Modern Digital Communication Techniques
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Lecture - 27
Memoryless Modulation (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. So, we have been discussing pulse amplitude modulation. In the previous lecture we have seen the details of the mapping procedure and why do it call it a symbol mapper. We have also identified that the difference between the levels at the output of the quantizer and the levels at the output of the symbol mapper. So, we should never get confused with these 2 things.

The symbol, the quantizer output levels are used to choose a bit sequence. These bit sequences are used to re choose another set of amplitude levels. And the number of amplitude levels you now choose from is quite different and has no relationship to the number of levels at the output of the quantizer. These are completely different things. So, we should never get confuse with that.

Now when we talk about choosing the quantizer amplitude levels sorry the pulse amplitude levels and that is why we are pulse amplitude modulation what we are trying to discuss is how to map the bit sequence to the amplitude levels it is just not a very simple mapping because we are saying that these waveforms that are getting selected would go into the channel. So, if they go into the channel it should be able to resist or pass through reliably through the channel.

So, of course, there are several ways to improve reliability and one of the standard techniques is forward error correction codes and I have always said that this is not going to be discussed in this particular lecture, but we would definitely cover some essential components which are necessary and which are hardly bit complex, but would add a lot of value to the modulation techniques. So, let us get back to our discussion and we have identified that.

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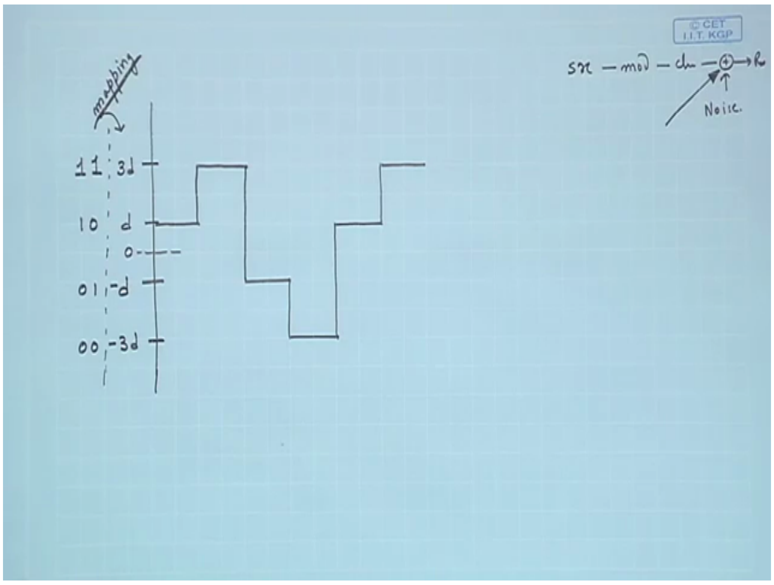

Suppose we have the pulse amplitude modulation levels. So, let this be 0 and let this be a level minus d let this level be minus 3 d let this level be d. So, we are trying to follow a dimension which makes sense and we said that suppose when a 1 1 comes I would map to the level 3 d when a 1 0 comes I would map to level this when a 0 one comes this when a 0 0 comes this.

So, this is the mapping. So, this is the mapping that you do mapping bits through or symbols equal to these are symbols through waveforms or bits to symbols whatever way you want identify it. So, now, we said that suppose we have a waveform: for example, as this an arbitrary waveform example that I am taking and not following the earlier example that we had taken. So, this has just purely for the sake of discussion right and then we say that noise. So, this is the waveform that goes through the channel and of course, we have said that there is it is possible to have a carrier less modulation; that means, you could use a carrier and you may not use a carrier if you are not using a carrier in that case your signals here fc simply 0.

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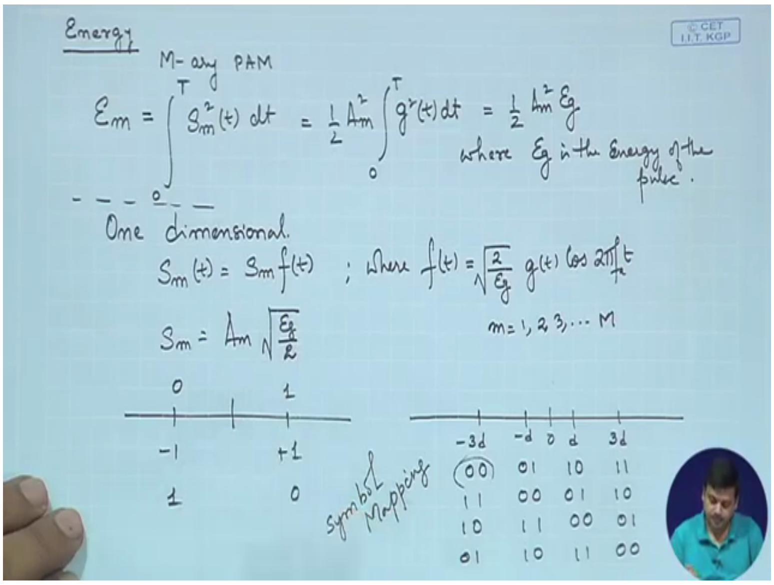

And you have ft equals to root over 2 by eg times gt which is the pulse duration simple wave value of gt could be one for the duration of 0 to capital T and S m t would be S m times f t f t is this part and S m remains as this same. So, it is not a big big deal to think in terms of carry less because we have already always said that we are going to discuss in terms of baseband communication right. So, then we say if you recall our earlier discussion there is source there is modulator then there is the channel and there is this noise getting added before it enters the receiver. And we also said that this noise is because of all kinds of components that are present in the receiver and we one of the simplest channel models that we can think of is the additive wide Gaussian channel noise.

So, when we said it is white Gaussian noise we mean it is additive in nature right and we did describe that additivity does not is not is not a problematic, because this noise that gets added you may not be able to determine and recover the original signal, because this noise is independent of the source symbols that what that is what is important to note. And we said its Gaussian because of the probability density function of noise that we had described in the previous lecture.

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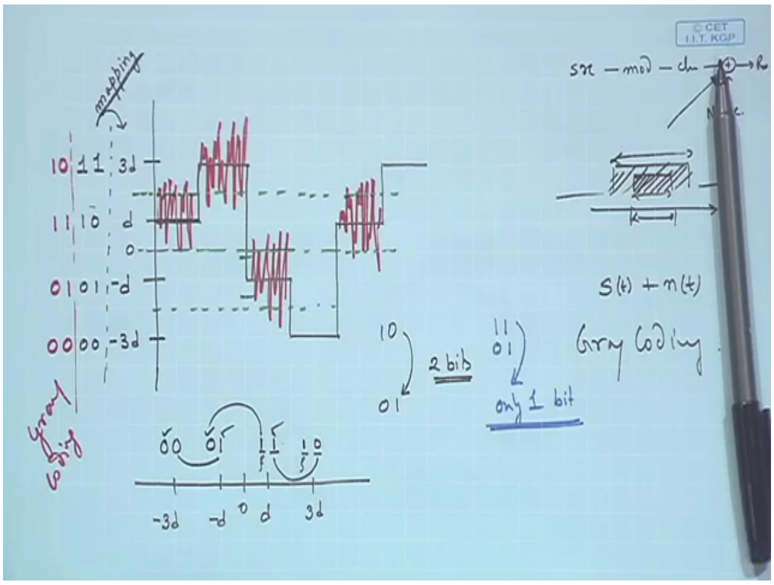

And we said it is white and the spectral properties we did discuss and at that point we have said that you can think of narrowband white stochastic process in the sense that I have a filter which is wide enough to pass my signal right this is the filter width this is the signal bandwidth if this is f and this noise also passes through this portion and noise is not pass through this portion.

So, in the portion of the signal whatever noise is present is white we could say that- so that is a narrow band and white noise which has Gaussian pdf and it is white within the band of our concern. So, getting back additive white Gaussian noise could produce a time domain waveform which is like this which is 0 mean, we already we have always said that is 0 mean. That means, if I am sending a voltage let a d then noise of 0 mean gets added to that.

So, for example if I send s of t which is 0 mean I am going to add noise to it; it is 0 mean. So, therefore, the mean is that of the mean of S t and if the mean is that level d; that means, combined signal will be around level d right and here the noise is basically random right and so on and so forth and we also said that there is a decision level right there is a decision level and there is a decision level. So, these are decision levels. So, what do we mean by decision levels is that when I receive the signal.

So, suppose something has been sent what does the receiver do receiver simply observes this compares it with a threshold right if the received signal goes above this threshold it would say that a plus 3 d is the receiver amplitude, because in this part it is no resolution if the received signal voltage is below this decision and above this decision. So, if it is within this band if it is within this band you would map it to d. If it is above this you would map it to 3 d if it is within this band you would map it to minus d if it is below this band would map to minus 3 d, right

So, now, if the signal goes above this decision point it would map to 3 d. So, what it means is if 1 0 was sent and because of the noisiness it would detect it as 3 d you are going to detect it as 1 1. Similarly here you have sent minus d. However, it could be detected as plus d or it could be detected as minus d depending upon whether noise has shifted this level in the upward direction or downward direction beyond the threshold if noise is restricted to within this level within this level in that case minus d you would be detected as minus d. So, what I am trying to say is that when d is sent it could be detected as d or 3 d or minus d.

Now you could kind of guess from this we are not going into all details which will do at an appropriate time at a later part of the different modules of this lecture you can guess from these pictures what I have almost try to influence to you is that a d can be recognized as the d itself or 3 d with certain probability and minus d with a certain probability right because and detecting 3 d instead of d is the probability that it crosses this threshold and detecting it as minus d is the probability it crosses this threshold right and there is a less probability that it gets detected as minus 3 d. Similarly, this one has a more chance of getting detected as d compared to minus 3 d which is even more compared to minus 3 d.

So; that means, 3 d getting detected as minus 3 d is very less chance 3 d rejected at minus d its probably little bit higher and 3 d getting detected as d is a little bit higher chance. So, all I am trying to tell you is that there is a high probability of getting detected as a neighboring amplitude compared to further away amplitudes right. So, this is one philosophy which is used in mapping these bits to the amplitude levels so that we can get less number of bits going into errors. For example, if we take these particular examples. So, what we have over here is a d getting detected as 3 d you have 1 bit in error or d getting detected as minus d would also now a d getting detected as minus d look at this particular is interesting.

So, suppose I have noise right noise is this. So, this is a decision threshold this is the decision threshold. So, if d gets detected as minus d it gets it is minus d when this threshold is crossed. So, it is 1 0 and it is 0 1. So, a 1 0 gets detected as 0 one; that means, when this symbol goes into error 2 bits goes into error, right. So, there is a concept known as gray coding the idea behind gray coding is that that you would try to map the particular bits to symbols in such a way that when these changes occur or this neighboring amplitude levels are mapped by bit sequence in a way that the neighboring bit sequence differ by at most 1 bit.

So, what I mean is if we look at these particular possible options. So, if we see this we have 0 0 followed by 0 one. So, this and this are the same only 1 bit difference, but here this becomes a one this becomes a 0. So, there are 2 bit differences. So, this does not qualify as gray coding. So, if we take a look at this again the same thing happens over here 0 can become a 1 1 can become a 0, because it is a neighbor, right. Whereas, if I look at this particular sequence here also you have a same situation right a one becoming a 0 or 1 becoming a 0. So, we need to find a mechanism by which you could avoid this and let us try and do that. Suppose I can just make it horizontal for ease and use better use of space. So, you can have a d minus d 3 d I am only taking four levels all right. So, let us try doing that let us put 0 0 into this then we need to make 1 bit change.

So, let us make 0 1 from minus d 2 plus d we can make 1 bit change. So, 0 could remain as 0 this could become a 0 this we already have. So, we do not select that option. So, with other keep of these to 1 which is fixed and this we change to 1 right. And now we need to make another option and that option is clear. So, this one would remain a 1 and this 1 remain a 0. So, what we see between these 2 neighbors this is common only there is a difference over here right here we see that this and this are common and there is a difference only within these 2.

And similarly between this and this we see that this and this is common only there is a difference between these 2 and 3 d and minus 3 d are not neighbors we do not need to worry 3 d n minus g are not neighbors we do not need to worry this and this are not immediate neighbors we not do not need to worry. So, let us remap these things minus 3 d is 0 0 minus d is 0 1. So, here we flip. So, what we do is instead of this you make it as 0 0 0 1; now we make 1 1 and 1 0.

So, with this new mapping what we are going to get is that if you are making a mistake here we take the same example you have a 1 0 that was sent as d. So, look at this we are just simply changing the mapping. So, suppose I had sent a d and I detect it as minus d as we took in this example. So, we would have a 1 1 that is 1 1 over here being detected as 0 1.

So, these we see differ by only 1 bit. So, whenever you make a error in detecting a particular symbol or an amplitude at most you would make 1 bit error that is the philosophy of this particular method of coding, right.

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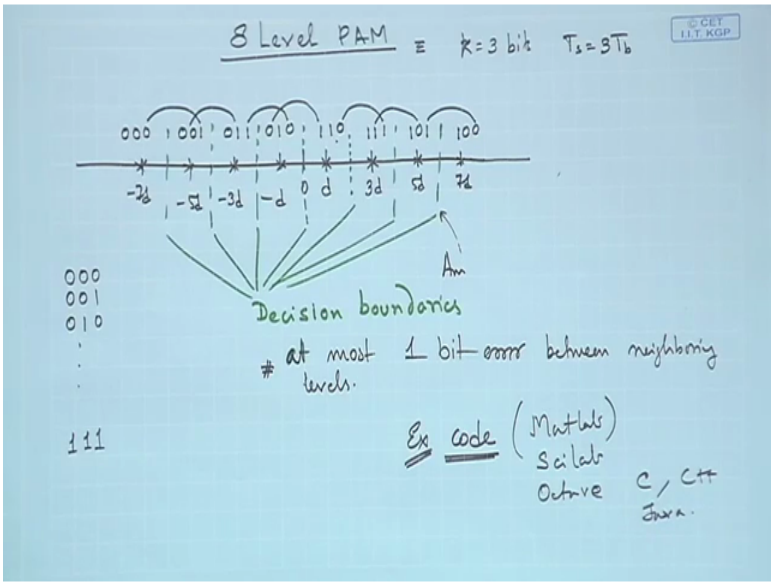

So, if we move on further we can try and take another example and or you could try it yourself in different forms we would do it. So, let us take that you have eight level PAM right just a small note at this point I am discussing gray coding here in pulse amplitude modulation. We will not discuss gray coding in these details as we discuss future modulation techniques. We would assume that you simply extend this philosophy to this other modulations right. So, moving on to eighth level PAM this simply implies that you have eight amplitude levels; that means, k is equal to 3 bits; that means, ts the symbol duration is 3 times tb.

So, if there is a 0 and this is d you have 3 d you are going to have 5 d you can do this calculations of finding am right these are the Ams that you will get minus d minus 3 d minus 5 d we have 6. So, we have 7 d and minus 7 d so, now, you 1 2 3 4 5 6 7 8. So, you have 8 levels right. Now, the problem that is with us; so these are the amplitude levels the problem with us is you have these bits up to 1 1 1 because they have to be mapped to these amplitude levels right. So, there could be many solutions let us try one we will begin with 0 0 0 you could then go to 0 0 1 and then we could say 0 1 we have kept this to same and I have only modified this then you could have a 0 a 1 a 0; that means, you are modified only this bit.

So, between this and this you modified this bit and then we could have a one in the middle a 0 on the right one over here; that means, we have modified only this bit right further moving down we could say 1 1 and 1. That means, you are modifying this bit then we could say 1 1 0 make this and then you could have 1 0 0 modified displayed. So, in this way what we see that these 2 neighbors differ only by 1 bit this and this neighbor differ only by 1 bit in the middle bit this and this differ only in the extreme right bit this and this in the extreme left bit and so on and so forth. So, by this way if I am sending level d and by mistake I go to level 3 d; that means, this is my decision point these are my decision boundaries right these are my decision boundaries.

Then the neighbors; that means, if I am sending minus j and detecting as d you are actually sending 3 bits per amplitude levels. So, whenever you make a mistake at most you are going to make at most 1 bit error between neighboring levels right. So, that is what we have achieved you could try for yourself as an example or as an exercise that can you find some other mapping instead of the one that we have displayed over here and could you write a short algorithm or a script by which you could automatically assign these bit sequences to the levels in a gray coding format.

So, you can try out an exercise where you can develop a MATLAB code let us take MATLAB right you could use Scilab which is a freeware you could use octave you could use C or C plus plus or java or whatever you feel like, so that you write a small program of your own by which you would take bit sequences and then you would map this bit sequences to amplitude levels right. So, what I am trying to hint to you at this point is you should start your activity from now on a particular programming language through which you should be able to simulate the performance of a communication system in future and since we have started the communication part here. So, the digital modulation this is the appropriate time to start it and of course, you could have done similar things for source coding.

But we could assume that there is a source of bits which may have come through a compressed a compressor or may not, but now we are at the stage where we are actually sending these bits into the channel. Therefore, this is a very crucial part that we are into and I would highly recommend that you would start using some software where you could develop this as the first algorithm in the domain of digital communications all right.

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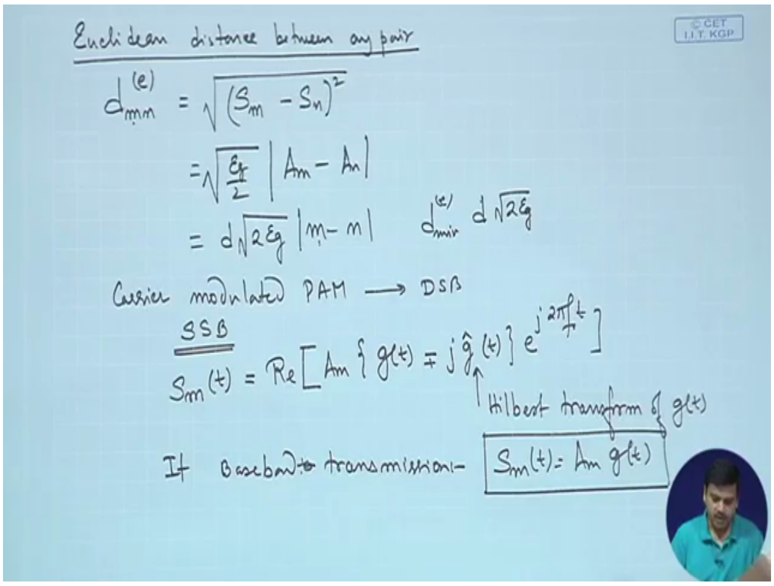

So, with this we move forward that hope we have understood some of the basic concepts of gray coding and at this point we are also interested in writing or calculating the Euclidean distance between any pair these things would be important when we start discussing error probabilities.

Now we have already given indications of how an error can happen we will do details of calculating the probability of getting into an error; that means, if you send an amplitude d what is the chance that you will detect it as some other symbol and the moment you detected or some other symbol it is an error. So, overall we will try to calculate on an average; what is the chance of making a mistake. So, that is going to happen at a later time, but when we do that whatever we are writing now would turn out to be very very handy; so, d mn e that is what the notation that we write Euclidean distance between the level m and level n.

. So, we define it as S m is the symbolic coefficient minus S n. So, this you can think of the connotations squared and S m we have already described how we should write S m that is we have it here right. So, from this you could calculate this as you could calculate this as root over eg upon 2 times mod of am minus 1 which would turn out to be d square root of 2 eg into modulus of m minus n, right. So, this would be useful and for m and n being neighbors. That means, if you would say that I want to calculate the minimum distance which is minimum. So, m and n would differ by at most a value of 1. So, you would get d root 2 eg; that means, the minimum Euclidian distance.

Between the 2 neighbors that is the closest right is between any 2 m and n could be any 2 amplitude levels further at this point I would also like to point out that the carrier modulated PAM that you have is basically a double sideband that is very very clear because you have am gt cos 2 pi fct. So, if you look at the spectrum it will be double sideband right and what we could do is instead of selling double sideband you could send a single sideband because its spectrally more efficient and in that case the S m of t could be written as real part of Am gt plus or minus j g hat of t where g hat would be Hilbert transform e to the power of j 2 pi fct.

So, this is how it would look like and this is of course, the Hilbert transform of gt and you could we have already said that if you are talking about baseband transmission then we would write S m of t is equal to Am gt and that is also clear from here because fc 0 cos 2 pi fct is 1. So, we have S m which is this multiplied by this portion. So, this and this cancels out you have am times gt. So, we already have seen this expression.

So, this is not very different and we have also described, we have also described; how does the signal look like. So, still we can draw a representative picture of the particular waveform that we have with us.

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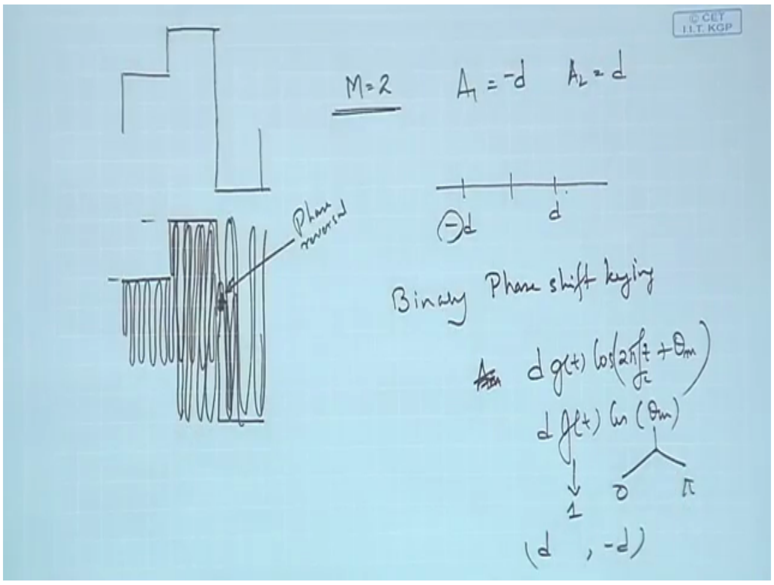

So, if for example, we have a waveform let us say; which is of the shape, right. So, if it is baseband it is just this particular envelope and if it is with carrier modulation I need to redraw this picture. So, if it is width carrier and you are going to get sorry I made a mistake here this will go up till here and here there will be a phase reversal in the phase reversal and after the phase reversal you are going to get this. So, this is with carrier this is in baseband and generally this looks like the amplitude modulation.

However you have this discrete levels that we have already discussed and the special case when you have m is equal to 2 we have already discussed this and we should be careful over here we have said that a one is equal to minus d and a 2 is equal to power plus d. So, I just need to give a reminder at this point. So, if you look at these 2 levels what you would also encounter is phase shift keying which will study later. So, this is also known as binary phase shift keying, because if 2 signals and the difference between 2 these 2 signals is this phase which is 180 degrees there is a minus in front of it. So, in other words you could also look at this particular signal and you could say that there is a plus theta is a 2 pi fct plus theta.

And theta is 0 at one point and theta is pi at another point and if it is. So, so; that means, fc would be 0, but you are going to get am gt cos of theta and am is constant in that case which is d only theta changes. So, just briefly let me write down you are going to get am or rather I would write d times gt cos 2 pi fct plus theta m. So, if fc is equal to 0, you would get d times gt cos theta m and theta m could take a 0 or pi and if this is equal to one for 0 you are going to get a d for pi you are going to get minus d; so which is same as this; which you are going to encounter at some later stage.

So, all I want to point out here is although we have discussed PAM at a very special case where m is equal to 2 the PAM would look very similar to a binary phase shift keying which we will start discussing in the upcoming lectures.

Thank you.