

**Modern Digital Communication Techniques**  
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**Lecture - 33**  
**With Memory Modulation**

Welcome to the lectures on modern digital communication techniques. In the previous lecture we have discussed M-ary orthogonal frequency shift keying, where we have brought out some of the important criteria for maintaining orthogonality and how would you decide, upon which particular frequency was chosen and how would you implement your receiver. And now for that case we have of course, seen that you are choosing some kind of binary PAM and all that you are doing is choosing one particular frequency.

Now, while we do that we had used the correlation function  $\rho_{km}$  and we have also used the real part of the correlation function.

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The image shows handwritten mathematical derivations and diagrams. On the left, a vertical axis is labeled  $\text{Re}(\rho_{km})$  with tick marks at  $f_1, f_2, f_3, \dots, f_m$ . To its right, a diagram shows a complex plane with a horizontal axis labeled 'I' and a vertical axis labeled 'Q'. A point is marked with a dot and a horizontal arrow pointing left, and a vertical arrow pointing down. The text in the center and right includes:

$$|\rho_{km}| = 0$$

$$|\rho_{km}| = \left| \frac{\sin \pi T(m-k) \Delta f}{\pi T(m-k) \Delta f} e^{j 2\pi f(m-k) \Delta f T} \right|$$

$$= 0 \quad \text{if } \Delta f = \frac{1}{T}, m \neq k$$

$$= 1 \quad \text{if } \Delta f = \frac{m}{T}$$

Below the equations, there is a note: "Frequency Division Multiplexing". At the bottom right, there is a crossed-out expression:  $\frac{\sin \pi(m-k) \Delta f T}{\pi(m-k) \Delta f T}$ .

So, when we use the real part of the correlation function, we are concerned only with the selection of a particular frequency and not further; however, if you note we did mention this briefly in the previous lecture. That if you are selecting this frequency, if you are selecting these frequencies you are not using all possible dimensions that might be available.

So, once we have selected one particular frequency and you could continue to use that particular frequency by choosing, by instead of making it a binary PAM you could use a QAM kind of a signalling and you could send more signals in that particular frequency.

Now, what we are trying to indicate is that let this frequency be  $f_1$  this  $f_2$   $f_3$   $f_4$  up to  $f_m$ . So, when we do this if any one particular frequency chooses a QAM signal then, we are effectively saying that you are sending more number of bits even after choosing a particular signal right.

So, the other direction that you can go towards is not only choose, but also you can send more bits on a particular chosen stream. So, if QAM has  $k_2$  bits and this requires  $k_1$  bits. So, together you can have  $k_1$  plus  $k_2$  bits communicated not only by selection, but also by further modulation.

So, when we do this since we are talking about QAM like signals you cannot just use the real part of  $\rho_{k,m}$ . Because when you say QAM the QAM signal is having an I/ component as well as it has a q component; that means, if there is a constellation point you have on the free in phase, as well as the quadrature phase that is there is a real component, as well as there is an imaginary component.

So, when we are sending this and we are interested in  $\rho_{k,m}$  what we should be interested in is when does the absolute value of  $\rho_{k,m}$  go to 0, right. Only when the absolute value of  $\rho_{k,m}$  goes to 0, then only we will be able to decipher between whether the signal is present in one of the components or not and will be further able to decode how many bits were used in choosing a particular QAM constellation.

So, if you would look at the signal the  $\rho_{k,m}$  you are getting it as  $\sin(\pi T m - k \Delta f)$  upon  $\pi T m - k \Delta f$  into  $e^{j 2 \pi m - k \Delta f T}$ . Now this particular signal would go to 0 if you have  $\Delta f$  is equal to  $1/T$  and  $m$  not equal to  $k$  and this would be equal to 1 if  $\Delta f$  is of course,  $1/T$  and  $m$  is equal to  $k$ .

So, our condition would change at this point and what we have is that the modulus of this would go to zero; that means, when you are taking the absolute value you are taking the absolute value over here. So, absolute value would mean this has absolute value of 1. So, interested in  $\sin k$ , the  $\sin k$  would go to 0 clearly by the earlier condition that we had is  $\Delta f$  is equal to  $1/T$ . Because if  $\Delta f$  is equal to  $1/T$  what you are left with

in the numerator is  $\sin \pi m \text{ minus } k$ . And in the denominator of course, you have  $\pi m \text{ minus } k$ , which is a sinc function and which goes to 0 for  $m \text{ not equal to } k$  and which is equal to 1 for  $m \text{ equal to } k$  right.

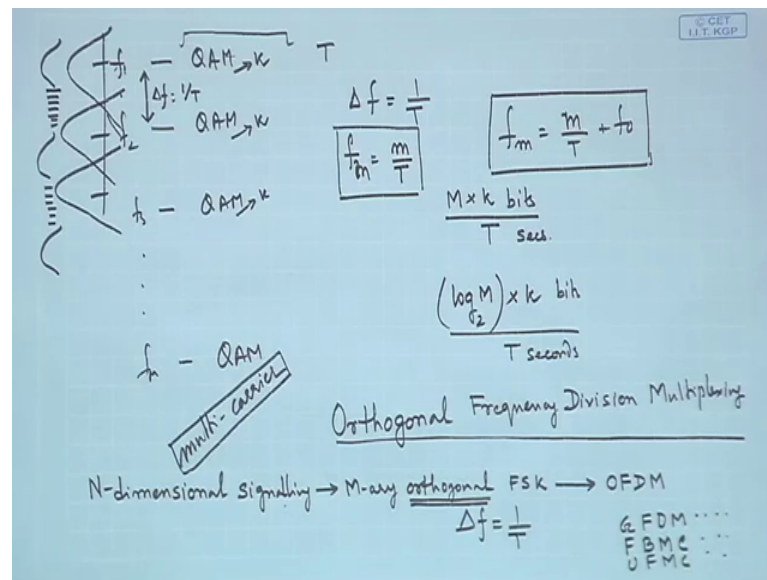
So, rather you could say that  $\Delta f$  is equal to some  $n \text{ upon } T$  right. So, for all  $n$  integers  $n \text{ not equal to } 0$  you will be getting this thing going to 0 So that means, for all frequency separations multiples of  $1 \text{ by } T$  that is what we actually have in the previous case we had  $2n \text{ upon } T$  in the earlier case for real  $\rho k$ , right. I will just strike it off for to remove confusion. So, we should remember earlier we had because we have taken the real, but now since you are taking the absolute value, this one will go to one you are left with this is the relationship that holds. So,  $\Delta f$  should be equal to  $1 \text{ by } T$ , right.

So, now since we have got this relationship we can think a bit further and we can say that we are choosing one frequency and then we are sending a few bits on one particular frequency. Now one could use this configuration and could go to one of the improved version of transmission whereby, instead of choosing this you could say that I would like to send across these frequencies simultaneously.

So, when I say simultaneously, so I am kind of doing frequency division multiplexing. Please note that, we are simultaneously sending it. So, we are multiplexing in frequency, right. And in each of the frequencies you are free to send QAM signal right. So, in each of the frequencies if you are sending PAM like signals you would be having you need a separation of  $1 \text{ by } 2 \Delta T$  and if you are sending QAM like signals you will be having  $1 \text{ by } \Delta T$ .

Let us proceed on to see what does this kind of situation leads to.

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So, what you are going to have is in this frequency you will have some QAM. So,  $f_1$  in this frequency  $f_2$  you are going to have another QAM in this, you can have another QAM signal and so on. So, what we had studied is a situation like this where there was one particular frequency and you could do QAM.

Now, we are saying that there is another frequency and you could do QAM on top of that as well. So, if this require  $k$  bits, this could also use  $k$  bits or any other possible number. So that means, in the first one you could choose let us say, 8 possible constellations in the second one you could choose 16 possible constellation, you are free to do that. Of course, you will be constrained by system and operating conditions.

So that means, you are able to send simultaneously all these bits. So, you have in other words if there are  $M$  such things you have  $M$  multiplied by  $k$  bits that you are sending per symbol duration per  $T$  seconds, right. If you would compare with what you did earlier; that means, if you chose one of the frequencies and then you send QAM you would send  $\log_2$  of  $m$  bits by selecting multiplied by  $k$  bits because of QAM, right. Every  $T$  seconds.

So, definitely what you can see is this number is greater than or equal to this number right. So, this number is greater than this number. So, by doing this you can actually send more bits per second than by doing a selection. But of course, we will see at a later time there is another advantage of doing the frequency shift keying which will be evident

when we discuss further on error probability and compare the different modulation schemes.

So, now we have mentioned that we could do this provided the  $\Delta f$  is equal to  $1/T$ . So that means,  $f_1$  is or  $f_2$  is equal to  $f_1 + 1/T$ , or you could say  $f_2$  is equal to  $f_1 + 1/T$  to avoid the 0 that is not a problem because  $2\pi f_1 t$  carry out is there. So, if you are having frequencies of this fashion or you could say  $f_m$  is equal to  $f_0 + m/T$  plus  $f_0$ ,  $f_0$  is the base frequency then it would also mean the same.

So, you could have orthogonal frequencies to transmit and we have already said that this is frequency division multiplexing and you would have the very, very famous orthogonal frequency division multiplexing scheme. Now you may have studied frequency division multiplexing schemes earlier and in frequency division multiplexing you would remember that it requires some kind of a guard band between 2 corresponding frequencies. Whereas here, you do not have a guard band. In fact, the frequencies they overlap with each other.

So, you have as frequency spectrum which looks like this and provided our time domain pulse is a rectangular pulse. So, what we have in this case is the smallest frequency separation while doing frequency division multiplexing. You can not do any better than this any smaller frequency separation while maintaining orthogonality amongst the carriers or sometimes called sub carriers. So, we will not continue the discussion on orthogonal frequency division multiplexing beyond this point, except if there are doubts we can always clarify. Because OFDM has many other details which we do not find it pertinent to discuss in this particular course. So, we just like to send across the message that in OFDM you have the standard frequency division multiplexing, where each frequency carries some message signal by means of let us say quadrature amplitude modulation. And the neighbouring frequency is selected in a way that the frequency separation is inversely proportional to or is equal to  $1/T$  upon this symbol duration, the symbol duration for the QAM.

So that means, it is very simple to remember you have any one frequency and this particular frequency is the QAM, whatever you have studied for QAM there is no problem with this. And  $T$  was a symbol duration. So, all you need to do is choose this second frequency such that this separation is equal to  $1/T$  because, at the receiver

you will be observing this whole  $T$  interval in order to decode the signal right. So, if you maintain this separation in this whole interval this frequency component and this frequency component would be orthogonal, by virtue of the proof that we have described over here.

So, in one step actually we have in a very simple way, we have covered from multi dimension signalling to orthogonal multi dimensional signal signalling which you have used to explain M-ary orthogonal FSK. And that we have exploited in discussing some of the important aspects of, one of the important aspects of OFDM.

So, the way we have travelled is N-dimensional signalling followed by M-ary orthogonal frequency shift keying. And then we have exploited whatever we have discussed to do orthogonal frequency division multiplexing, right. And what is common in all of this is orthogonal by virtue of  $\Delta f$  being equal to  $1/T$  for QAM signals. And  $1/2T$  for PM like signals. Why did we extend our discussion to OFDM? The reason is OFDM is very, very efficient because of the frequency packing, that we have briefly described over here.

More details will require much more number of lectures. So, we had like to avoid that over here. So, since we are doing orthogonal we are doing the closest packing possible  $1/T$ . And by means of this you are saving a lot of bandwidth and that is one reason why this is this has been used in many wire line modems to provide very high spectral efficiency many bits per second per hertz. Because if you had a frequency separation you would have used a frequency over here some guard band and then another frequency some guard band then another frequency. So, your efficient efficiency of using the spectrum would have been less.

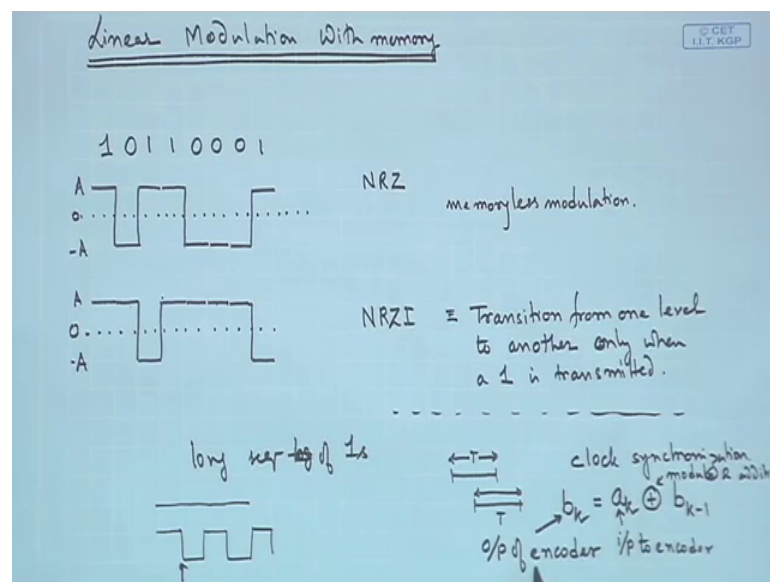
Now, after being used in wire line communication it has been extensively used in wireless communications. You may have you may be well aware of the technology known as Wi-Fi or wireless LAN. And in the modern version of wireless LAN it uses OFDM as the transmission technology. You may be also aware of the fourth generation wireless communication systems known as 4G and also available in terms of LTE or LTE advanced also known as 4G+ advanced uses OFDM as the transmission technology, including WiMAX also uses OFDM as a transmission technology. And just for the sake of discussion of information sake the 5th generation communication system are also

expected to use some minor variations small variations around OFDM. And these kind of techniques that we have just described which uses these things are also known as multi carrier. Simply because this is one of the carrier frequencies, this is one of the carriers it carries one of the QAMs, this one carries another QAM. So, these families are known as multi carrier signals.

So, OFDM is one form of multi carrier. As you are prepared with this now, you might encounter many further multi carrier techniques which have been recently explored for the fifth generation communication system. Just to name a few, there is something called generalized frequency division multiplexing, I would not go beyond just this acronym then there is a filter bank multi carrier then there is universal filtered multi carrier and so on and so forth. So, all these are called multi carrier techniques and the way these multiple carriers are placed is dependent upon the particular technique.

So, whenever they placed orthogonal you have orthogonal frequency division multiplexing otherwise, you will have a different form of frequency division multiplexing and they are known as different multi carrier techniques.

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So, we move further and we would like to briefly discuss one of the other modulation techniques that we need to cover. And this comes under the category of linear modulation with memory. So, we have talked about linear modulations without memory

Now, we are moving into one of the schemes there of course, many other schemes we will just see one of the schemes which is linear modulation and it uses memory. So, suppose we have bit sequence suppose we have a bit sequence like this. And if you are doing NRZ; that means, non return to 0 signalling that would require whenever there is a one you send a high amplitude whenever there is a 0 you send the negative amplitude. So, this is plus A this is minus A and again whenever there is a plus 1 you send plus 1 plus A whenever there is a 0 you send a minus A and so on and so forth.

So, this is the 0 level. The, this is memory less modulation clearly, right. And then there is another modulation which is very interesting which is NRZI and we would see what it is. So, what it does is whenever there is a one there is a phase transition and where there is a 0 there is no phase transition. So, suppose we begin with plus A for one suppose you are in this state

So, now since you have a 0 you continue on that same state and then since one you have a 1 you would change your state from A to minus A. And again since you have a 1 you would go back to plus A and here since you have a 0 you continue on your state and since you have a 1 over here you again switch your state, right.

So, in this case you are basically there is a transition from one level to another only when a 1 is transmitted. So, what the receiver does receiver suppose there is some communication it sees there is no transition. So, if there is no transition the receiver will note there is a 0. Then the receiver sees there is a transition; that means, there is a 1 there is a transition there is a 1 there is no transition there is 0, no transition that is 0, no transition there is 0, a transition; that means, there is a 1 which would match with the transmitted sequence.

So, why would one go for this particular scheme? The need to go for this particular scheme is that if you have a long, if there is a long sequence of ones; that means, you have 1, 1, 1, 1, 1, 1 suppose you have that what would this give this would give continuous signals of A, A, A, A, A, and So on. There is no transition whereas, if you have NRZI or NRZ 1 whenever there is a 1 if you have a long sequence of 1 you would experience continuous transition, right. So that means, the long sequences of 1 in this case would produce a continuous value of plus A whereas, in this case it would produce continuous shifting between these.



Now, why this is important? The reason this is important because just imagine the receiver. The receiver needs to detect the signal in the interval 0 to capital T which is the symbol duration. Here the bit duration and symbol duration are the same in this particular situation.

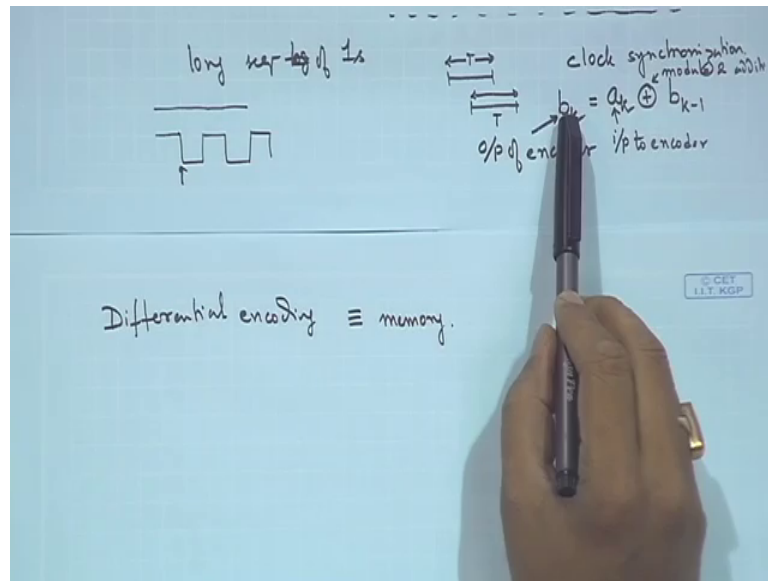
So, the question is how does the receiver know about T, the capital T. Well you could argue that the receiver has a clock we generates T, that is true. But now if you look at this let this be the duration T. And the receiver generates a duration T. Although this length these lengths are the same, but there is no way to identify what is the starting point of the bit, the receiver can only produce a duration of T, but it needs to know the starting point.

Now, if there are transitions the receiver can easily identify that this transition means the bit has changed. So therefore, whenever there is a transition it will synchronize the clock. So, it will help in clock synchronization. So, then one could say that well in this case if there is a sequence of zeros then you have a problem, but of course, those could be handled in many other ways by mixing the zeros and ones by xor and many other operations.

So, this is highly beneficial in clock recovery there are many other techniques like the miller code and there are many others, we just show one of the schemes over here to identify of this works. And in this case we could say the output bit  $B_k$  is equal to  $A_k$  the input bit, modulo 2 addition of the previous output right. So, this is the output of encoder and this is the input to encoder and this one is of course, modulo 2 addition.

So, if you would follow this you could clearly generate this kind of a pattern right. So, this kind of operation which we could also identify as one with differential, differential encoding. So, this is also known as, this is also known as differential encoding the reason.

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Because of modulo 2 operation over here and this operation introduces memory into the system, right.

So, what you see over here is that the previous output is used to generate the current output. So, when we discussed memory less and with memory modulation we had stated that memory less modulations are those which where the current output does not depend on the memory; that means, the past; that means, whatever happened in the; that means, whatever signals have been transmitted in the past symbol intervals whereas, the ones with memory are those which use reference to the past; that means, which use information about signals that have been transmitted in the past.

So, if we look at this we find the signal to be transmitted now is dependent upon the current input of course, but it is also dependent upon the signal which has been transmitted in the past. And amongst various benefit one of the benefit is that it gets is it is able to help the receiver recover clocks by allowing more transitions even if there is a long sequence of ones that is present in the sequence.

So, with this we will we will not take up any further modulations or linear modulations with memory. We will of course, go with other non-linear modulations with memory which are a very important class of digital modulation techniques in the next few lectures.

Thank you.