Modern Digital Communication Techniques
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Lecture - 28
Memoryless Modulation (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. So far we have discussed a various aspects of communications, amongst which we are currently in the module where we are talking about a linear modulations which are memoryless.

And in that category we have completed or we have covered this pulse amplitude modulation, we have given you the signal representation, and we have also been able to connect the way you represent the signal to the vector representation from where you would represent it in terms of the components. The PAM signal we have identified it is a 1 dimensional signal and where the basis vector or the dimension that we are talking about is the cosine of 2 pi fct, and the signal that we have said is the amplitude or the component.

So, we have also discussed that this particular communication mode can be represented in the pass band and base band and so will be all communication systems. And we have also discussed in a few lectures back that we would like to represent the low pass equivalent form.

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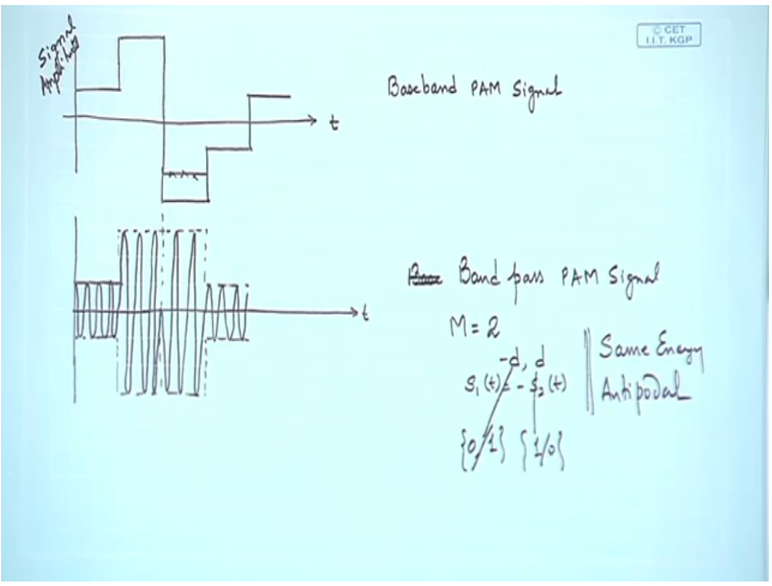

And we would just try to complete our earlier discussion on PAM by trying to see the waveform in the pass band and base band with given a rough picture earlier we will try to give a clearer picture this time.

So, suppose we have the signal amplitude as per this particular figure do not consider this suppose we have a waveform which goes like this and this is the direction of t and this is the signal amplitude. So, this we would clearly call as the baseband PAM signal which you are now aware of we have been drawing this picture and if we would like to draw the corresponding PAM signal in pass band the envelope would appear as in this and there is a separation, but it will continue.

So, since we are talking about the modulated carrier it will be the envelope and will of course, just keep a restriction diagram limited to this. So, what we could do is we could now put a carrier in this right since there is phase continuity this will go on. Now there is a phase reversal at this point. So, we would consider a phase reversal and here there is no phase reversal the phase is continuing. So, this we could say is the pass band or the band pass PAM signal. So, that is how they would look like and we would if it is a carrier less on base band representation it will appear like this if it is with carrier this is how it is going to appear at the receiver which you will see in a few lectures down the line that you remove the carrier and that is what we have been discussing for quite some time.

So, in the pass band pass signal there is a carrier which is put on this or the carrier amplitude is influenced by this signal and at the receiver when this carrier is removed what you get back is this kind of a signal. And in the last lecture also we have identified the special case of m is equal to 2. So, if m is equal to 2 we found that the 2 amplitude levels are minus d and d. So, in that case you could also say that S 1 of t is equal to minus S 2 of t and what you can clearly see these signals are of the same energy and since they are of opposite sign you can call it antipodal as well.

So, these are some of the interesting characteristics which we would like to remind or with which we would like to conclude our discussion on pulse amplitude modulation at this point I would also like to remind you that we have discussed a special way of mapping; that means, these amplitudes that we have decided are selected based on the big sequence. So, if you have 2 amplitude levels only you do not have a problem because

2 amplitudes could be selected as for example, these values minus d could be mapped to A 0 or this could be mapped to A 1 and vice versa.

So, that does not affect much. So, this could be A 0 or A 1 and correspondingly this will be A 1 or A 0 and that is it there is not much of a difference its symmetric whereas, if we had 4 amplitude levels, right. So, if you had let us say 3 d d minus d and minus 3 d then you would need 2 bits 0 0 0 1 1 0 1 1 and we have described that there is with a particular way of mapping you could reduce the possibilities of errors. So, some of these things would be consistent throughout a discussion of other modulation techniques.

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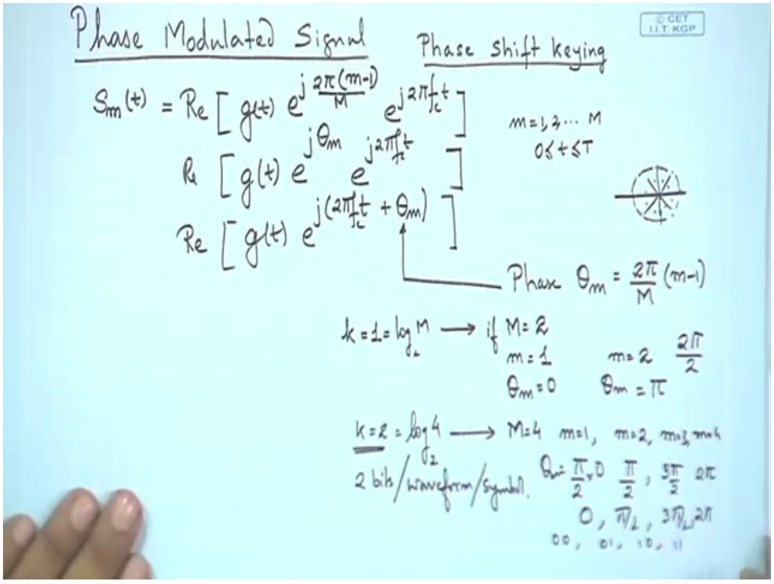

So, at this point we would like to move over to the discussion of the next modulation technique which comes under the category of phase modulation or phase modulated signal you can say right and rather we could call this a phase shift keying, because there is a more appropriate term as will be clear like in the previous case we would call amplitude shift keying here we would call it the phase shift keying.

So, in this particular case the S m of t; that means, the signal which is going out you could write it as real part of gt times e to the power of j 2 pi m minus 1 upon capital M times e to the power of j 2 pi fct where m is equal to 1 2 up to capital M and this is valid for the interval t in the range of 0 to capital t a very simple form of g would be 1 in this range and 0 in the range beyond that.

So, where the definition of gt we would not change the definition of a gt would remain the same as we have used for pulse amplitude modulation where we said gt is the real valued signal. So, there is no much a variation on that it is a pulse shape as we have said, but here if you would note that we have not given any amplitude in the pulse amplitude modulation there was an am where as it is reflected in this form well you could even write it as e to the power of j theta sub m e to the power of j 2 pi fct and of course, the real part of that.

You could also combine these and you could say j 2 pi fct plus theta m. So, when we write it in this form we clearly see that here we have a phase where theta m is equal to 2 pi upon m multiplied by m minus 1. So, if we have m is equal to 2 and then m would take a value of 1 and m would take a value of 2 for m equals to 1 your theta m would get 0 and for m equals to 2 your theta m would be m equals to 2 your theta m would become 2 pi upon 2 which is equal to pi right because 2 minus 1 goes out.

So, it is pi it is one eighty degrees. So, similarly you can calculate m equals to 4 and so on and so forth. So, what we see from this is that 2 pi upon m. So, if this is covering 2 pi it is divided in equal phases and then as you keep increasing m you get the different angles, right. So, what we have over here is the carrier is remaining as it is there is no change in amplitude only m is getting selected right. So, as we did in case of pulse amplitude modulation here in this example we have k that is number of bits to be selected is equal to 1 which is equal to log base 2 of m.

So, for this case m equals to 4 your k is equal to 2 which is log base 2 of 4: that means, k equals to 2 implies you have 2 bits per waveform or per symbol. So, clearly you can write down m equals to 1 equals to 2 m equals to 3 and m equals to 4 theta m we have 2 pi upon 4. So, that is pi by 2 multiplied by for m equals to 1 it is 0 for this case you have pi by 2 for this case you have 3 pi by 2 and this case you have 2 pi.

So, that is how you covered the whole range 0 pi by 2 3 pi by 2 and 2 pi so. Now, it is a matter of mapping the bits. That means, you have to map 0 0 0 1 1 0 1 1 to these. So, if you have to map these bits again we would work out some kind of a gray coding as we have discussed earlier when we look at the signal space diagram.

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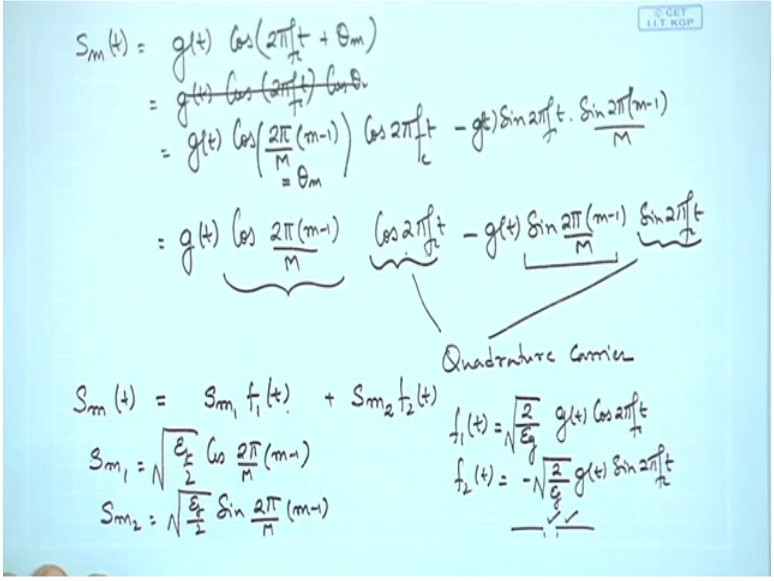

So, moving ahead with this representation; so, you could now rather write the expression of S mt we continue on this particular format that we have is gt. So, we have this whole expression. So, we could write it as gt cos 2 pi fct plus theta m.

So, now you could be using your earlier knowledge of analog communications where you studied angle modulation in an angle modulation you would modify the phase or the angle of the signal in accordance with the source signal. In this case you are modifying or you are choosing a particular angle that is what we have described here that is what we have described here that these are a few angles that are available and as we said in digital communications, we are mainly interested in choosing one of the options. So, here is the same thing that you are doing we are choosing this based on the bit sequence. And this bit sequence is an outcome of a sampling the analog signal followed by quantization followed by source coding and these bits or these levels have nothing to do with the levels of the quantization output I repeat this statement once again.

So, these bits you take whatever bit stream come in. So, even if it is not an analog source it could be a discrete source you just select 2 bits at a time select the phase if you have 4 phases to choose from if you have 8 phases to choose from you would select 3 bits at a time that would help you in choosing one of the 8 possible phases. So, getting back to this expression you could write this as g of t cos 2 pi fct times cos theta m rather I would write g of t cos 2 pi upon m times m minus 1. So, this thing is equal to theta m times cos 2 pi fct minus gt sin 2 pi fct times sin 2 pi m minus 1 upon m.

So, what we have is this gt there is a phase rather if we write it clean you would get a gt cos 2 pi m minus 1 upon m times cos 2 pi fct minus gt sin 2 pi m minus 1 upon m times sin 2 pi fct the reason for writing in this form is that it is going to give us an idea that there is a cosine carrier there is a sin carrier. So, what we have brought into now compared to the earlier the pulse amplitude modulation is we have quadrature carrier components. So, we have quadrature carrier the cosine and the sin and apparently it may be that we are using the same carrier fc. So, these 2 would interfere with each other, but since these are quadrature; that means, at ninety degree phase when we recover the signal we will not have this component interfere with these component things will be a little bit more clear very soon.

So, if we look at this representation we have no change in amplitude only change in phase right whereas, if we look at this particular representation we have a carrier and there is some amplitude which is because of the phase there is another carrier where there is another amplitude being selected and these 2 carriers are of the same frequency, but they are at 90 degrees offset with respect to each other right. So, this is a representation that we can think of. So, we can write this signal in a form that is what we desire as S m 1 f 1 of t plus S m 2 f 2 of t. So, what we mean by this is that we are interested in finding out the basis functions or the orthonormal functions f 1 and f 2 and would like to represent the signal as orthonormal expansion in terms of basis function.

Now, by simply looking at this we can identify that this a cos 2 pi fct and sin 2 pi fct can serve as the orthonormal functions because these 2 are orthogonal with respect to each other and then we could use a similar form as we had done before we could say f one of t is equal to root over 2 upon eg where eg is the energy of the pulse times gt cos 2 pi fct and f 2 of t is equal to minus 2 upon eg gt sin 2 pi fct; I would like to mention at this point that for the interval 0 to t the cos and sin are making them orthogonal to each other and for time intervals beyond. That means, if we are considering time intervals let us say this is 0 to t this gt is making the symbol here and the symbol there orthogonal to each other these things will be clear again when we take up multi dimensional signaling and in that case we could say that S m one would be square root of eg by 2 cos 2 pi upon m small m minus 1 and S m 2 is eg by 2 sin 2 pi upon m m minus 1.

So, this is fit in this multiplied by this these coefficients cancel out gt cos 2 pi m minus 1 times cos 2 pi fct we have the first term and when these 2 are multiplied we get the second term and of course the minus sign is there.

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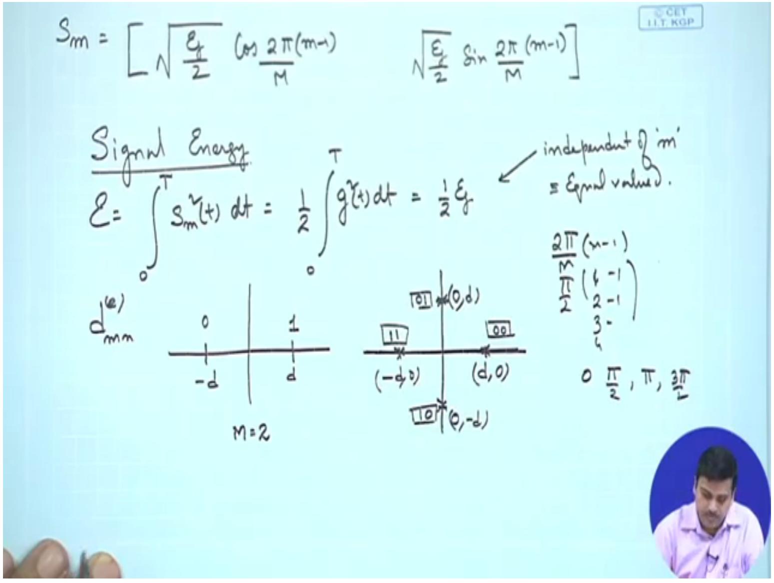

So, continuing on this expression mode we could write that S m which is the signal vector form it could be written as 2 components the first component is eg upon 2 cos 2 pi m minus 1 upon capital M and the second component is eg by 2 sin 2 pi n minus 1 upon capital N.

So, let us say; so, if you look at this whole configuration or the setup that you have created. So, what we have got is from this expression we have been able to create the vector notation that we are generally interested in. So, once we have this vector notation or we can use our typical vector rules to be able to proceed with the computations for this particular signal model. So, as you choose a value of capital M and you change values of small m you can create all sets of these coefficients as we did for pulse amplitude modulation we would be interested in the signal energy.

So, signal energy calculation e would be 0 to t this is the symbol duration S m t squared dt and which would turn out to be half of 0 to t gt squared dt, that would be clear from these expressions. In fact, it can be clear from this expression right because you have a real. So, there is a cosine you have this. So, from this these relationships come and which is equal to half of eg right. So, there are a few observations to be made at this point one of the observations is that this is independent of m; that means, it is equal valued clearly this is in contrast with pulse amplitude modulation because in pulse amplitude you have different amplitudes. Since you have different amplitudes for example, we take this the energy in this will be different from energy in this particular waveform.

So, since these 2 are having different energies there is an average energy consideration which is there, but here all of them have the same amplitude and this is also clear from the expressions that we have and we have also described there is no amplitude is a constant amplitude only phase changes. So, these are all consistent. So, these are equal valued and quite distinct from the pulse amplitude modulation and the other important thing to note over here is that since the amplitude is a constant.

Therefore, it is a constant modulus you can say; that means, the amplitude is not fluctuating. So, typical whatever advantage you had for angle modulation or phase modulation even to get similar advantages in this also and the only difference is that there you take a continuum a set of angles or you take all possible angles between 0 to 2 pi, but here you have only fixed angles to select from. So, you have quantized that is the major difference in this.

The next important calculation which we should make is the d Euclidean distance between 2 points. So, before we do this calculation we can have a look at the signal space diagram. So, if we choose m equals to 2 we have already done the calculations before we have done the calculations before. So, in the signal space diagram we have at, let us say there is an amplitude d; that means, at 0 angle and another at pi angle. So, 0 and pi d n minus t right so that means; it is clear? Now that for m equals to 2 we said that pulse amplitude modulation and a phase shift keying they would appear the same and that is what we have over here and this you could mark with A 0 this you could mark with A 1 and vice versa.

This you would identify for m equals to 2 for m equals to 4 again we had worked it out before. So, these are the angles that you have. So, one is at 0 degrees this is the other one which is pi by 2 another is 3 pi by 2 sorry A 1 is at 0 pi by 2 sorry we have made a mistake over here 3 pi by 2 and over here. So, these are the 4 phases that you would turn out that that we would get. So, we will just need to work it out 2 pi upon m multiplied by m minus 1. So, that is pi by 2 multiplied by 1 2 3 and 4 minus 1 that is what you get 0 you get pi by 2 then you are going to get pi. So, we made a small mistake over here pi and then you are going to get 3 pi by 2. So, these are the angles that you are going to get.

So, there is a small change over here right. So, these are the angles that you would get and now you note that it is no longer A 1 dimension signal and that is also clear from the expression that we have over here from the signal space also; it is clear? Because, you are going round in 2 pi you are selecting these angles you could also say that if this is my I axis this is my q axis then I have 2 options in the I axis I have 2 options in the q axis.

So, this notation would be useful and you could mark you could say that d comma 0 is this point in the Cartesian coordinate this coordinate is 0 comma d this coordinate is minus t comma 0 this coordinate is 0 comma minus t and then this you could mark it as 0 0 bit sequence this could be A 0 one bit sequence this could be a 1 1 1 bit sequence and this could be A 1 0 bit sequence.

If we do this matching or mapping what we would find is that any 2 neighboring phase they differ by 1 bit if you would compare. So, inherently we have made a gray coding out of this and for A 8 phase; that means, when m is equal to 8.

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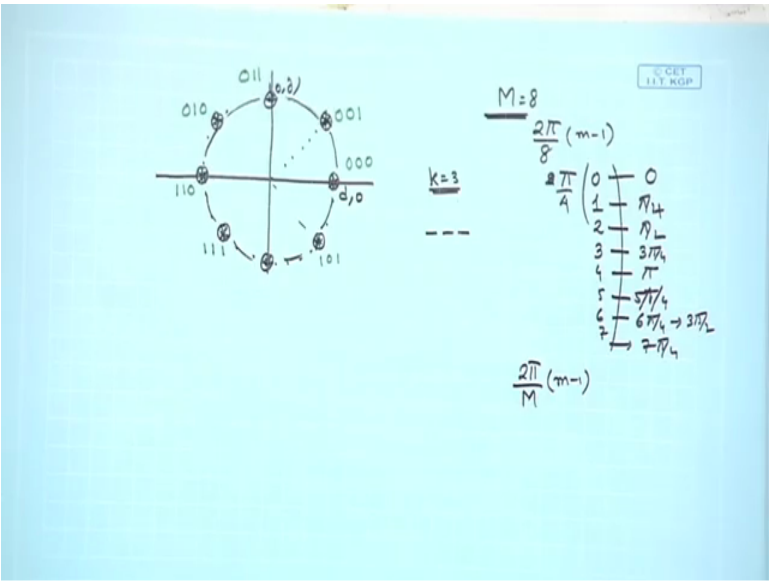

So, you are going to get 2 pi by 8 multiplied by m minus 1 or you are going to get pi by 4 multiplied by 0 1 2 3 4 5 6 and 7 that is what you are going to get leading to 0 leading to pi by 4 leading to pi by 2 leading to 3 pi by 4 leading to pi leading to 5 pi by 4 6 pi by 4.

That means, 3 pi by 2 and leading to 7 pi by 4. So, these are the angles that you are going to generate considering these angles 0 is one of the angles and then you have for 1 it is pi by 4.

So, the second is pi by 2. So, this is another coordinate because your amplitude is constant and then you want to get there this is one of them that is 3 pi by 2 and this is another; this 7 pi by 4 1 2 3 4 5 6 7. So, this is 7 pi by 4. So, this is I do not remark them and you could claim this as d this will be scaled accordingly with square root d comma 0 0 comma d and so on and so forth.

Now, comes the issue of mapping this with a a particular bit sequence. So, since there are 8 levels 8 possible phases. So, definitely your k is will be equal to 3 and when k is equal to 3; that means, you would select 3 bits. So, one of the possible ways of mapping would be we could say let us put 0 0 0 to this then we could put 0 0 0 0 0 1 to this and then we could have 0 1 1 and followed by this 0 1 0 1 1 0 1 1 1 and so on and so forth and 1 0 1 and so on and so forth.

So, with this you could again create similar to a gray coding and you could carry forward. So, what we could see here the difference between the analog angle modulation and the digital modulation scheme is if we look at this particular diagram we have only a fixed set of angles to select from. And we will be choosing only these angles based on the bit sequence and not a continuum set of angles where is an angle modulation in phase modulation in analog communication you are selecting a continuum. So, you would select any angle there as well; whereas, in this case you are selecting only the angles which you have marked right a fair note at this point would be that in case of a amplitude modulation as well as angle modulation we have defined a specific way of selecting these phases and in this case of course, we have said 2 pi upon m times m minus 1.

so; that means, we have uniformly divided the total angle available into equal intervals of angles and we have selected the angles now this is not restrictive. That means, you need not necessarily follow this rule you are free to choose a other combinations and same with the amplitude, but when we discuss a QAM very soon things will be clear that there are certain criteria based on which your choice should be made. So, that you get a particular desired capable waveform.

So, with this particular discussion we conclude this lecture, and we would like to continue on this in the next lecture.

Thank you.