

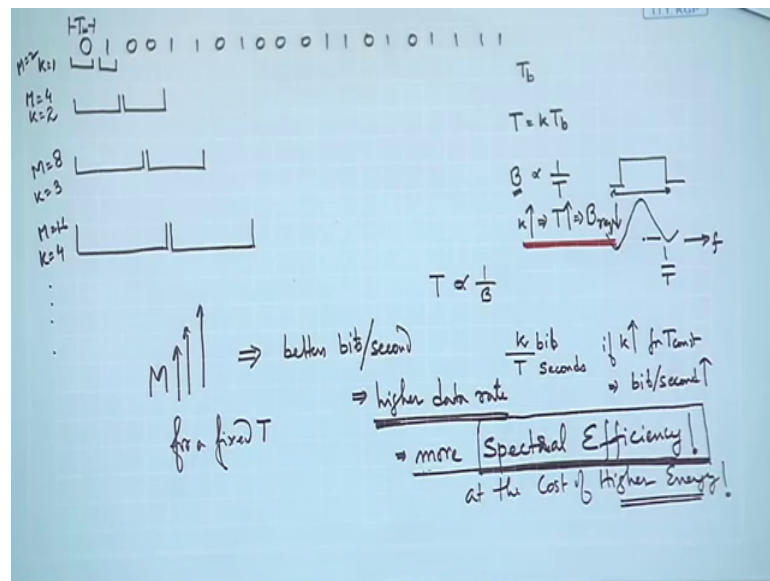
Modern Digital Communication Techniques
Prof. Suvra Sekhar Das
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Lecture - 31
MemoryLess Modulation (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. So, we have been discussing quadrature amplitude modulation. And we have seen in the previous lecture how you could use a scalable constellation to change the number of bits per symbol as and when needed without much alteration to the bits constellation that we have.

At this point it is important also to note that how does this play a role. And this will be used in some later lecture.

(Refer Slide Time: 00:57)

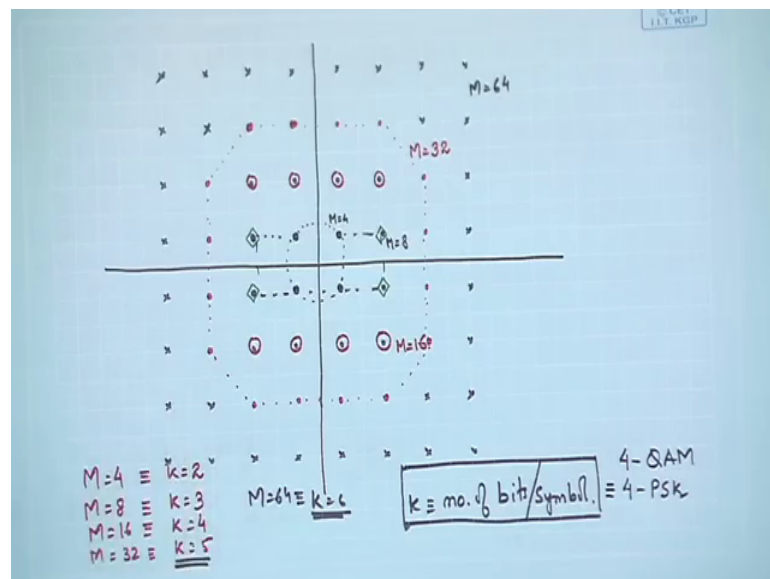


So we have been saying that let there be a bit sequence I am drawing random taking a random bit sequence right. So, let that be a certain bit sequence and we have to do some kind of digital modulation using this bit sequence of the send it across using digital modulation techniques. So, if we are using 2 level amplitude then it is simple that I would choose 1 bit at a time because M equals to 2 means k equals to 1. So, here this would map to let us say minus d this would map to plus t and so on and so forth.

If so, this is k equals to 1 and M equals to 2. And if we now have M is equal to 4 and k is equal to 2 suppose we have this combination in that case we have to choose 2 bits at a time right it could be pulse amplitude modulation it could be phase shift keying or it could be quadrature amplitude modulation if it is pulse amplitude modulation you have 4 levels if it is PSK then it is 4 constellation points in the circle if it is QAM you have 4 constellation points. Again they could be arranged in your according to your own choice if you have M is equal to its a 8 in that case k equals to 3 or in other words you choose 3 bits at a time.

If M is equal to 16 your k should be equal to 4 and in that case you would choose 4 bits at a time and so on and so forth and so on dot dot again M equals to 16 would mean that you have 16 amplitude levels this would mean you have 8 amplitude levels at this point we can take a quick look if you are doing PAM you would need 8 amplitude levels.

(Refer Slide Time: 03:08)



So, 1 2 3 4 5 6 7 8, so, your range of amplitude is this one if you were to do let us say QAM we would restrict our self to these few where we have a larger amplitude swing in the I channel and you have a smaller amplitude swing in the q channel overall the peak power with respect to the average power should be smaller than the case of PAM.

So that is another justification or reason why we could go for a QAM signaling now if you would go for a QAM signaling you are efficiently using your signals because you have the complex signal transmission if it is spam then it is a double side band and you

need to resort to SSB. In order to single side band transmission in order to achieve a certain spectral efficiency if you are doing QAM you would achieve the same spectral efficiency as you do with the single side band system. So, now there are 2 important aspects that we need to look at one where we compare that if I have a certain fixed bit rate t_b that we have to use.

So if we have a certain fixed bit rate t_b in that case if it is k equals to 1 our symbol duration should be equal to that of t_b . So, this is t_b , let us say right now when you are using 4 level your symbol duration is not twice when it is 8 level your symbol duration is thrice when it is 16 level your symbol duration is 4 times. So, what has happened is the symbol duration has become larger compared to the earlier case.

So, we have t is equal to k times t_b we will of course, discuss a particular aspect at a later time. Roughly speaking, as of now the bandwidth that is occupied is proportional to 1 upon t because over this symbol duration your amplitude would remain constant if it is pulse amplitude modulation. If it is phase shift keying your phase would remain constant if it is QAM the amplitude and phase together would remain constant for that particular duration right.

So, what we see is that the bandwidth required reduces to transmit the same bit rate. So, that is one way of looking at the advantage that you get in sending over higher order modulation techniques the other way to look at the same thing would be that suppose I have fixed t . That means, we are saying that I have a fixed bandwidth; that means, I have a fixed bandwidth So, there are 2 different views that we are taking one we have just mentioned that we compare I have a fixed bit stream coming into the system whenever fixed bit stream coming into the system I have to choose different modulation techniques.

So as I increase the modulation order; that means, I go from M equals to 2 to M equals to 4 16 and higher and higher orders what I am doing is I am grouping more bits together and sending them every time I send it across the channel. So, every time means every symbol right every waveform that I choose and that waveform duration will be over a longer duration of time to match the bit rate what you said over here is that this time or the bandwidth occupied by a signal which has a certain time duration is inversely related just a quick description if my pulse is rectangular then the bandwidth or the or the

spectrum would be sync in nature right and as I increase this is $1/t$ as I increase this right as I increase this; this will shrink right this is the f as I increase this is this is going to shrink. So, this portion is going to reduce. So, if this portion is going to reduce the bandwidth occupied would be less right if the bandwidth occupied is less; that means, I am choosing a lesser bandwidth as t increases.

So we can say as k increases implies t increases implies b required decreases this is important right the other view is. So, what is the summary? The summary that we are trying to say is that if I am choosing a higher order modulation every time I send a waveform I am actually sending more number of bits. For example, if I am choosing 1 of 8 possible levels I am communicating 3 bits if I am choosing 1 out of 16 possible waveforms I am actually communicating 4 bits. So, this is the notion that should be with us. So, every time I choose a waveform my waveform duration should be dependent or is to proportional to the number of bits. So, as this becomes longer my bandwidth becomes smaller.

So, with a smaller bandwidth I can send across overall I am increasing capacity of the system period the next way of looking at it is suppose you have a particular bandwidth right so; that means, your symbol duration is decided by the bandwidth which is $1/B$ upon the bandwidth. So, the symbol duration is fixed in that symbol duration if I send 1 bit I am sending 1 bit per symbol if I choose 2 bits then I am sending 2 bits per symbol duration. So, so my bit rate would be k bits per t seconds the k bits per t seconds.

So, in the second discussion we are saying that if k increases for t constant implies bits per second increases and in the first notion of discussion we say that as you increase k your t decreases right in that way bits per second also increases. So overall what we could summarize and say is that by this notion of increasing the modulation order you are packing more bits per symbol thereby you are using your channel or system more effectively than by sending 1 bit per symbol.

Now, here you can clearly distinguish the difference between an analog communication system and a digital communication system in this particular communication system you can if things permit which will see with reasons later on we could practically make M very very large. So, as you increase your M for a fixed t in let us say right what you are

effectively getting is better bits per second right. So, what implies higher data rate right you can say more spectral efficiency.

Now as communication engineers we should be updated with the latest requirement in terms of wireless communication system especially the more number of bits you can send per second for a given communication system the better it is like for example, you if you are given 1 megahertz of bandwidth and you are choosing 1 bit per symbol duration compared to another system which uses- let us say 6 bits per symbol duration the second system is actually transmitting 6 times higher bits per second compared to the first system. So, the question that could come to our mind is does it come for free.

Now clearly as I increase M going by this particular picture what we have is as I increase M my constellation becomes bigger and bigger right. So, if the constellation becomes bigger the clear thing that we can see is we need to spend more energy if we have to keep this distance the same this is the minimum distance between the constellation points. And I had told you I need to discuss this in details later on that this minimum distance for between the neighboring constellation points decides there are probability. So, if we have to maintain a certain error probability and increase the spectral efficiency; that means, increase M you have to spend more energy.

So, that gives as an hint that if you spend more power into the system or if you send more energy into the system you can achieve higher spectral efficiency; that means, at the cost of higher energy right. So, initially we were discussing about the choice of constellation. So, you can see that the choice of constellation is dependent upon another factor which we did not mention there is the bits per second that we have over here right.

So, if you have a higher bits per second requirement you would go for a constellation which is be which will be more efficient in terms of spectral efficiency. And spectral efficiency is a terminology which is one of the important terms that you would encounter in a digital communication system in contrast to an analog communication system the analog communication system will not be able to provide you with any such benefit whatever signal comes you will be modulating onto the carrier.

So, the bandwidth occupied would be dictated by the bandwidth of the original signal and you can say there is a modulation index associated along with that whereas, in this case it is clearly at with the control at the control of the system designer. So, the system

designer can choose: what is the amount of spectral efficiency that it wants to achieve, but subject to the constraint of achieving a certain amount of error probability and the constraint of total amount of power that is available to the designer. So, this is something very critical which we should be aware of with this we have more or less completed our discussion on linear memory less modulations and we have given you some of the important tenets some of the important ways of describing the signal. And we will be using this particular framework in all future setups.

So you would; so, this is very important that you go through all this all the models and all the setups all the; the notations that we have used, because the entire remaining portion of the communication will be built on the particular expressions of signals or the relationship we have got with respect to the spectral representation the properties of narrowband noise process which we connected to a white noise narrowband filtered white noise its power spectral density and its autocorrelation, because these will be used when we are studying error probability when we are designing receivers in the next few lectures.

So, once again I reiterate that this forms the basis or the foundation of what will be doing in the next few lectures. So, although again I would like to say we have briefly covered some of the important aspects, but with this you are equipped to move forward and read up other relevant materials on these kinds of communication techniques. And we would not cover anything more on memory less modulations in this particular module. So, moving down further if we look at the constellations that we have discussed what we see is that even QAM is 2 dimensional: one is the I channel 1 is the q channel.

So any point can be projected onto these 2 and going by the signal constellation or the signal model you can see that these are the 2 basis functions which we had described earlier. So, any signal could be having components on the cosine and the sine and clearly here.

(Refer Slide Time: 17:05)

Quadrature Amplitude Modulation

$$S_m(t) = \text{Re} [(A_m \cos 2\pi f_c t + j A_m \sin 2\pi f_c t) g_m(t) e^{j 2\pi f_c t}] \quad m=1, 2, \dots, M$$

$g_m(t), 0 \leq t \leq T$

$$A_m \cos 2\pi f_c t = A_{ms} \sin 2\pi f_c t$$

$$\text{Re} [V_m e^{j(2\pi f_c t + \theta_m)} g_m(t)] \quad V_m = \sqrt{A_m^2 + A_{ms}^2}$$

$$\theta_m = \tan^{-1} \frac{A_{ms}}{A_m}$$

8 levels \equiv 3 bits

Amplitude Phase

---	000	-
---	.	-
---	111	-

(Refer Slide Time: 17:35)

Multidimensional Signals

T_1

T

T_1

NT

N

$x = \log_2 N$

$f_1(t)$

$f_2(t)$

$f_3(t)$

$S_1 = [1 \ 0 \ 0 \ 0 \ \dots]$

$S_2 = [0 \ 0 \ 1 \ 0 \ 0 \ \dots]$

These are the components so; that means, that we have been dealing with 2 dimensional signals there is a possibility since we have talked about vectors and signal space there we have hinted at that point that we need to go to multi dimension and therefore, we would like to look at multi dimensional signals.

So, there are various ways of doing generating multi dimensional signals we will look at some basic ones and then you can expand it to some other modes. So, for example, if we have let us say T ; T_1 as the total time duration that is available; so, let us say this

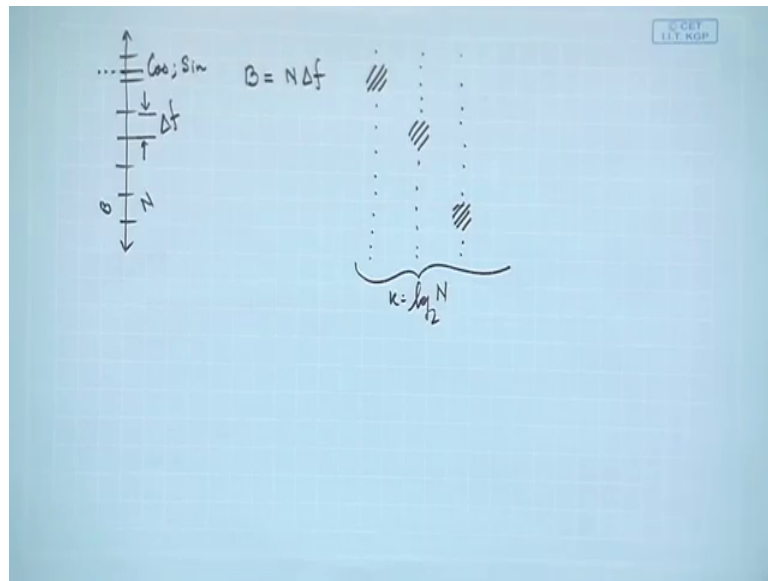
duration this is the t axis this is 0. Let us say this duration is T available for signaling what we could do is we could break it up into smaller durations, and let us say we have n times T such durations, right. And one of the ways that we could use in selecting a waveform is that we would select let us say one of the intervals and send that is a PAM signal in that and not send anything else in any other part of time.

Another signal could be send do not send anything and then you send a PAM signal here and so on and so forth. So, in this manner you could have n different signals that you could choose. And each one of the signals as you can see over here are orthogonal to each other orthogonal means if I multiply this and this, and integrate over the period from 0 to T I am going to get 0 because a product of this and a product of this is going to be 0 because this is non0 and it is 0 over here it is non0 this is 0 over here.

So again at the number of bits that I would require to represent this would be log base 2 of n because there are n possible choices and. So, this should be equal to k in this case right. So, we would do some kind of PAM signaling on each of the time slots when we have a group of time slots available. So, the signal that we could represent in this case would be of the form let us say- in this particular case this particular case would be 0 0 let us say some energy E_0 0 dot dot dot dot to here let us say this is my signal number s_3 then I could have s_1 represent it as square root of E_0 0 and so on and so forth and so on.

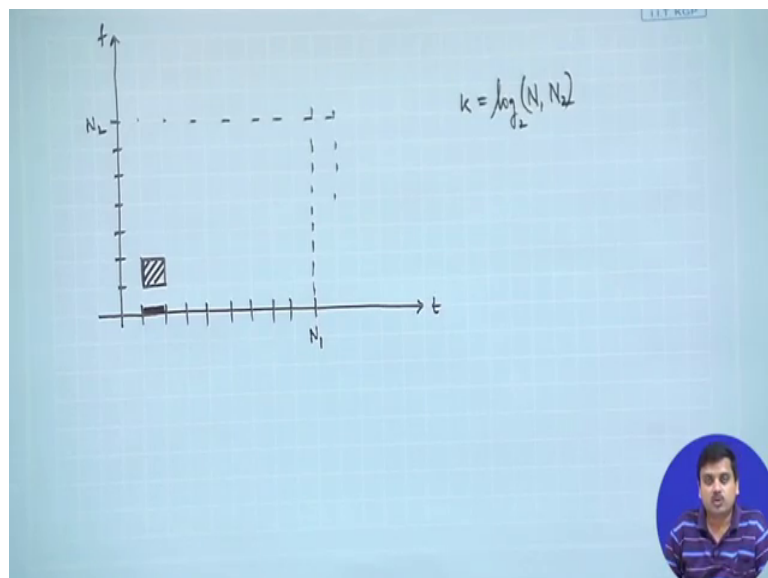
so these could be my signals where you can see this is the component of the signal on the basis vector which is this one; that means, my basis vectors could be a 1 in a whole bunch of 0s this is my first basis vector my second basis vector could be this way a very simple basis vector third one could be this way and so on and so forth; so if I project a signal and which has a component here. That means, I have chosen this basis vector which is my f_1 of t and this is let say f_2 of f_3 of t and so on and so forth; we could do a similar thing in the frequency domain and we say that suppose you have a bandwidth b in total then you could split up the bandwidth into n parts right and you could say that each of these small sections of the frequency have a bandwidth of Δf and then you could say b is equal to $n \Delta f$.

(Refer Slide Time: 21:56)



In a similar fashion as we have done here you could have n different choices because you could have one of the signals as some energy in this and 0 in all the cases you could have another signal which could have energy there and nothing over here, you could have another signal which is having energy over there and nothing in the other frequency bands. So that means, again you have n possible choices and you could use log base 2 of n as k .

(Refer Slide Time: 23:25)



So, you could do a multi dimensional signaling in time or frequency and you could also combine the time and frequency and you could have. So, if this is your time axis this is your frequency axis. So, if let us say you have N_1 such time units and if you have N_2 such frequency units then each of these time frequency units could be used to send a signal.

So, in this case you are going to have a total of $N_1 N_2$ possible combinations and \log_2 of $N_1 N_2$ would be the number of bits that could be represented by choosing any one of these options that are available with you. Now one might be a bit 1 might have a question that how different is this from having M_1 amplitude selections and M_2 phase selections as we did in QAM if you look at the signal space a consider diagram there we had 2 basis functions the cosine and the sin right.

Here we have a different thing here we are saying that if you look at this time dimension this unit of time is orthogonal to this unit of time because it is not present over here similarly this unit of frequency is orthogonal with any other space in frequency right. So, together we have $N_1 N_2$ orthogonal functions in this particular combination in this particular space and you could choose any one of them in conveying information.

That means, you have total of $N_1 N_2$ possible options and you would select \log_2 number of bits you will be using to convey this; this option this is one way of doing it another more efficient way of doing this could be that if we would get back to our picture here you could say that why not I choose one of these frequencies and I would send a PAM signal on top of it; that means, not a binary PAM, but M-ARY pam.

So I not only have n different options over here, but each one of them has another M different options. So that means, I have M options per frequency band and I have n different frequency bands which orthogonal to each other right. So, together we have M multiplied by n number of options where there are n orthogonal space n orthogonal frequency bands and each frequency band can have M different signals on the same dimension because we are doing PAM that is $\cos 2\pi fct$ our basis function only the amplitude levels are different, whereas if you are doing QAM.

That means, if I am choosing this center frequency I would use a cos and I would use a sin in that case each band would have 2 dimensions and we have n dimensions over here. In that case we are going to have $2n$ dimensions available for us for signaling and in

each of the dimensions you can choose a pair modulation which would further increase your possibilities.

So, by combining these you can go from 2 dimensions to n dimensional signaling, we will look at one such n dimensional signaling in the next lecture.

Thank you.