Modern Digital Communication Techniques

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Lecture - 30
Memoryless Modulation (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. So, we have travelled quite a bit in this particular course; and currently we are in a stage where we have discussed few linear and memory less modulation techniques. To name a few we have covered pulse amplitude modulation, we have also covered phase shift keying, the earlier one could be also called amplitude shift keying.

And then we have moved into combining the pulse amplitude modulation with the phase shift keying; and together we have come across the new modulation scheme which we call as the quadrature amplitude modulations. And I would also like to remind you that we have been discussing about carrier less modulation and with carrier modulation. So, just a quick revisit at the quadrature amplitude modulation and let us look at some of the interesting parts of how do we generate it and what are the few special ways of doing quadrature amplitude modulation before we move onto few more interesting things.

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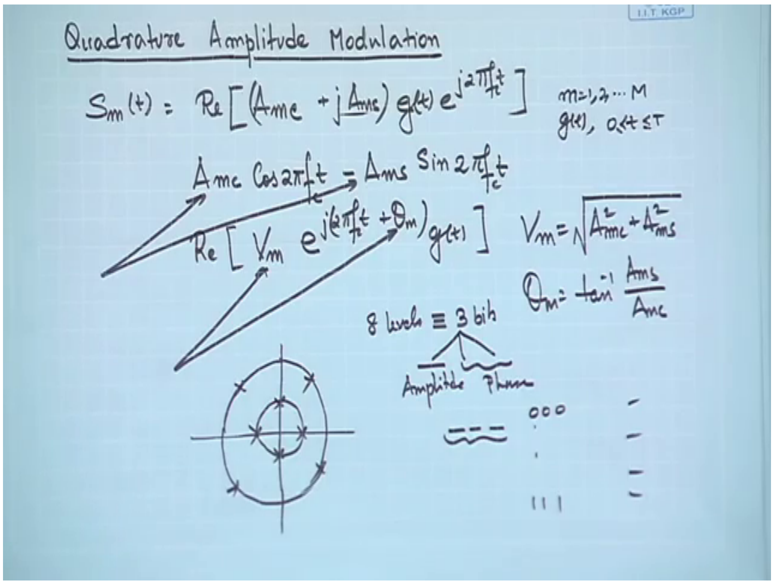

So, when we looked at quadrature amplitude modulation, we did describe the meaning in terms of the words that are used over here. So, modulation we said that in our context it would mean symbol mapping amplitude is of course, amplitude and quadrature is in terms of quadrature carrier and we had written down the signal expression S m t as real part of A message with the cosine plus j A m with the sign that means, the quadrature amplitudes. So we have the quadrature amplitudes. And there is a g t e to the power of j 2 pi f c t and that is the modulation part for m is equal to 1, 2, to capital M, and g t being non 0 for t less than equal to T within 0 to T.

And we said that this you could easily expand it as A mc 2 pi f c t minus a ms sine 2 pi f c t. And you could also write in terms of real part of V m e to the power of j 2 pi f c t plus theta m and also another g t. So, where we said that you could view it as two amplitudes or as an amplitude and phase and of course, we had connected V m with the amplitudes A mc squared A ms squared and theta M has tan inverse. So, these are some of the things that we have discussed.

And then we said that what would be the corresponding signal space diagram or the constellation diagram. So, if we would combine the amplitude and phase modulation then we said one could use M 1 level amplitudes as an example we took two amplitude levels. And then one could use 4 phases, the other phases could be here or it would be here; and together you would get 8 levels and that would be in 8 levels or 8 symbols would be in 3 bits. And we said these 3 bits be broken into 3 parts; one for the amplitude and the other two for the phase and there could be a coupling that for a certain amplitude phases are these and for the other amplitude these are the phases.

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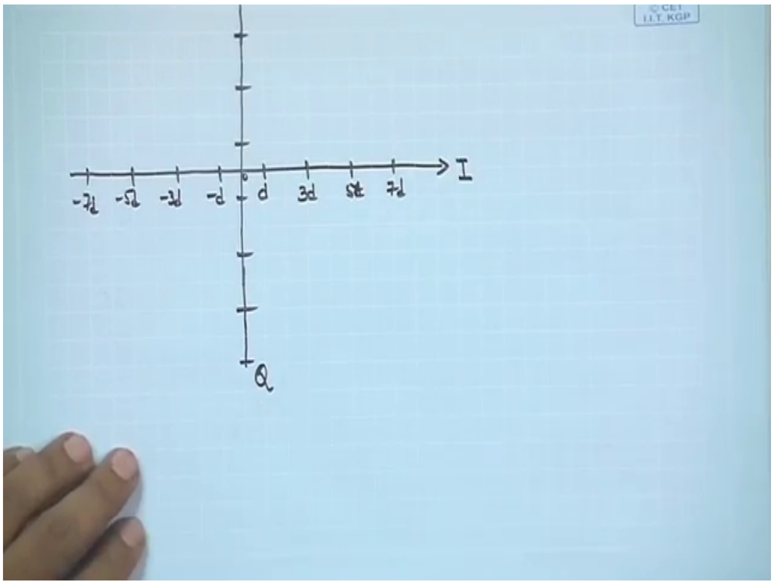

Along with this, we have also said that you could also think in terms of amplitude modulation only on quadrature carriers. So, for that we said that suppose we have two pulse amplitude modulation and this way you could continue doing that; and of course, here this will go little bit higher. So, in this way we could say this is the I axis and this vertical one is the Q axis.

And whatever amplitude we select here will be used in modulating the cosine carrier. So, A mc that means, whatever level with d, 3 d, minus d, minus 5 d, minus 7 d or and whatever amplitude we select here could be used to modulate this sine component of the carrier.

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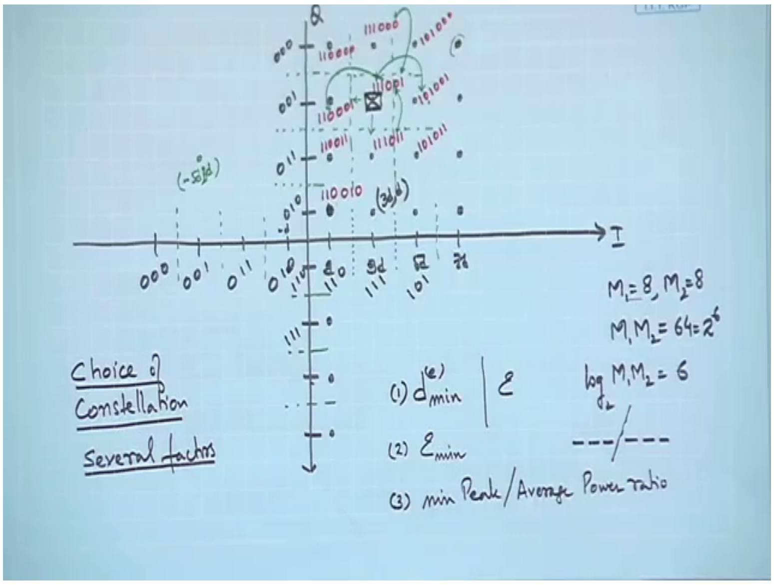

So, this is a bigger picture I think it is wiser to draw it a little bit bigger space. So, this one covers the space that we need. So, what we have over here is 4 and 4 - 8 levels that means 3 bits on the I channel; and 4 and 4 on the Q channel as well. So, in this way if we focus on how to go about this, so total there are 8 levels on the I channel, 8 levels on the Q channel. So, you could say M 1 is equal to 8, M 2 is equal to 8. So, a product would give M 1 M 2 as 64, so that would mean if I have to take number of bits log base 2 M 1 M 2 which would turn out to be 6 because this is 2 to the power of 6. So, there are 6 bits that are involved. And going by the description we could say that these PAM on the I channel could be selected using 3 bits; and PAM on the Q channel could be selected using another three bits.

So, in this diagram, that we have taken we could split it into one bit for amplitude onto 4 phase or we could group all three together and select any one of these. So, because it is a mapping of 8 three bits which means 8 possible combinations 000 down to 111 to eight possible amplitude and phase combinations. Similarly, here also you can think of I have 6 bits and all, and there are 64 levels that could be generated and you could choose these 6 bits to map to any of the 64 levels or you could split it into two.

So, if for example, we have to select one amplitude on the I axis, suppose we choose it as d and in the Q axis also we choose it as d, so together the constellation point would come over here. And if it is d on the I channel and 3 d on the Q channel, so it will be there and so on and so forth. So, you could choose all these points in the signal space diagram. Similarly, you could fill up the entire space and you could form constellations. So, these dots indicate the constellation location. So, for example, this particular dot would have the value 3 d and d that means, if we would compare with this peak this particular expression we would find A mc as 3 d and A ms as d, so that is how this particular mapping would happen.

Now, an important question here could be that whether the gray coding that we had discussed earlier would still be applicable in this case. As I had mentioned in case of phase shift keying, you could still do break coding by means of allocating the bits in a pattern where the neighboring two constellation points have difference at most one bit location; same thing can be done in this particular case as well. So, if you would do gray coded constellation here, you could begin with 000 or 001 and 011 or 010 and so on and so forth. And you could also do a gray coded mapping on this axis. In that manner, so basically one could have 110 going from there and then one could go to 1 1 1 and so on and so forth.

So, similarly if I would do the mapping over here, we could for example, say 000, 001 following the same pattern that we did here 010, and this one would be 110, this one would be 111. So, this particular location that have selected would have a mapping of 110, 010. So, this location would have a mapping of 110 followed by 010; 110 from I channel, and 010 from Q channel. So, we have 6 bits mapping to this constellation. So, if we have to select let say this particular constellation, this would have 111 and 000. This particular constellation would have 111001. And if I am to take this it is 110, because it is 110 here and 000. Similarly, this one would be 110001, 110011 this one would be 111 followed by 011 and then we could map this to 101 and so on. So, if it is 101, we could map this as 101 followed by 011101001, this is 101000.

So, now trying to discuss what we have seen before suppose our constellation of interest is this particular constellation. So, this is how far interest. And as we had done before, so for PAM our decision boundary more details on decision boundary will be discussed later. Let us choose halfway between the amplitude points. Similarly, decision boundaries on the Q channel would be halfway between the amplitude levels. So, if this one is selected, the way to demodulate would be to find component on the I as well as find component on the Q. And then find in which decision region does the I lie and in which decision region does the Q lie. So, this would go down there and on this axis its goes down there.

So, if there is no disturbance then this constellation would be rated the receiver to lie between this locations. In this location and I axis the first three bits would be 111; and here in this decision boundary the second three bits would be 001. So, you have this combination decoded. However, if because of noise your I gets disturbed and Q gets disturbed. In that case, your decision boundaries have to be extended to visualize what is going to happen. So, this of course we will see later.

So, what we find that as long as this symbol is within this region which is the decision region now it is an area as long as it is within this decision region, you would decode it correctly. However, if this constellation has moved to this side or to this side, due to addition of noise or to this side or above you would decide on which of the constellation points of the receiver based on which decision region this constellation has moved to. Now, let us take that if because of noise, the decision has been made as the neighboring symbol.

So, these four are the nearest neighbors clearly by distance; this is not the nearest, but it in the near neighborhood. So, if it moves there we would find that these two locations differ by at most one bit which is the central bit, this 0 is a 1 over here. This 0 is a 1 in this case. And if we compare these two, we will find that this 0 is a 1. And again if we compare this with this neighbor we will find that there is a difference this is 001; this is 001, it differs in this particular bit. And if you go down then naturally it is differing in this particular bit in the middle bit. So that means, if it moves to any of the neighboring symbols although there are 6 bits involved compared to any other situation. You are still making at most 1-bit error by virtue of gray coding. And we could achieve this simply by extending our understanding of gray coding whatever we have done before from PAM to the QAM.

So, there is a clear advantage in terms of simplicity that if we are using if you are visualizing this as two PAMS then you could use all your things that you have developed in case of PAM directly into this. The technique to be applied is you would take the component of the signal on the I channel component of on the Q channel and you could proceed in the receiver. At the transmitter side when you have to generate the signal, so we again would go by the same logic that we have used. So, we would take two PAM signals. So, we can say that I have a bit stream coming.

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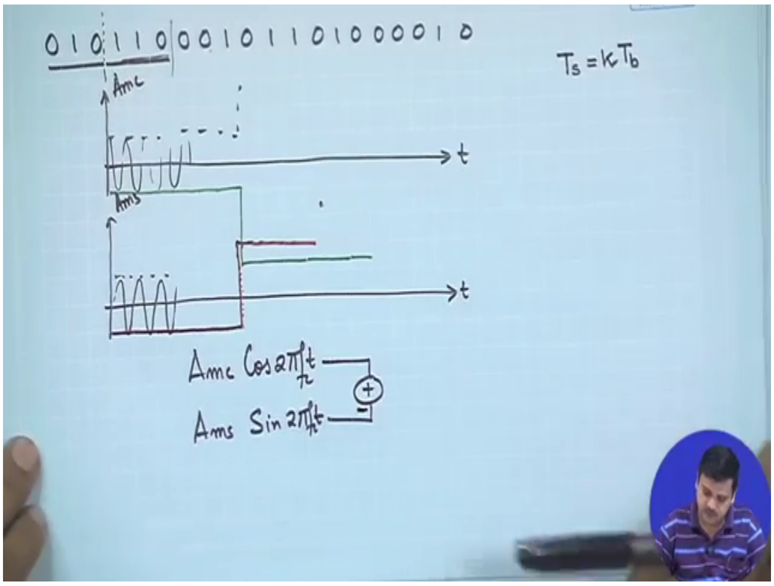

Let us I have this bit stream coming or I need to write few more bits let say these. So, now, the question is how do you do a QAM constellation generation with this. So, what we do is suppose we have to take this constellation and you have to take 6 bits. So, you can say I would take these 3 plus these 3, so that is 6 bits to be choose my first symbol. Once again just to remind you in this case the symbol duration T s would be equal to k times T b. So, whatever is T b is k 6 then in 6 times the bit duration. And then you could split them into first three bits mapping to I channel, second three bits to Q channel in the sense that first three bits are used to select the amplitude from this PAM, second three could be used from this PAM.

Another simplicity that could be clearly visible you could use the same PAM mapping that means, you do not have to reproduce a PAM for the Q because the same mapping. So, we would use the same PAM mapping once for the I, use the same mapping for the Q. However, you can see if it is 010, so 01 0 we have it over to 010 we have it here so that means, it is minus d. And 110 we have it over here which is again a minus d that means, this constellation location would be a minus d and a minus d, minus d and a minus d this constellation location would be what is generated over here.

So, in the I waveform will be minus d in the Q it will be minus d. So, if we try to draw the waveform we will have the time line and here we have to write A mc. And similarly we have to draw the timeline and you are going to get A ms. So, what we will getting is in the first that means, still here you are going to get a minus d and here also you will get a minus d let us look at the next bit sequence you have 001. So, if I would choose 001, so that is minus d, minus 3 d, minus 5 d. So, I have to go down to let say somewhere there and I have to continue and then you have a 011.

So, if you choose a 011 that is over here which is a d so that means, you have this constellation point, this constellation point is minus 5 d coma d. So, in the Q channel, you are going to have sorry 011 is a 3 d so that means this will go somewhere there. So that means, what we are trying to show by this picture is that the I channel and the Q channel or the cosine carrier and the sinusoid carrier or the in phase amplitude and the quadrature phase amplitude would be different. And then you will be getting, so you will be getting a cosine wave and here you will be getting a sine wave of the same frequency, right.

And when they are mixed together is what will be generated at the output, so that means, at the output you are going to get this amplitude and you will be also getting the second amplitude. And then you are going to add it, but of course, there will be a negative sign in front of it.

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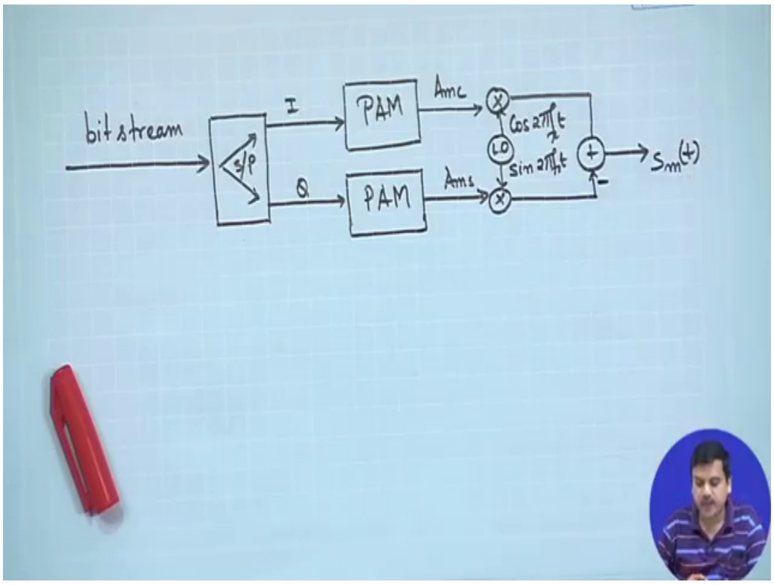

So, a block diagrammatic representation that we can think of for this particular generation would be bit stream. Then you could say split into the I channel and split into the Q, channel and there will be PAM this is the logical flow diagram. So, your diagram could be drawn different depending upon the implementation. So, if you think in terms of hardware implementation, you could use the same table and iterate through them.

So, what you are going to get there is A mc, what you want to get out of this is A ms. So, this is split. So, it is serial to parallel conversion you can think of and then there is a local oscillator which sends cos 2 pi f c t and there is a multiplier and which sends sin 2 pi f c t. So, this could be generated by passing this through a Hilbert transform. There is a multiplier and then we are going to add this with a negative sign and then you are going to send it out, so that clearly means you have two separate channels: right one is the I channel one is the q channel and when you do operation like this you are going to get s m of t. In this way, you can generate a quadrature amplitude modulated signal.

There is another advantage of doing it in this way that instead of taking this of course, there could be certain advantages of doing it, but there is more advantage in this in terms of ease of implementation clearly of course, there could be other reasons of going for constellation which looks like this. So, you have your decision boundaries which are clearly symmetrical and this of course, appears for the case of memory less modulation and by symbol by symbol detection.

So, if you are doing this and in this pattern as I have already mentioned you can use a single PAM lookup table for generation and you could use a simple decision threshold for detection as well. Because you can clearly see whatever we have in the I-axis, you could use a similar thing on the Q-axis. You may note that in this particular diagram I have reversed the order. So, 000 maps to minus 7 d, but 000 maps to 7 d. So, it could be reversed and I could start from 000 here and end up with some other number over there, so that means that there is a lot of simplicity in decoding.

Whereas, if you are choosing this kind of a constellation, there is no straightforward simple way of mapping amplitudes and phases or the reverse direction of the mapping. However, the choice of constellation, so when you have to choose a constellation would depend upon several factors among several factors one of them could be d Euclidean distance minimum that means, we are talking about the minimum Euclidean distance between two constellation points constrained by a certain energy of the constellation.

So, we would like to keep the distance between two constellation points as far apart as possible with a certain energy constraint. There could be another condition that you would like the constellation which has minimum energy it depends upon the particular requirement. You could also have a requirement of minimum peak to average power ratio, one could visibly see that if one uses a constellation like this the peak power with respect to average power is lower. When I mean average power the average power of all these constellation points; and when I mean peak power the peak power is the maximum power compared to a constellation of this shape, where the peak power would be obtained by a constellation at the corner.

So, extreme corners will have the highest power; whereas, the average will be an expectation over all of this which would be somewhere here. So, in this one for a similar sized constellation, there could be possibility of a higher peak to average power ratio. So, depending upon the requirements one would choose a particular constellation. So, there is no fixed decision that one particular constellation is better than the other.

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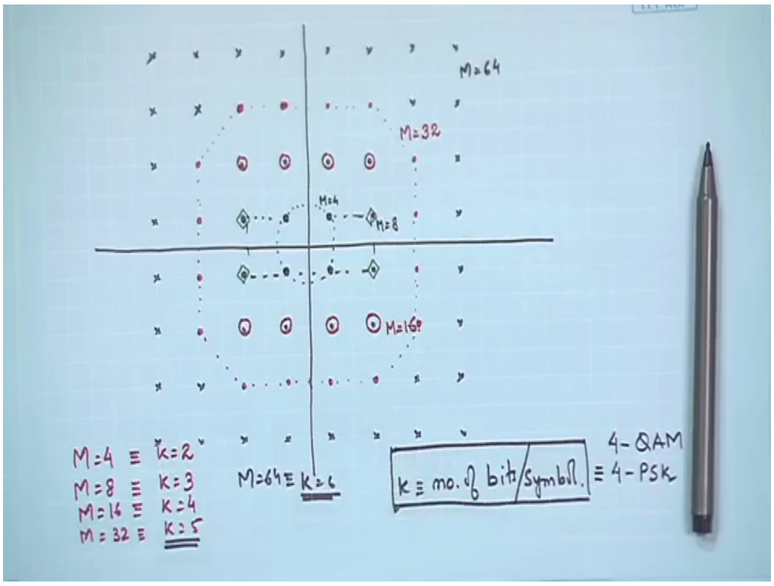

Finally, I would like to take you through to the way that you could generate a generalized constellation generalized in the sense I will draw this once again. So, if I would select a constellation set here let us say these 4 points. Clearly, what I have created is the constellation with 4 constellation. So, I would call it 4-QAM, 4-QAM as well as you could claim that this is 4-PSK. So, this is a simple the next level that we could have is to generate 8 constellation set.

So, we could add a couple of more points in this and what we have is now 4 and 4. So, this is now 8 constellation set. So, M equals to 4, whereas, if we add these constellation points we get M is equal to 8; and now if we add a few more constellations points. So, you would like to demarcate them little bit differently, because these have been added these have been added to the existing set and what we have created is 8 and 8, M equals to 16 constellation point.

So, what you can see that we are easily scaling from 4 to 8 to 16 without changing the earlier constellation. Similarly, you could extend this particular constellation to the next level by adding a few more constellations. So, we have added 4, 4, you should add one more here 4 over here and 4. So, we have added 16 more constellations. So, if we would group these, so then we have the next constellation size of M equals to 32. So, again you can see clearly that we have been able to extend this for M equals to 4 means implies k equals to 2-bits. M equals to 8 implies k equals to 3-bit, M equals to 16 implies k equals to 4 bits, and M is equal to 32 implies k is equal to 5 bits because 32 is 2 to the power of 5.

And now you could extend it further and you could say that I will add a few more points. So, if you do this then you end up with M equals to 64 and when M is equal to 64 this would mean k equals to 6. So, what we have done effectively is we have been able to easily scale the constellation for 2 bits per symbol to 3 bits per symbol to 4 bits to 6 bits and so on and so forth. So, this would be very, very helpful when we are doing dynamic constellation changing that means, we would be looking at situations where you could change the number of bits per symbol. So, this k that we have over here these k values that we have over here is effectively number of bits per symbol.

So, in this fashion if your communication system is dynamically changing the number of bits per symbol which we will see at a particular time you could use a scalable constellation or you could use a flexible constellation by virtue of which your transmitter receiver architecture could remain as simple as possible.

We stop our discussion on quadrature amplitude modulation for this particular lecture here. We continue in the next lecture.

Thank you.