Modern Digital Communication Techniques
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Lecture - 32
Memoryless Modulation (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. And we would like to just remind what we have done so far. We have done modulations which are linear and without any memory, and most of the modulations which we have worked with are 1 dimensional or at most 2 dimensional. For example, the PAM was 1 dimensional, then we had phase shift keying which was 2 dimensional, we also did quadrature amplitude modulation which was 2 dimensional. And then in the previous lecture we have started discussing multidimensional signal.

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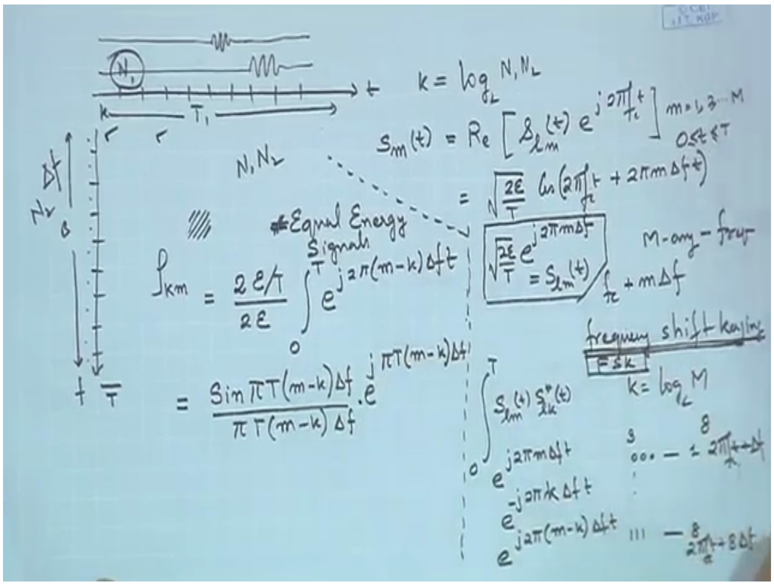

I have briefly in multidimensional signal what we have covered is a situation where you might have time direction which is with the length of T 1 or correspondingly you might have a frequency axis with a total bandwidth of B. And we said that you could divide this time into smaller slots or it could divide it into slots N such slots and you could select any one of these slots using binary pam. So, the presence or absence of the signal would denote which particular signaling mode is used this could be one of the options and this could be another option.

So, what we said is that if there are N possible set selections then log of N base 2 is the number of bits that would be required because you have N different things to choose from similarly in the frequency axis we said that you could divide it into chunks of dimension delta f and there could be N such chunks or N 1 and N 2 that for easier and then this whole time frequency space could have could be of N 1 N 2 dimension just for the explanations sake. If I am choosing this particular band then I could have N 1 possible options over here. That means, I could have a signal here I could have a signal there or anywhere else and they could be N 1 possible options which is indicated here and for any one option. That means, if I have selected this I could choose N 2 possibilities and these are all orthogonal to each other.

So, if we would select this particular entry; that means, a particular frequency block and a particular time block then we would get one such possibility and that could be chosen by using log base 2 N 1 N 2 number of bits that is what we said and we also described a possibility where you could choose any one of the dimensions and then you could do PAM or QAM accordingly.

Now, before we move into this second mode let us look at what are the options or what are the details of multi dimensional signaling and let us proceed with that. So, when we have this we could write this particular signal that S m of t; that means, if we are talking about m array signaling S mt is equal to real part of S l t S l m of t e to the power of j 2 pi fct. Now this is a standard mode, of course m equals to 1 to up to capital m indicating. So, here when I write m I mean m possible options.

So, I have used N 1 and N 2 this m could be any one of them and valid for t less than or equal to t being 0 so; that means, any one particular section could have this and this you could easily write it has 2 e that is the energy upon t cos 2 pi fct and if I am using in the frequency domain; that means, I am using one of the different frequency options in that case you could write 2 pi m delta f t; That means, I am talking about m array orthogonal signals in the frequency domain right and we will we will see what does it mean.

So, depending upon the choice of m; the carrier frequency would be 2 pi fct that the total frequency of this would be 2 pi fc plus delta f or 2 pi fc plus 2 delta f or in other words you could say fc plus m delta f is the choice so; that means, depending upon the value of m you would be selecting any one of these frequency bands. So, we are talking about only the selection of one of the frequency bands over here and since you are talking about selecting one of the frequency bands. So, we are in other words talking about frequency shift keying matching with the terminology that we have been using for other dimensions.

So, we have seen till now the amplitude shift keying which is comparable or analogous to what you have studied as amplitude modulation we have seen phase shift keying which is somewhat similar or has a lot of similarity with phase modulation and now what you are seeing is frequency shift keying which has somewhat similarity to frequency modulation.

So, here instead of choosing the continuum set of frequency; that means, instead of modulating the frequency directly by the signal what you have is you are choosing one of the frequency by virtue of m and this m would be chosen by using this k. That means, you would have k goes to log base 2 of m. So, if you have eight possible frequencies you would have three bits to choose from. So, 0 0 0 could mean the index 1 and 1 1 1 could mean index eight so; that means, 2 pi fct plus delta f to 2 pi fct. So, this will be 2 pi fct plus delta f and this one would be 2 pi fct plus eight delta f.

So, the choice of the frequency whichever frequency you are choosing indicates; what is the bit that is getting communicated. So, this is the way you could represent such a signal and in short you would call it fs k right. So, once we have this one of the interesting properties that we should look at is the signal correlation and we define it as rho k m now why we study this would be very clear within a few minutes.

So, rho k m we define it as the correlation and you could also see from here comparing this with the phase shift king the amplitude is fixed. So, if amplitude is fixed. That means, you have equal energy signals. So, this is one of the important properties that we have and therefore, we are interested in the cross correlation of the signals. So, the cross correlation of the signal would be worked out as this expression which is integral 0 to t. So, this is normalization with respect to energy. So, you can see 2 e upon t and this is normalized by the energy e to the power of j 2 pi m minus k delta f t.

Now, how do we get this if you look at the phase band of the signal if the basement of the signal is e to the power of j 2 pi m delta f. So, this will be S l m and of course, you are going to have root over 2 e upon t. So, this particular expression is equal to S l m of t.

That means, when we are doing cross correlation will be integrating from 0 to t. So, this is valid for an interval of 0 to t 0 to t S l m t times S l k conjugate t would give rho k m. So, that is what we have done and of course normalized by the signal energy. So, if you do this you have e to the power of j 2 pi m delta f t and the conjugate would give you e to the power of minus j 2 pi k delta f t and this multiplied by that of this would give you 2 e by t and this is the normalization term that we have over here.

So; that means, if these 2 are multiplied you have e to the power of j 2 pi m minus k delta f t right. So, this is how you get this particular expression and if you work this particular expression out you will get these some of these things cancel out and you going to get the expression of sin pi t m minus k delta f, because integrating from 0 to t 0 term would give a 0 and the t term is what remains with us upon pi t m minus k delta f multiplied by e to the power of j pi t m minus k delta f. So, from this step to this step if you would work out your going to get e to the power of j 2 pi m minus k in brief you can just write it and of course, you should be able to do it yourself.

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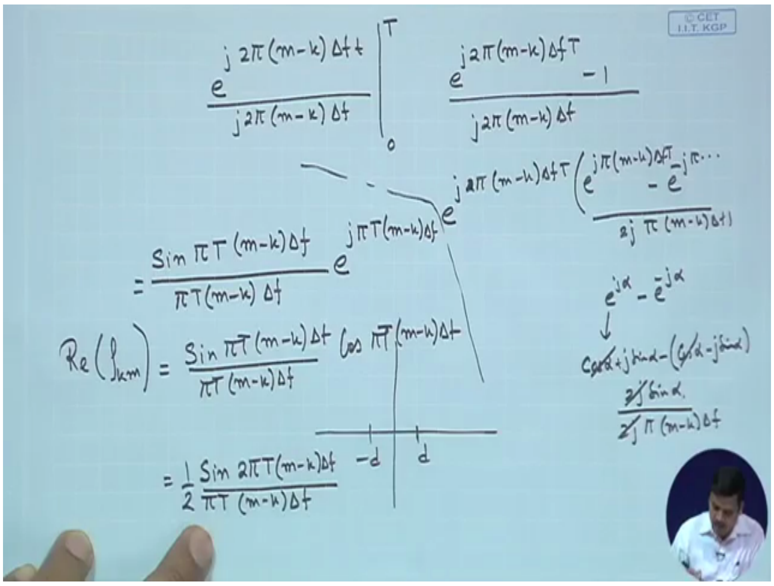

E to the power of j 2 pi m minus k delta f t and of course, there is a dt upon j 2 pi m minus k delta f integrated from 0 to t. So, if you do it you going to get e to the power of j 2 pi m minus k delta f t and the other term would be 0 0 would lead to a 1. So, this will be e to the power of j 2 pi m minus k delta f t times j 2 pi m minus k delta f minus 1 and all the other terms would cancel out. So, you could take out e to the power of j 2 pi you could take out e to the power of j pi m minus k delta f t from this you will be left with e to the power of j pi m minus k delta f t minus e to the power of minus j pi dot dot dot and by the denominator you have 2 j pi m minus k delta f.

So, this expression is e to the power of j alpha minus e to the power of minus j alpha this is cos. So, cos alpha plus j sin alpha and this 1 would be cos alpha minus j sin alpha. So, cos and cos works cancels out; so, you are going to get 2 j sin alpha in the numerator term denominator you have 2 j pi m minus k denominator you have m minus k times delta f.

So, 2 j 2 j cancels out you are left with this term right. So, that is the term that we get over here and with multiplication of t on the numerator. So, this t that is that is there this t that is present in the numerator comes down in the denominator. So, this t that is available, this t that is available would come to the denominator and the expression you would land up with would be sin pi t. So, we getting alpha if you look at alpha this is the expression pi t m minus k delta f and upon this numerator has a t term a 1 upon t term. So, you have pi t m minus k delta f term and you have this extra term which is their j pi t m minus k delta f right. So, this is this is the term that you end up with.

Now, if we are selecting binary PAM; that means, we are selecting between 2 amplitude levels or you could also take on off keying that is presence or absence. So, binary PAM would have only 1 dimension. That means, if you would recall the constellation there is a d and there is a minus t right. So, if we are doing frequency shift keying when we are shifting the frequency or when we are selecting the frequency we are choosing one of these frequencies with the presence of a signal.

And what we are interested in is the signal being present or not and not for the details of it and in any case this does not have a q component. So, we can be interested in the real part of rho km. Now this correlation that we are doing at this point would be a bit more clearer when we work with receivers and where you will find that correlation receiver is one of the important receivers that we do.

Another interesting fact over here that would come out is when decoding we are correlating with different frequencies that we can explain shortly and what is the component of 1 frequency on the other is what we are trying to find out. So, if you are taking the real part of this. So, you will be getting sin pi t m minus k delta f upon pi t m minus k delta f and real part of this would be cos pi t m minus k delta f and sin and cos would work out to be half sin 2 pi t m minus k delta f upon. So, you again have in the denominator 2 pi t m minus k delta f and clearly this particular term that we have with us would go to 0.

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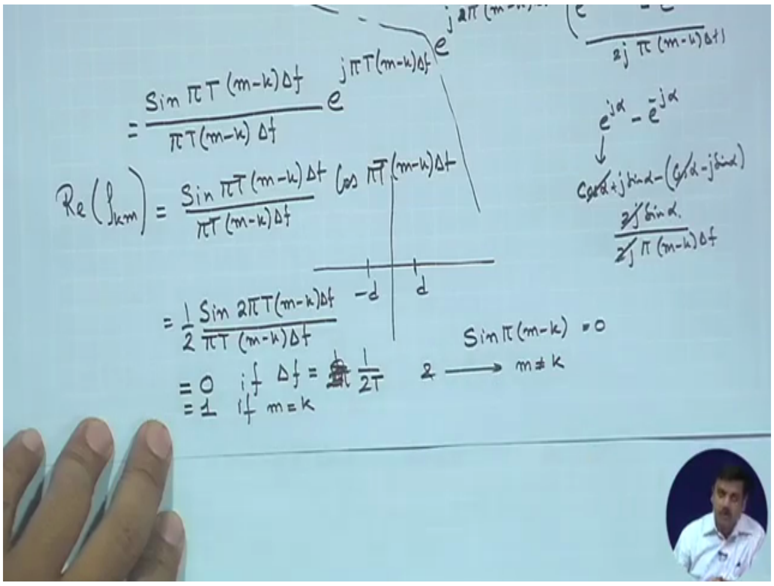

If delta t or if delta f that is here is equal to 1 upon 2 t right so; that means, if delta f is equal to 1 upon t t then what you are left with is sin pi m minus k, because delta f equals to 1 minus 1 upon 2 t. So, sin pi m minus k m is an integer k is an integer is equal to 0 so; that means, this part would go to 0 if delta f is equal to 1 upon 2 t.

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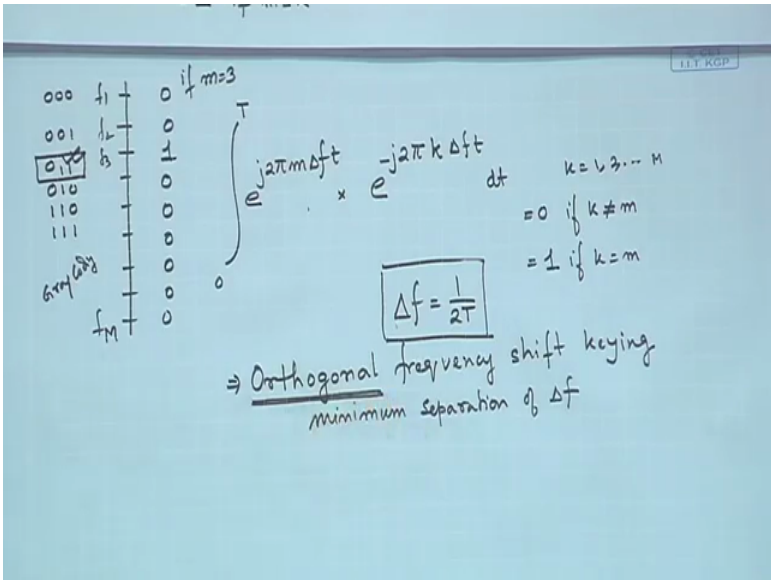

Now, what is the significance of this the significance of this is like this that I have a few frequencies. So, let this be f 1; let this be f 2 f three and. So, on let this be fm right and what we have sent is e to the power of j 2 pi m delta f t at the receiver, how we demodulator, how we identify which particular signal is present or not what we do is we can correlate this with another frequency; that means, all frequencies. So, k is equal to 1 2 up to m. So, we will correlate. So, here we should write for all values of this thing. So, only if; so, for m not equal to k, of course because you have this denominator if in the denominator you have m equals to k and 0 upon 0.

So, it does not work out and if you could recognize this; this is a sink function and a sink of 0 is equal to 1 so; that means, which is equal to 1 if m is equal to k and this thing so; that means, using these and we apply over here what we get is when k suppose m is three and k is equal to 1. So, m is not equal to k this correlation result and when we do the correlation we basically multiply this we integrate from 0 to t dt.

So, this result is what we have actually obtained over here by doing the correlation that is how we started that is why we started the correlation right and this is the outcome of the correlation that is what we have with us. So, this would be equal to 0 if k is not equal to m and this would be equal to 1 if k is equal to m. So, when I correlate with f 1 when I have sent f three I am going to get a 0 here with f 2 I am going to get a 0 over here I am going to get a 1 and all others I am going to get a 0 if m is equal to 3 right so; that means, at the receiver I would be able to identify that this particular frequency was sent now this particular selection should definitely map to a particular a bit coding pattern that we have discussed. So, if this 0 0 0 maps to selection of f 1 and let us say 0 0 1 maps to this 0 1 1 let it map to this and 0 1 0 would map to this and so on 1 1 0 would up to this would map to this and so on and so forth.

So, now, since we have identified that f three has been sent we would immediately decode that the bit sequence at the transmitter must have been 0 1 1 and if you would note carefully I have been trying to generate a gray coding pattern over here. So, you could still use gray coding if you are if you are very keen about it right and will also find out the minimum distance of course.

So, what I am trying to tell you at this point is that along as long as we maintain this criteria; that means, delta f is equal to 1 upon 2 t in that case and we integrate between 0 to t. In that case I would be able to very definitely identify which particular signal is sent. So, it is also giving us a form of separation of the frequencies of choice and connecting it to the symbol duration of our interest. So, t is the symbol duration we should be careful.

So, in this way what we are trying to say in other words is that when this integration is equal to 0 you would be able to identify from our vector notation that or and our signal expansion this is a projection of one signal on another or the inner product of 2 signals right. So, we said that if the inner product is 0. That means, the signals are orthogonal to each other right for k not equal to m.

So, we could also say this is orthogonal frequency shift keying right and another important fact to mention at this point is if you would make the separation larger than this you can always find out another set of orthogonal frequencies, but you will be using more bandwidth than needed, whereas if you make delta f smaller than this, then you would no longer be able to maintain this orthoganality right there will be interference amongst these signals and you will not get a correlation which is 0 you will get some values of correlation.

So, therefore, choosing delta f equals to 1 by 2 t would make it orthogonal and this is also the minimum separation that is minimum separation of delta f that is possible in order to maintain orthoganality amongst the signals. Remember at this point we have stated that we are interested in choosing one of the frequencies and we have m possible frequencies. So, you can choose a frequency send a particular PAM signal on that particular frequency over interval of 0 to t. Once you have done that at the receiver you must decode. So, in order to decode you would correlate this signal with all possible signals we will see this thing a later on; so all possible signals in the set of detectable signals.

So, we have m possible signals capital m possible signals will be correlating with all such things as indicated here and will find that if the trans the received signal is not equal to the particular signal against time which correlating may outcome would be 0 and it will be 1, if my chosen signal at the receiver is same as the signal that I have received from the transmitter and this would clearly produce an outcome of correlation with signal 1 signal 2 signal three signal 4 and so on.

And thereby I would be able to identify which particular frequency was chosen. And accordingly once identify I will be able to select the particular bit sequence. And what we have also obtained is if we maintain delta f equals to 1 by t 2 t. Then only this condition holds true and these whole set of conditions hold true; that means, we are saying that by this separation of frequency amongst the different options we are maintaining an orthogonal frequency shift keying frequency shift keying we have already explained.

Now, why we said orthogonal because this signal and this signal when integrated over 0 to t with the conjugate this is the projection of 1 signal. On the other or inner product of 2 signals and the inner product of 2 signals being 0 indicates that the these 2 signals are orthogonal similar to the vector dimension and when these 2 signals are identical the result is 1.

And therefore, we claim that this is an orthogonal frequency shift keying and we have also stated that you could probably make delta f larger. That means, you could make twice the spacing or three times on. So, on and. So, forth, but that would not lead to an efficient communication, but if you on the other hand you could make delta f smaller, but if you would make delta f smaller then you would not find this correlation term going to 0 and then this orthoganality will not be valid anymore.

So, what we say is that this is the minimum separation in frequencies that is possible in order to maintain orthoganality amongst the signal and efficiently communicate emery frequency shift keying.

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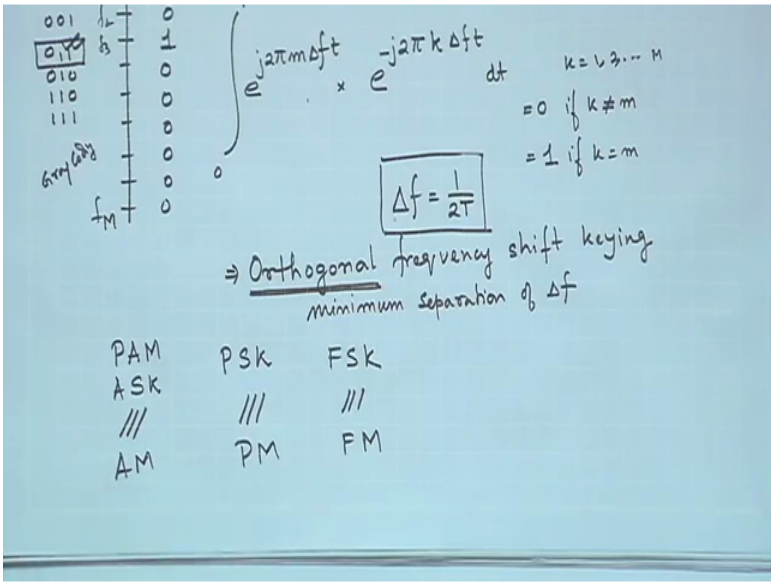

So, in summary at this point what you can state is that we have done pulse amplitude modulation which you could also say as a amplitude shift keying we have done phase shift keying. And we have done frequency shift keying which is analogous to amplitude modulation this is similar to phase modulation. And this is similar to frequency modulation. So, we have covered some of the basic modulation techniques which you have also studied in analog communication.

However, we have looked at their corresponding digital versions and have also given the appropriate representation. And if you note carefully we have been consistent in using the notation that we have started when describing characterization of signals.

Thank you.