Modern Digital Communication Techniques
Prof. Suvra Sekhar Das
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Lecture - 29
Memoryless Modulation (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. In the previous lecture we have presented the phase shift keying or the digital version of angle modulation and we have represented the signal model. We have calculated the signal energy; we have also given the signal space diagram. So, we will exploit whatever we have discussed in this particular lecture and carry forward some of our discussions which we have left there.

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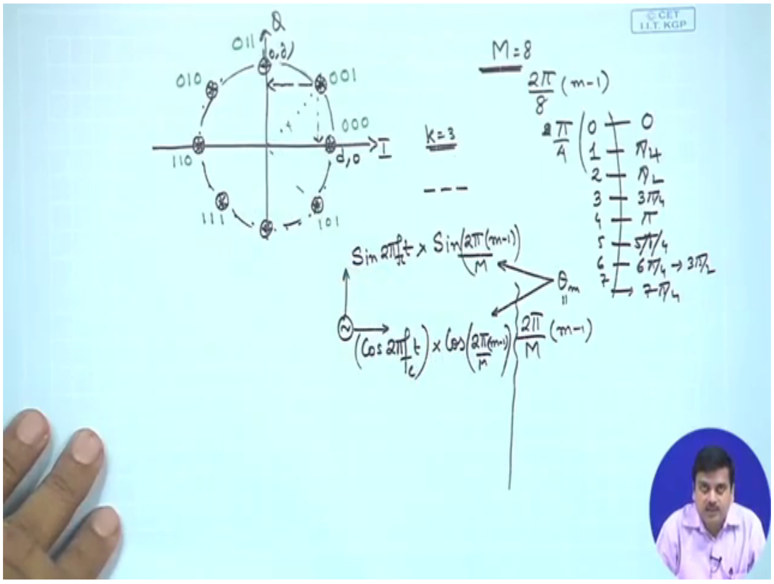

So, if we would look at the typical signal space diagram that we have been drawing.

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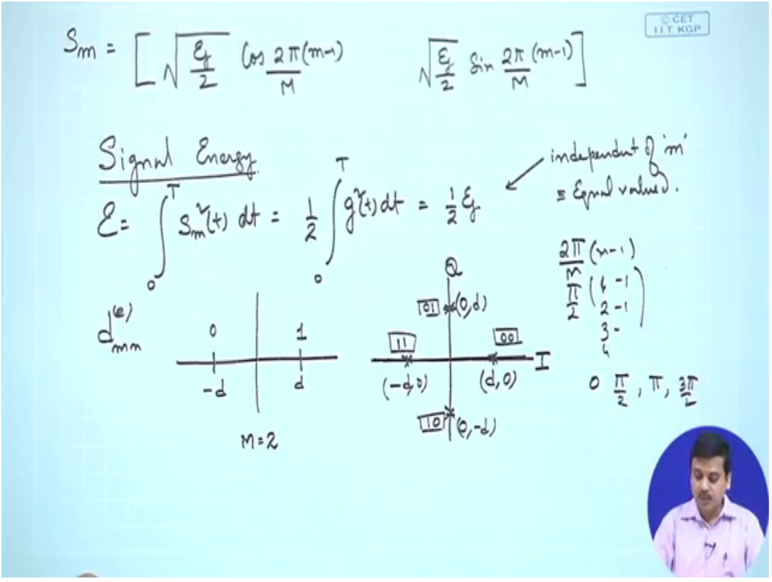

So, what we could think of is: in this case clearly the way to send the signal would be.

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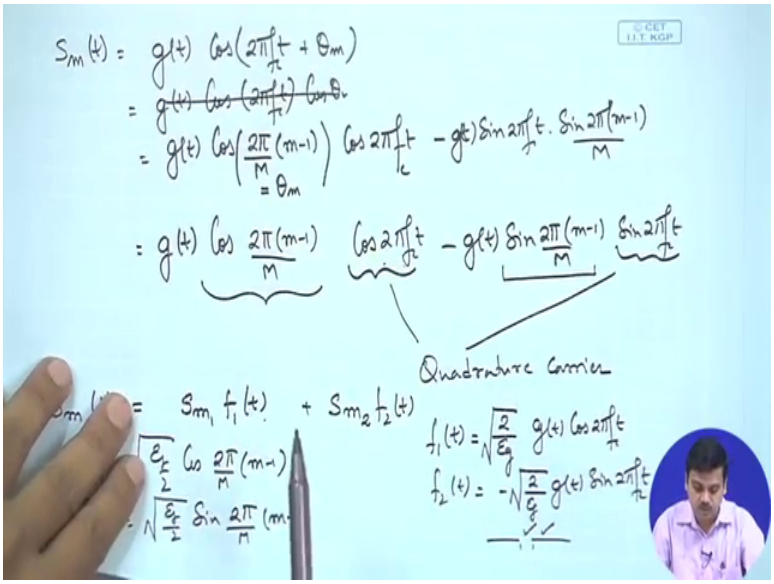

You need a simple cosine a carrier cos 2 pi f c t you would choose the angles and you would do away with that or you could simply modify the amplitude to be plus d or minus d, and you are done with that.

Whereas, if you choose m equals to some higher value; that means, if I choose m equals some higher value as in this particular picture. We see that you have a cosine carrier and a sin carrier. That means, the i axis and the quadrature that is the q axis, right. And if we move there you have a similar thing, only thing is that you have more values in it, but still they could be represented by a representation on the 2 dimensional planes. That means, you would need a cosine carrier and a sin carrier and with components on these carriers you would be able to send the signal.

So, typically during transmission you would generate you would have a local oscillator you would generate a cos 2 pi f c t. And you would also generate ninety degree phase shifted sin 2 pi f c t. On this you are going to do you can think of doing an amplitude modulation with cos 2 pi m minus 1 upon capital M. And on this you can think of doing an amplitude modulation of sin 2 pi m minus 1 upon capital M. So that means, we can think of there is a pulse amplitude modulation going on here there is a pulse amplitude modulation going on there, on 2 different carriers which can generate this particular signal space.

So, if you choose different values of m you are going to get these locations, and in the vector form they could be easily represented as in here right. So that is one important thing which we should keep in mind.

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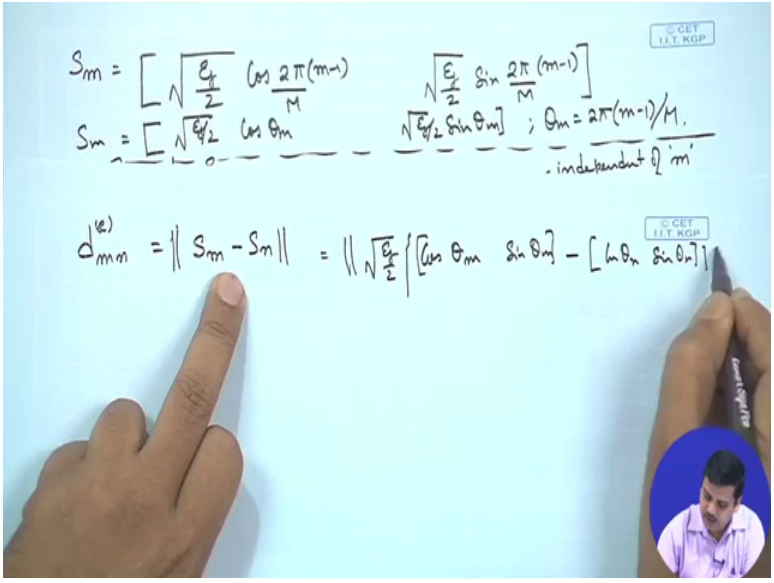

The next important thing which we are supposed to discuss as we did in PAM is the Euclidean distance between 2 signal points; that means, the 2 phases. So, if S m 1 and S m 2 are the signals. So, this would be norm of S m minus S n.

So, if you would do this you could calculate this as root over E g by 2 because this is common in both of them all you need to do is to take this form. Take this form E g by 2 is common in both of them to keep it here and we can continue. So, what you are going to have is cos theta m. So, remember this you could write it as theta m, where theta m is 2 pi to m minus 1 by capital M. That is it. So, if cos theta m because of the first signal and sin theta n because of the first signal that is this one, first component second component minus cos theta n and sin theta n and norm of that .

So, if you continue on this you could get this as So, we just like to recall how do you evaluate the norm.

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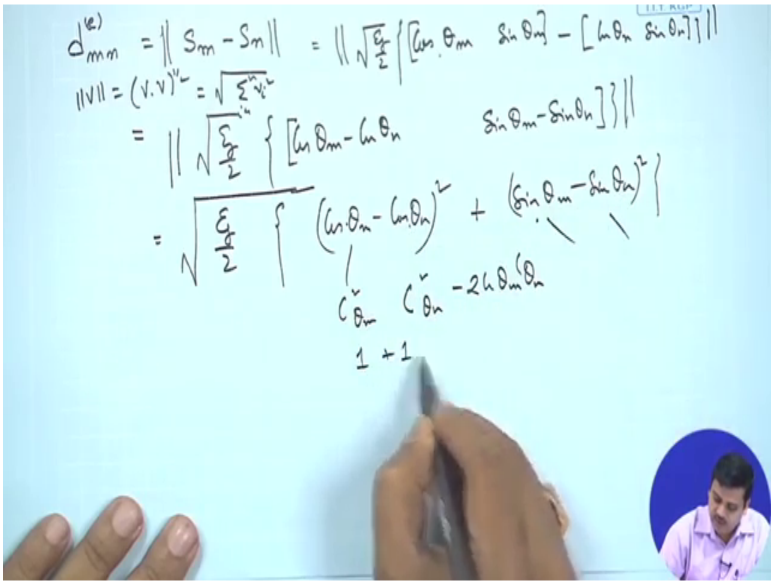

So, if you would remember the norm of this was v dot v half or square root of i equals to one to n v i squared. So, since we have this as a vector we are going to take the subtraction of the first component and the second component. So, we still have to calculate the norm of E g by 2 times the first component is cos theta m minus cos theta n, is the first component and the second component is sin theta m minus sin theta n. So, the norm of this vector this is the first component, that is the second component.

So that means, going by this we are left with square root of, square of this E g by 2 and you have this term squared plus this term squared means, cos theta m minus cos theta n squared plus sin theta m minus sin theta m squared, right. And a square root across the whole. So, if you carry forward this you are going to get a cos squared theta m you are going to get a cos squared theta n, and you are going to get minus cos theta m, cos theta n and same with these terms. If you would collect the cos squared theta

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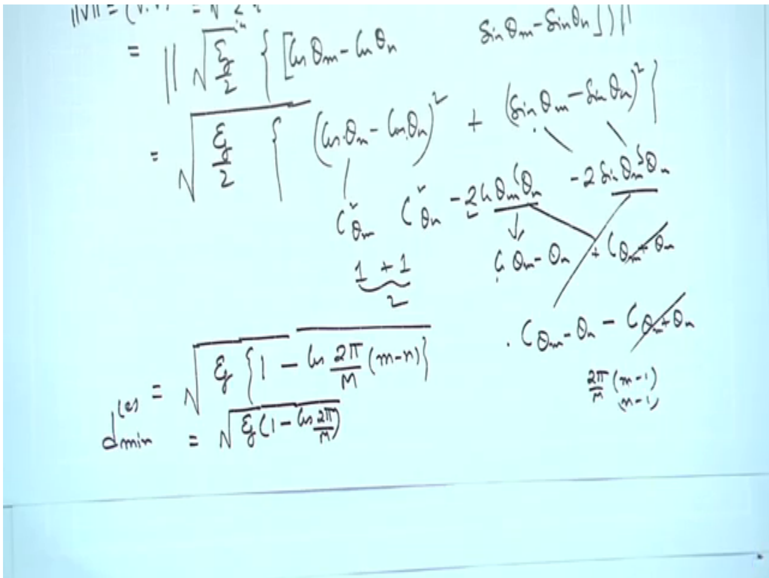

And sin square theta even get a 1 you are going to collect cos squared theta n and sin square theta n, you are going to get a 1 then you will be getting a term 2 cos theta m theta n and you are also going to get minus 2 sin theta m sin theta n.

So, here again you have a cos A cos B and a sin A sin B this would expand to cos of theta m minus theta n and you are going to get plus cos of theta m plus theta n. And this will also result in cos of theta m minus theta n minus cos of theta m plus theta n. So, these terms are going to cancel out you are going to left, we will be left with these 2 terms and this would reduce to the expression. So, there is a 2 over here there is a 2 additional over here. So, what we will be left with is root over which will cancel out with this 2 E g. So, I would urge you to do this particular work by yourself, and square root of 1 minus cos theta m, minus theta n.

So, that will result in 2 pi upon m because, theta m and theta n nothing but 2 pi by m, m minus 1 and the other case it is n minus 1. All the terms cancel out except this m and n. So, you are left with this term. From this you can calculate d min the Euclidean distance the minimum Euclidian distance which will turn out to be root over E g times 1 minus cos of 2 pi by m. So that means, this is the expression of d min we will be using d min squared at some point when we will be calculating the error term and you can compare this with respect to the pulse amplitude modulation and a small note, that for given signal energy the d min squared is 1 which determines the error probability, which will come to seattle at a point you can compare and see which of the pulse amplitude modulation or phase shift keying has a better minimum distance for a given energy of the signals from these sessions.

So, till this point we have covered 2 important modulation techniques. That is, the pulse amplitude modulation where you are selecting an amplitude based on bit sequence. The number of bits you would see group together to select an amplitude would depend upon the number of amplitude levels in the pulse. In the phase shift keying you are choosing the phase based on the based on the incoming bit sequence and philosophically they are similar in terms of you choose one of the possible options. In the first case you choose amplitude in this case you choose the phase and we have given the signal space representation we have given the energy computation we have given the minimum Euclidian distance between 2 constellation points, which will again play a crucial role when we do error probability calculations.

So, with these 2 we would like to move forward to the next important cons modulation technique in the domain of digital communications which is the phase shift keying.

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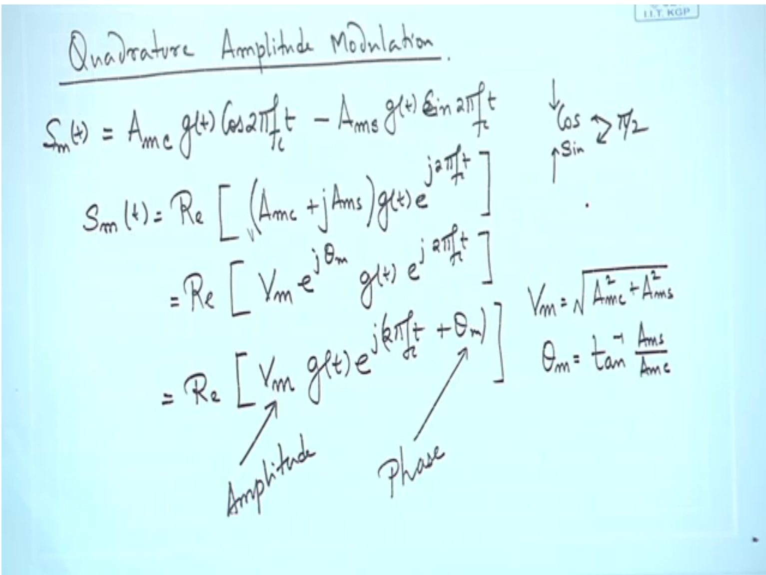

Sorry, which is the quadrature amplitude modulation right. So, when we talk about quadrature amplitude modulation by now all the terms are known to you. So, by modulation what we mean is already known to you in terms of when you study analog communication you studied modulation and what we mean by modulation in the digital communication we have also explained, which consists of 2 parts the symbol mapper followed by the up conversion. So, generally up conversion is what you study in the analog communications here symbol mapper is a special part which we have described.

So, modulation part we have no issues if we talk about amplitude we are clear. So, if you talk of amplitude modulation you already know about it. If it is quadrature we have introduced the term quadrature in the previous lecture and in a few minutes back also we have talked about the quadrature. So, quadrature for us would mean the carrier in phase which is in quadrature with each other; that means, there is a cosine and there is a sin, which are at ninety degree degrees with each other. So, being at ninety degrees, they do not interfere with each other. So, now, when we have quadrature amplitude modulation, although we know the terms individually or in some groups, this when put together has a different meaning altogether.

So, this quadrature amplitude modulation is all about modulating the amplitude of quadrature carriers. We have already seen the modulation of amplitudes of quadrature carrier when we studied phase shift keying. And while doing that, we have actually discussed the expression where we said that if you look at this expression, you have quadrature carriers which we identified and this is the amplitude that is what we had explained. And in the signal space diagram also we had explained the same.

However we may note that the amplitude is dependent on a single parameter that is theta m; that means, the amplitude of the i component and the in phase and the quadrature phase component. So, if we call cosine as the in phase then sin is the quadrature phase, they are dependent only on the angle and nothing else, right. Whereas, if we talk about the, if you are going to talk about the quadrature amplitude modulation, here we are talking about modulating the amplitudes through independent data streams which is contrast, quite in contrast with what we have studied before.

So, we could write the signal representation in various forms. And one of the forms well we could change the order of writing would be the amplitude modulation of the cosine or the in phase component times the pulse shape. And the carrier as well as, the amplitude of the sin component that is why we have an s g t and sin 2 pi f c t. You might recall a similarity with the expression of the low pass equivalent form that we had arrived initially.

So, there we also had x cos 2 pi f c t minus y sin 2 pi f c t. If we see the pulse, if we see the phase shift keying the x and y are related by their phase; that means, we have single parameter by which we can control, but here we are saying that we choose x and y independently and then we can get a phase out of it that is. So, we are getting independent phases. So, independent amplitudes while we are modulating this thing. Effectively we should write S m of t has real part of A mc plus j A mn. So, this is x plus jy according to what we discussed before, g t e to the power of j 2 pi f c t. And this would result in this expression. And we could also write this as real part of some V m e to the power of j theta m where, V m is square root of A mc square plus A ms squared and theta m is the tan inverse of this over this.

So, times g t e to the power of j 2 pi f c t you could also write this as real part of V m g t e to the power of j 2 pi f c t plus theta m. So now, you can see that from this particular expression there is an amplitude and here there is a phase. So, what we can get from this expression is that, the amplitude V m could be selected independent of the phase and vice versa. This could be one view of this the other view could be that I have quadrature carriers and I could select phases independently. And of course, to complete this set of expressions V m is equal to square root of A mc squared plus A ms squared and theta m is equal to tan inverse A ms 1 A mc.

So, if I would choose A mc and A ms I get V m and theta m. Or if I choose V m and theta m I get A mc and A ms in a in either way. So, whatever it is in the amplitude in the pulse amplitude modulation you had only one signal to choose from; that means, you would choose the amplitude only and there was only a cosine so that was the basis function. In case of phase shift keying again you had only one parameter to select that is the phase and you chose from a whole set of phases available one of the phases, right. That did result in a generation of different amplitudes in the i and the cosine, but they were constrained by the phase. Whereas, here in this case you have 2 things to select; that means, amplitude and phase or 2 different amplitudes.

So, these are the 2 different views of quadrature amplitude modulation that is usually available. So, we will take one of the views first and then we will look at the other view after a while. So, we can think of the quadrature amplitude modulation as m 1 level pulse amplitude modulation.

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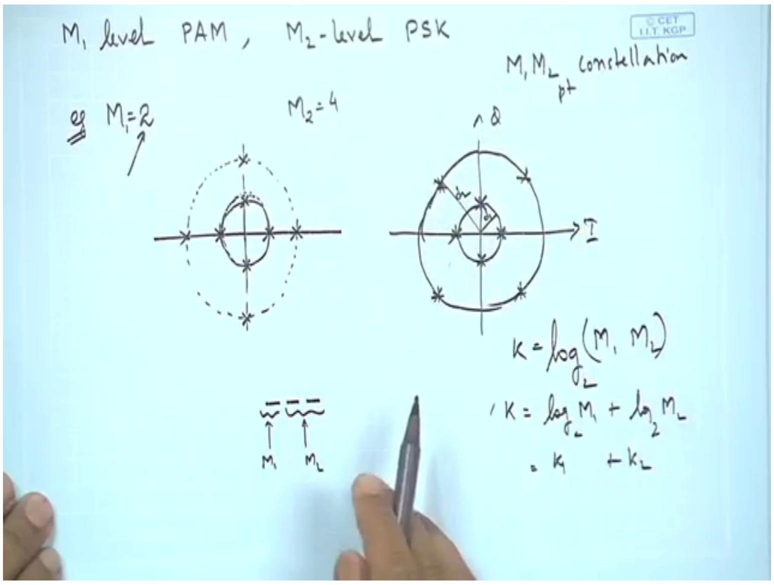

That is here, as well as you have m 2 level of phase shift keying. So, I could give m 1 level amplitudes, I could combine m 2 level phases. So, as an example suppose, I have example, I have m 1 equals to 2 and if I have m 2 equals to 4, let m 2 equals to 4. So that means, I have 2 amplitudes, right. And I have 2 different I have 4 different phases.

So, if we try to draw our signal space diagram. So, have 2 different amplitudes. So, if I select a d and a minus d we are actually taking phase and amplitude together. So, it is not so easy to plot it directly based on this 2 phase. So, what we do is let us suppose we have one amplitude level covering a radius as the amplitude. So, going by this there is a certain radius, right. V m and there is another amplitude which covers another radius, right. And now we have 4 phases.

So, we have discussed 4 possible phases. So, we can mark these as 4 possible phases as well as, the second amplitude could also have these 4 phases another alternative another different option could be that, you have this is one amplitude, this is another amplitude, these are the 4 different phases of the first amplitude and these are 4 different phases of the second amplitude, right.

So that means, our constellation points. So, we have probably used a new keyword at this point constellation points. Constellation points would mean these locations. So, this could be let us say d 1 and this could be let us say d 2, and these are 4 different phases these are 4 different phases. So, overall we have 8 different possibilities. So, when we multiply these 2 we have a total of m 1 m 2 constellation, m 1 m 2 points in the constellation, right. And the number of bits that it can represent would be log base 2 m 1 m 2. And this would be our choice of k. And then you could simul you could easily separate it out as k is equal to log base 2 of m 1 plus log base 2 of m 2, and that you could say is probably k 1 plus k 2 right.

So, what we are trying to do over here is that, you could probably imagine that k 1 bits. So, here if it is 8 so there would be 3 bits involved in this. So, out of 3 bits one bit could be used in selecting the amplitude. So, I have 3 bits because there are 8 possible values. So, 8 possible values would be chosen by 3 bits. So, I have 3 bit locations and So, k is equal to 3 in this particular example and there I am seeing that if m 1 is equal to 2 k 1 equals to one and k 2 equals to 2. So that means, I would select using this I would select m 1 and using this I would select m 2 weather 2 bits possible 0 0 0 1 1 0 1 1.

So that means, I have 2 different sources you can think of it in that way or you can divide it in this fashion or you can look at it as a combination. And here again you could use gray coding in order to ensure that the nearest neighbors differ at most by one bit. So, again that is not a very straightforward and simple job and you could try to write an algorithm by which you would map these bits to constellation points or to these signal locations which would generate the mapping in a gray coding format. So, this is one way of looking at it and now we move forward and try to look at the signal. The way we write it in terms of vector notation.

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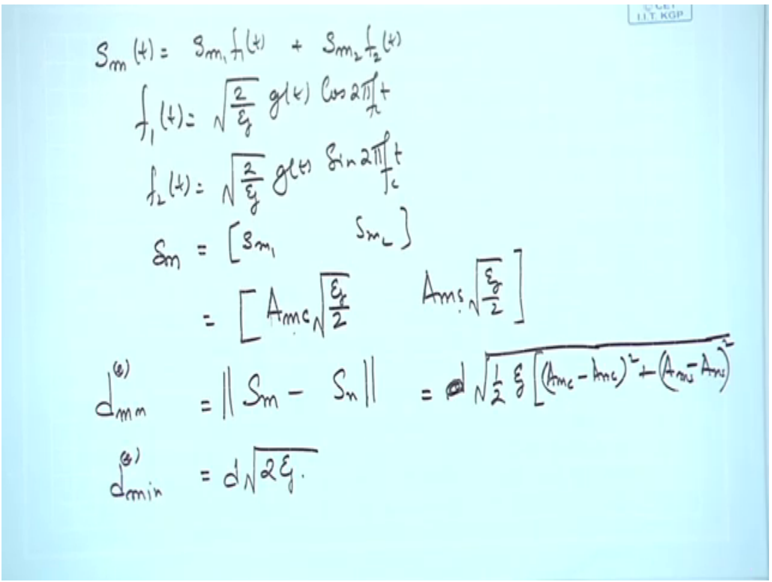

So, we could write s m t has s m 1 f 1 t, because clearly it is not one dimensional this 2 dimensional there is an i and there is a q. So, since it is 2 dimensional I have 2 phases sorry, for 2 basis signals which I would mark by f 1 and f 2. And I could write f 1 similar to the case of PSK as square root of root over 2 by E g g t cos 2 pi f c t. And f 2 t is root over 2 upon E g g t sin 2 pi f c t and S m could be written as S m 1, S m 2 in the vector notation. Where S m 1 would be the amplitude of the cosine components along with E g by 2 So that it normalizes and A ms with root over E g by 2.

So, this way it would be the vector representation and is the signal representation. So, what we have done here effectively combined pulse amplitude modulation along with phase shift keying to generate quadrature amplitude modulation. And this way we have generated a new modulation scheme where you have a lot more flexibility. So, in a in a similar fashion as we have done before you could calculate the Euclidean distance between 2 points as the norm of S m minus S n. S m is given S n would be s a n ca ns and you can keep on doing that and the end result you are going to get would be d and the minimum distance, sorry this would be equal to square root of half E g A mc minus A nc squared as we did before plus a m sign minus an sin squared and the d minimum; that means, the nearest constellation point given by d 2 root E g. Along with these kinds of constellations there is another important constellation way of representing the constellation which is the sometimes called this square or generally the rectangular constellation.

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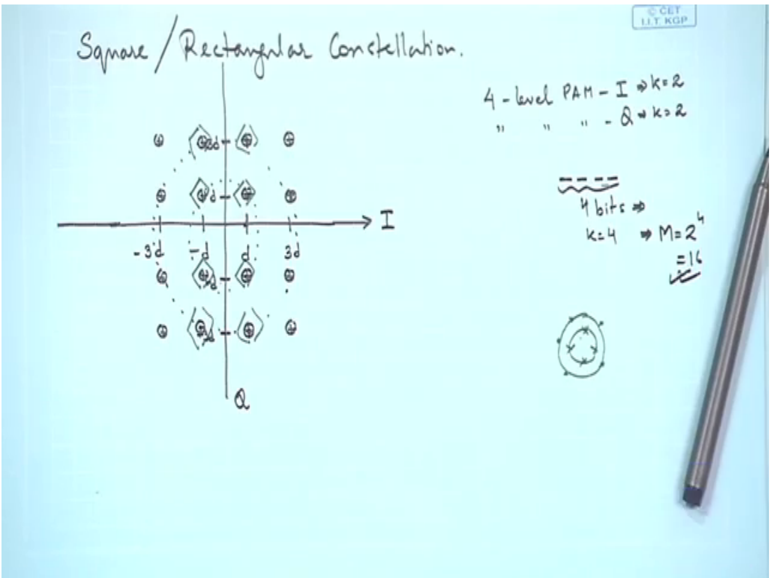

This is an outcome of seeing the signal as amplitude modulation on 2 quadrature carriers whereas, the one that we have represented here is a combination of PAM and PSK, appears I mean they ultimately are the same, but the visualization could be different. And here what we generally do, is suppose this is my i axis and let this be my q axis.

So, here we can think of PAM this is pulse amplitude modulation, if you see this carefully. This one is a pulse amplitude modulated signal, this is another pulse amplitude modulated signal right. So, we can think of a pulse amplitude modulated on the i axis a pulse amplitude modulated on the q axis. So, if you are doing pulse amplitude on the i axis you have a d, you have a minus d you have a 3 d, you have a minus 3 d if you have 4 levels. And similarly you can have 4 levels on the q. So, what we have done is we have combined of 4 level PAM in the i axis to a 4 level PAM on the q axis.

So that means, we have in the i axis and a 4 level PAM in the q axis. So, this would mean that k equals to 2 in the i axis this would mean k equals to 2 in the q axis. So that means, we have 2 bits and will choose these 4 levels we have 2 bits will choose these 4 levels. So, when we combine them, what do we get? When I have So together, we have 4 bits to from the I selection to from the q selection So that means, we have 4 bits So that means, k is equal to 4 combined k which implies m is equal to 2 to the power of 4 that is 16, right. So, let us see whether we get it or not. So, when I select 2 bits on the i axis let us say, have come to this d I all also have to select 2 bits on the q axis. So, I could select all of these points. And then a combination of d and d would be a point here similarly I will get a combination there I will get a combination there I will get a combination there.

So, we are able to select all these possible points. So, when my i is 3 d I will again get these 4 possible points when my i is minus 3 d again I am going to get all these possible points, there is a point here. So, with these circled plus are all possible constellation points that get generated if we think in terms of the PAM signal. So, if we count the total number of such constellation points it matches our number of 16. So, what we see is that this quadrature amplitude modulation can be created or the constellation for quadrature amplitude modulation can be created in 2 possible ways, one is where you select a PAM and you mix it with a phase shift keying. And the other is you could do a mixing of 2 PAMs on separate i and q axis.

So, if we look at this select a subset here. This is a phase shift keying with one level of amplitude. So, there are 4 possibilities; that means 2 bits. Well, done and if I select these, we have 1, 2, 3, 4, 5, 6, 7, 8. So, have 8 possibilities So 3 bits. So, now, if I would select let us say, these are the 4. Along with this I could select; let us say, these I could combine this as PAM with PSK as different phases and different amplitudes, but of course, the combination would not be as straightforward as we had drawn the diagram where we had 2 concentric circles and different phases.

So, here the representation was easier compared to if we represent over here. So, in both the cases we have 8 possible 1, 2, 3, 4, 5, 6, 7, 8 possible constellations, here also 8 possible constellations, but the way we do the mapping is different in both the cases with this different color we could represent with 3 bits and they would only the constellation shape would be different. And you could even form a PAM on i and PAM on q and you would still do a constellation which looks like this. It will be a bit more complicated, but end of the day you are choosing 8 constellation points using these combinations, right both the things are valid. Now what remains is which particular constellation you should choose.

We will continue on this discussion in the next lecture.

Thank you.