about this part and if you take the pass band component you have to take the real part of this if you take the real part of this; this will go inside the cosine which is here and this is some initial phase that you are left with.

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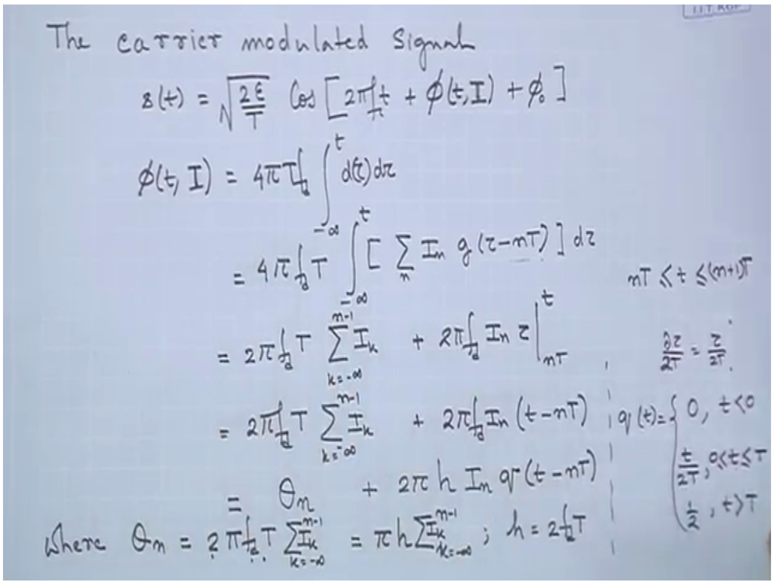

So, this is the basic representation of the signal that we have with us. So, moving down further you would have the carrier modulated signal. Now, we need the carrier modulated signal. So, again you could write looking at the above one as root over 2 e upon T cos of 2 pi fct. So, this is similar to the earlier expression that we had used. So, you would check this; this 2 pi fct and then we have the phase. And now we write the phase as a function of T and some function of the whole sequence of I plus let there be this phi naught.

So, what is phi I of T and what is phi of T and I phi of T and let us say capital I is 4 pi T fd integrate minus infinity to T d of tau d tau. So, this expression is the phase of the signal. So, we have captured it in this particular expression and you could write this particular expression as now we are going to replace d of tau we had written d of tau over here. So, we would use this d of tau in our expression this is 4 pi fd T. And then you have the integral minus infinity to T we are going to replace this with this expression that we had written earlier.

So, you are going to have summation I N summation over N g of tau minus nt d tau. So, look at this expression. So, this T has been replaced by tau because we have d of tau we had d of T d of tau g of tau we exactly have that particular expression over here right. So, we have to simplify this expression. So, you could easily get that g of T has 1 upon 2 t. So, this will cancel out some of the terms over here and you could expand this particular terms this set of terms in the set that I would like to take the time which is up to nt. And then I would like to take the term up to N plus 1 T. Because, now I am interested in this interval over which I am sending. So, I will split this expression up into a set which has N less than n; that means I will have up to N minus 1 and the set of n. So, we could write this expression as 2 pi fd T and summation, because once you have this integrate integrator operating this is 1 upon 2 t.

So, this will integrate to 1 upon 2 T and 1 upon 2 t. So, you have d of tau by 2 t. So, this would result in tau and 2 T and the moment you integrate for the interval 0 to T you have a T over there. So, this one works out you are left with I k I k k equals to minus infinity to N minus 1. So, I am taking up to N minus 1 plus now I will take the other part; that means, I have taken up to N minus 1 I will take the remaining part 2 pi ft, and then I am going to have the result I [noise], because this is the I nth part and you are going to have integration of d tau as tau from the limit nt to T.

So, up to this part your integration up to capital nt, but here it is from nt to t. So, this particular step you could work out and you would get these answers and we can add a few more steps anyway. So, this one will work out to be this part remains as it is there is no change plus 2 pi ft I N and this one definitely is T minus nt. So, this is the expression that you will expand to when you do this; this part I have done it almost orally, but if you work out it will turn out to be appropriately here.

So, this one you could summarize with theta N plus 2 pi then you have a h then you have an I N and then you have a q of T minus N T you could replace you could write this in a notation form in this way where you have theta N is equal to if you look at this term things would be clear 2 pi fd T and summation k equals to minus infinity to N minus 1 I of k. So, that exactly matches with this and this you could write it as pi h sum of k equals to minus infinity to N minus 1 I k where h you would put as 2 fd t. So, if you look at this expressions pi 2 fd T is basically over here. So, you have 2 pi fd T if I replace h by fd T and the rest of it remains the same.

Now, if you go back here you again get 2 fd t. So, we need to define; what is q in order to complete this expression and match with that. So, what we will have here is q of T is defined as 0 for T less than 0. So, when you have T minus nt you put T minus nt less than 0 that would work out to be for T less than nt right; That means, we are interested in this period. So, this function is 0 for T less than nt in this particular expression and this is equal to T upon 2 T for T in the interval T to 0 right.

So, again if you would put T minus nt you would find this is equal to T upon 2 T for the period nt to capital N plus 1 into T because T minus nt you would put T minus nt over here nt goes this side so; that means, T is greater than nt and when nt goes this side you will find T is less than equal to T plus nt which is N plus 1 into T and this is equal to half for T greater than or equal to 2. So, for 0 we do not have a problem for these sections if you put 1 upon 2 T and then you would find that this whole expression is valid for this interval and when you put it over here is half it could be valid for T greater than nt.

So, that way you could write this phi of T I which is a function of T and I in a clear form as phi of T comma I could write it as theta N plus 2 pi h phi N into q of T minus nt.

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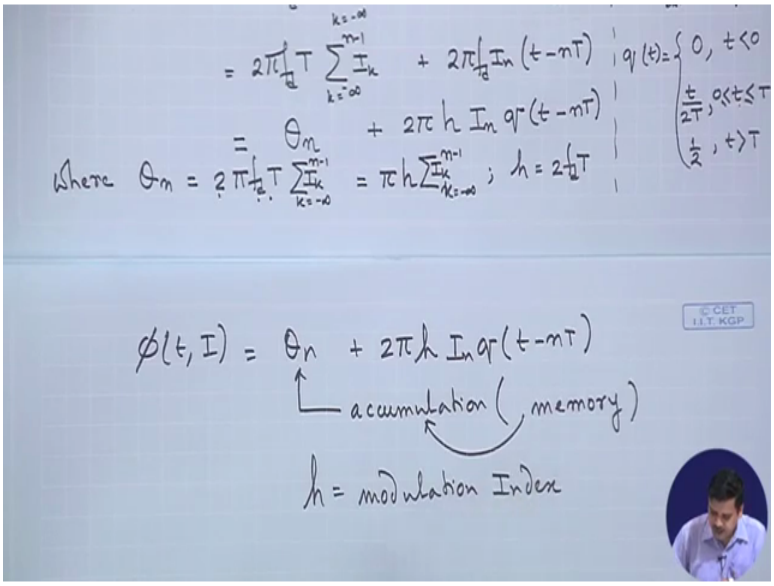

Now, why we write it in this form this would be of rotational advantage very soon where we identify this theta N as accumulation because theta N is the summation the moment we have accumulation you would call this accumulate accumulation as memory and this is phi it is the phase and this phase is present in the modulating signal. Therefore, you can clearly see that we have introduced memory in the modulation.

So, this is the accumulation up to all terms and you would also call h as the modulation index, because with h you have I N; I N is the modulating signal. So, when multiplied you are basically influencing the amount of modulating signal that would affect the phase of the signal.

So, hence it is appropriately call the modulating index. And therefore, we have been able to or we have written it in this particular form which is of our interest where we have not changed anything from the basic form they only reshuffled or we have worked around the expression show that, so that it helps us in writing some of the expressions in the next few important modules.

So, we stop this particular lecture here and we continue with this method or these derivations in the next lecture.

Thank you.