Modern Digital Communication Techniques
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Lecture - 26
Memoryless Modulation (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. So, we have started discussing modulation methods in which we have just started discussing about pulse amplitude modulation. In pulse amplitude modulation we have identified that there is a pulse whose amplitude gets selected based on the input bit sequence that is the information sequence. And we have also mentioned earlier in the beginning of a discussion on modulation digital modulation techniques that in contrast to analog communications in digital communication there is a choice from a finite set.

So, I would again like to revisit that particular aspect before we proceed further, because in the previous lecture we have just discussed that there is a sequence of bits which are used in selecting the particular choice of waveform. So, that is the part where you are mapping the bits to the waveforms or you are mapping symbols to waveforms or a basically mapping bits to symbols. So, that is why we identified that this particular part of the communication system could also be known as symbol mapper. So, this is sometimes a bit confusing. And hence I will try to re explain some of the things so that it clarifies that what exactly is being used to select the waveforms.

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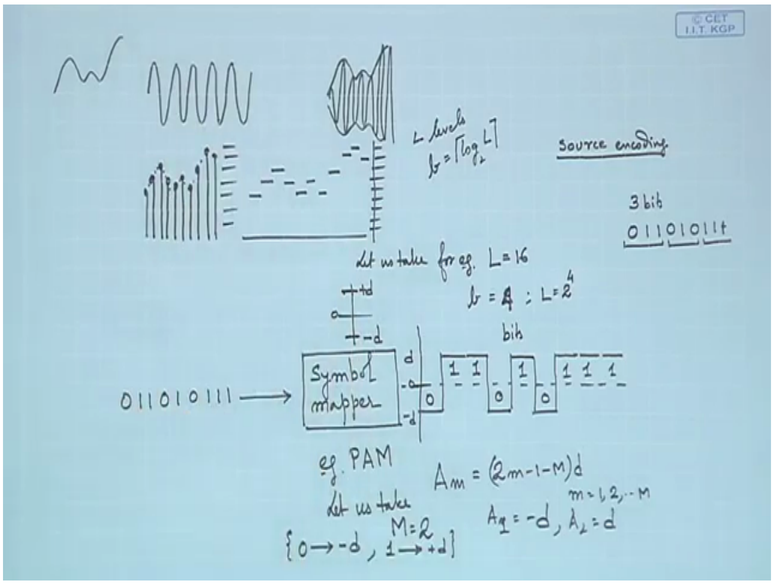

So, if you consider a typical analog waveform let us say this and we have a carrier let us say this. So, if you want to modulate the carrier with this waveform what you typically get is a envelope and you would get the analog amplitude on the carrier right. Whereas, if we are talking about a digital communication system the first step that we generally do is to sample the source; so when you are sampling the source let us take for example, the particular waveform as the source signal. And due to sampling you we are going to get let us say I will. So, we will be sampling at a rate which is more than twice the maximum frequency content. So, let us imagine that these are the samples.

And then these pass through a quantizer, so if we have certain definite levels. So, we could imagine as if you are at this level then you are at a level here then you are to level here then again back there back somewhere there again there again there something like this of the form. So, quantization means that instead of having continuum amplitudes you are now having discrete amplitude levels.

The next step in the process is that suppose there are certain numbers of levels. So, you would have to put some kind of coding method. So, if there are let us say L levels; L levels. So, you would need log base 2 of L as the number of left as the number of bits. So, let us take let us take for example, L is equal to 16 in this case this which could be used as b we would say that b would be equal to 2 because L is equal to sorry b is equal to 4 because L is equal to 2 to the power of 4 right ; that means, there are 4 bits required to identify each level uniquely in a fixed coding and then they could be source encoding that would depend upon the type of encoder as well as the variability of this. That means, the entropy of this let us assume that these 16 symbols would on an average require 3 bits let us take for example, because of redundancy suppose we have this.

So; that means, on an average we are going to get 3 bits per symbol; that means, per sample right. So, basically we are getting a sequence of bits where we are getting 3 bits on an average out now this bit stream would go into the symbol mapper that we have just discussed in the previous lectures. So, for example let us take pulse amplitude modulation and for simplicity let us take m is equal to 2.

Now if we would remember what we discussed that means for the amplitudes that we take the 2 m minus 1 minus m times d where m is equal to one 2 up to capital M. So, here m takes a value of 2; that means, m equals to 1 into the result we got was a 1 is equal to d; that means, 2 minus 1 that is 1 minus 2 that is minus 1 times d that is minus d and a 2 is equal to d; that means, we have at our disposal to select from 2 levels if this level is 0 this level would be minus d and this level would be plus d right and our symbol mapper could be designed in a way that 0 maps to minus d and a 1 maps to a plus d.

So, in this case the symbol mapper output would be a minus d followed by a plus d followed by a plus d followed by a minus d followed by plus d followed by a minus d followed by plus d plus d and a plus d so; that means, this is corresponding to 0 this is corresponding to 1 1 A 0 A 1 A 0 A 1 A 1 A 1. So, this is d this is minus d and this level is of course, 0 right.

So, what we can understand from this is that this these levels that the pulse amplitude modulation modulator selects has nothing to do with these levels right; not in an exact sense of course, these bit streams are generated because of these levels, but these levels these levels do not map to these levels. Because, clearly you can see that we have used b bits over here and we have taken the example where it is 4 bits, whereas this symbol mapper is taking one bit at a time you could also get things clearer.

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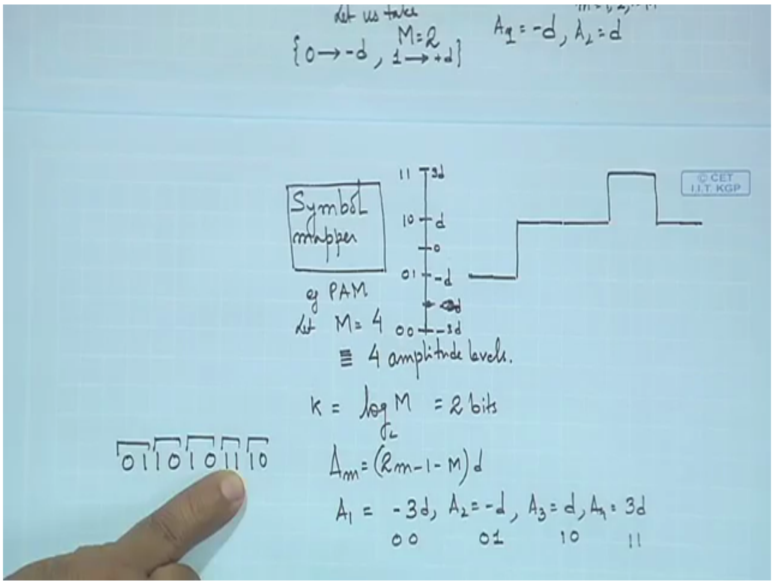

If you consider that let our symbol mapper that is with us again we will take the example of PAM because that is what we have learnt is let m is equal to 4.

So, when I say m is equal to 4 this implies that there are 4 amplitude levels right. So, if there are 4 amplitude levels; that mean, k is equal to log base 2 of m which is equal to 2 bits; that means, now we are going to select the amplitudes using 2 bits. So, if we are taking this sequence. So, we have the input sequence 0 1 1 0 1 0 1 1 1 suppose we have the sequence. So, we have to find what amplitude levels to we have. So, if we use this same method.

So, we are going to get A 1 is equal to 2 minus 1 that is 1 minus 4 that is minus 3 times d it is minus 3 d A 2 that is 4 minus 1 3 minus 4 that is minus 1 minus 1 times d is equal to minus d A 3 is equal to that is 3 to 6 minus 1 5 minus 4 it is one which is d and a 4 you can now put 4 over here 8 minus 1 7 minus 4 that is 3 times d is 3 d right. So, you have these levels. That means, we could say that this symbol mapper has levels if I mark this as 0 this would be minus d this would be minus 3 d let us say. So, ideally speaking this should be minus 3 d and if this is d then somewhere here should be 3 d right and in that case these sequence would map to certain levels like I would take 2 bits. So, these 2 bits have to be used in selecting an amplitude 1 0 has to be used in selecting that.

So, in the previous case we have seen 0 would map to minus d one would map to plus d. So, we could find a similar mapping we could say 0 0 would map to this amplitude 1 1 would map to this 1 0 would map to this and 1 1 would map to this. That means, whenever this is 0 0 I will select minus 3 d whenever its 0 1 I would select this whenever there is A 1 0 I would select this whenever there is A 1 1 I would select this. Now, what happens whenever there is 1 1, I would go there and I would select minus d.

So, now minus d would be selected now please take a note that we have discussed in the previous lecture that since there are 2 bits used for the symbol duration in that case the symbol duration would be equal to the duration of 2 bits right so; that means, this minus d would hold for a duration which is 2 td right followed by you have 1 0. So, then you going to get 1 0 then you have another 1 0 should be like this then you have 1 1. So, which will be here and then suppose we have a 1 0 let us say let us say 1 0; that means, you are back over there right.

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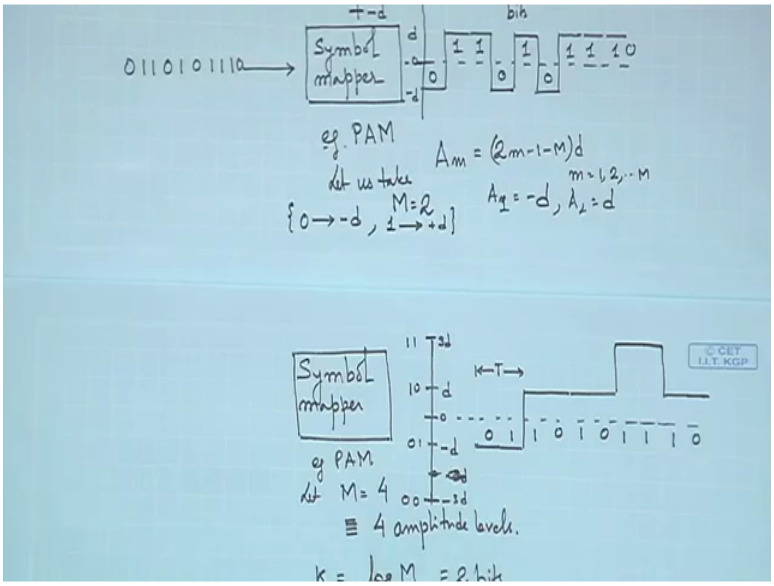

So, now in contrast if we compare the 2 waveforms that we have caught what we see is that this is the 0 level same as the dashed line and we have a 1 1 followed by 1 0 followed by 1 0 followed by 1 1. And let us say we have A 0 in all places right followed by 1 0. So, what we note is that this same sequence produced a different output waveform compared to this one and in both the cases since this bit rate was the same this bit rate would also remain the same the symbol duration is of course, changed

So, here this is t duration, but it is producing different levels. Now how we use it is another matter which we can continue to discuss. So, all I am trying to point out in this particular discussion is that the sequence that we get here which is due to this is used in a different way when it is sent to the symbol mapper all I am trying to say is that these levels do not mean the output of pam. So, they will get confused with this is a typical confusion that we get among students. So, we should clarify that these levels are the levels of the quantizer whereas; these levels are the levels of the symbol mapper. So, this is something important that we should remember right.

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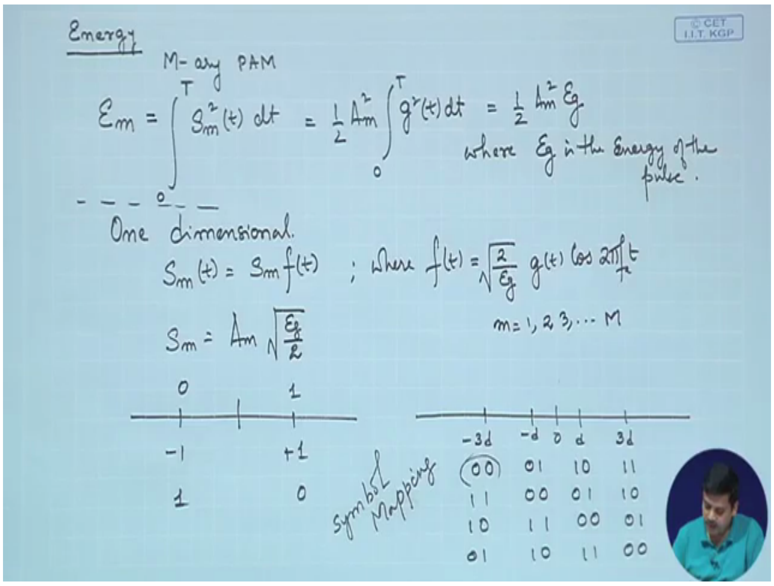

So, moving on further once we have discussed this particular thing what we can do is we can compute the energy. So, when you compute the energy of let us say M- ary PAM M-ary PAM if m is equal to 2 you have binary if it is 4 you have 4 ary let us say or 4 level PAM 8 level PAM and so on and so forth. So, if we are taking a look at the computation of energy for M- ary PAM we would say e m is equal to integrate 0 to t t is the symbol duration s m of t there is a symbol squared dt. So, this is the energy of the mx symbol right. So, this would turn out to be half a m squared 0 to t g square t dt and this is the energy of the pulse. So, you could clearly write it as half a m squared eg. So, where eg is the energy of the pulse?

So, now using this will be using this in one of the important calculations that is going to appear soon. So, moving ahead further we could say that these particular sing signals are one dimensional it will be clear very soon. Now why we are calling it one dimensional because you are selecting only the amplitude of the signal there is no other thing that you can select. So, there is a gt times cos 2 pi fct and you are having an a m on top of it. So, this a m is the only parameter that is changing gt cos 2 pi fct is remaining constant for the interval small t between 0 to capital T.

So, we would generally write down S m of t as some S m which is the coefficient you can think in terms of the coefficient times ft. So, we would like to recall our discussion on grams to the orthogonalization procedure and there we had orthogonalized a set of signal using a new set of signals which we would call it the basis signal and then you could expand any signal using the basis signal by projecting the original signal on to the basis.

So, whatever are the components on the basis would be used to denote in a vector notation. So, we have a similar form over here and we could say where ft is the function is root over 2 by eg, because we are normalizing the energy, because if you take ft square you are going to get eg in the denominator and this will be g gt squared. If you would integrate you are going to get eg in the numerator eg in the denominator and they would cancel out each other cos 2 pi fct and this would cancel out with this half. And in that case we could write S m is equal to am times square root of eg upon 2. So, if you would look at this signal we have S m which is am root over eg by 2 and ft where this. So, these 2 terms cancel out you have am gt cos 2 pi fct which was the expression of S mt in the earlier discussion. So, this is valid for m equals to 1 2 3 up to capital M.

Now, you could easily represent this S m on a diagram like this which we have been using and we could say if we have 2 levels where 0 would indicate to a minus d let us say and this would indicate to plus d. So, you could represent the signal in this form which we have already seen and of course, this is with normalized energy. Now at this point it is crucial that we look at what kind of mapping that we do because we are discussing the symbol mapper we have already talked about 2 different kinds of symbol mapper; that means, one which selects between 2 amplitudes and the one where there are 4 amplitudes levels.

So, in the 2 amplitude level case your life is simple. So, you could select the negative with 0 or you could also use this notation and then things will be quite manageable there will not be any differences, but if we look at the 4 level mapping; that means, if we take the example that is here then situation is pretty different, because you have several combinations. For example: if let us say this is 0, this is d, this is 3 d this is minus d minus 3 d we have already identified one particular combination or mapping right which is indicated in this particular amplitude selection. So, this maps to this level this maps to this level and it is also indicated here is also indicated here.

Now, one might ask that what if I do this kind of a mapping or what if I do this kind of a mapping right or maybe some shifted version; finally, you might have this. So, the question that comes up is out of these which mapping would one choose one could go for any other combination as well. So, I have taken 4 combinations right. So, if we look at this particular way of arrangement then we are basically talking about the mapping or we are talking about the symbol mapper right. So, which symbol? So, this is a symbol would map to which amplitude level is a issue of discussion.

So, you could do all possible combinations now we would like to come back to a discussion of white noise. So, during the discussion of white noise we talked about certain correlation properties we should also discuss about the pdf; probability density function.

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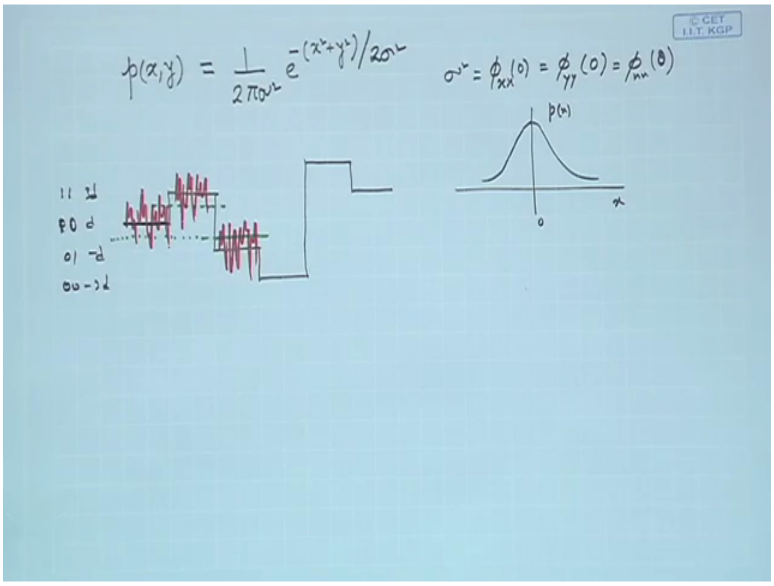

So, for quadrature; that means, the noise with inq components the probability density function would be given by one by for case of Gaussian noise which we had actually studied where x and y are the inq components or the quadrature components of the baseband equivalent noise and we said the pdf would look like this where sigma squared would be phi xx of 0 we have already discussed phi x x phi y y and I would not go back to them again notations remain the same. So, that is why we have been saying always will follow the notations.

So, if we go by this density function and since we have said that in case of Gaussian because of the un-co-relatedness you can say that they are independent you can treat px separately from py or px and py the joint distribution is the product of the distributions right. So, px would be the one without the y component and py would be the one without the x component and typically the Gaussian distribution would have a pdf which would look like this right let us say p of x and this is 0 right and this is x this is how it would look like.

Now, when we are sending these signals out into the channel this gets affected by noise we have already discussed that there is this additive noise. So, if we would see an example that let us say we have these levels these get affected by noise. So, noise gets added right and then what happens if we set a decision level which is half way. That means, if so, I should mark these levels of course that this is let us say minus d minus 3 d d and 3 d in this case if this is my decision level. So, if the signal falls below this decision level I would detect the signal level as d whereas, if it goes above this level I would detect it as plus 3 d similarly if it would fall below this level I would detect it as minus d if it goes above the level I would detect it as d.

So, we are landing into the problem of errors. So, what we would like to highlight at this point is because of noise one would make a choice which is not the right choice; that means, if you have sent a d, but because of presence of the noise fluctuations you may detect it as plus 3 d or you may detect it as minus d because there is a decision threshold here right. So, because of this these colored fluctuations if the fall below is threshold instead of detecting it as d and d had a particular symbol mapping.

. So, if we go by one of the particular mapping that we have used here that we have used here d had map to 1 1 and 3 d had map to 1 1 and minus d had map to 1 1 this at map 2 0 0 to the implication goes like this that if I make a mistake in detecting d as minus d I would instead of detecting this is 1 0 this is instead of detecting 1 0 I would detect as 1 1 now this would get me into error.

Similarly, if I go on the higher side instead of detecting 1 0 I would detect 1 1. So, at this point we would come to a conclusion of this particular unit. This particular lecture simply by stating that there is a possibility of mapping these symbols in a particular way whereby you could reduce the potential chance of making more errors. Of course, there are lots of techniques to reduce errors.

And we will start the next discussion of how to select these mappings whereby you can reduce the number of bits that go into errors.

Thank you.