# Modern Digital Communication Techniques

**Prof. Suvra Sekhar Das**
**G. S. Sanyal School of Telecommunication**
**Indian Institute of Technology, Kharagpur**

## Lecture – 21
## Characterization of Signals and Systems (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. In the previous lecture we have seen the relationship between the output of band pass channel when excited by the band pass signal. And that relationship is straightforward for us because it is the outcome of a channel whose impulse response is given by h t. That is what we have defined.

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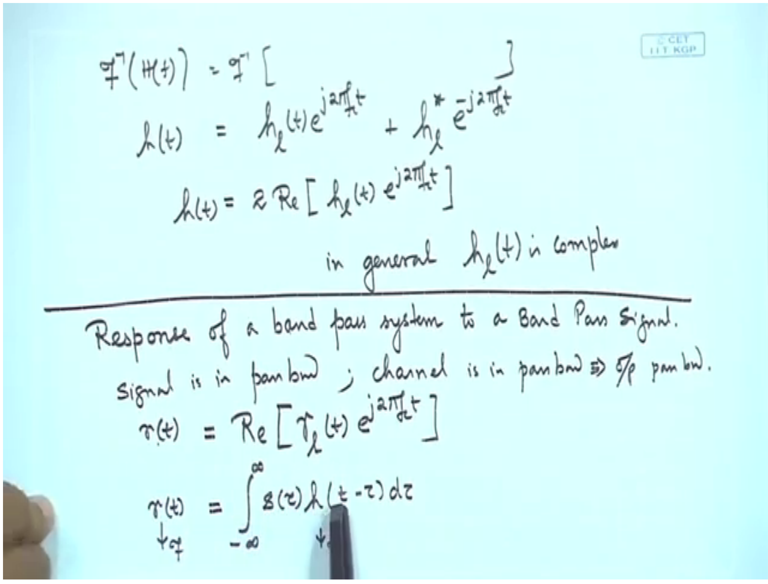


So, output r t is a convolution of S t and h t. And if we take the Fourier relationship it turns out to be this.

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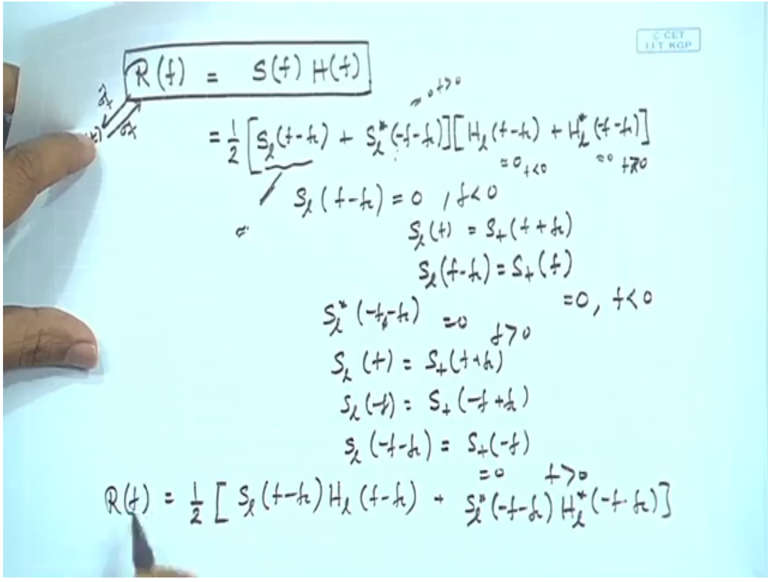

Now, we have already seen that S f has an expression which is S l of f minus f c plus S l conjugate minus f minus f c. And H f we have already defined here that you can see we have already defined in the previous lecture. So, we followed by the same thing. So, instead of the half you simply have H l of f minus f c plus h l minus f minus f c.

Now if we look into these expressions we could say that S l of f minus f c is very close to 0 for f less than 0. And one could argue in a way that in some form, one could say that S l of f is S f plus f c if you remember this. So, then you need to do this translation. So, one could have S l of f minus f c should be equal to S plus of f, I would simply replace f by new minus f c. So, I am going to get this so, this is equal to 0 for f less than 0 because we have taken only positive set of frequencies over here.

So that means, what we are saying that this value is very small or negligible a 0 for f less than 0. Similarly if you take this one S l of minus f minus f c this value is also 0 for f greater than 0 that is very, very small you could say. To argue on this front you could also set up in a similar way that S l of f is equal to S plus f plus f c and then S l of minus f is equal to S plus minus f, plus f c and then you could say that S l minus f minus f c is equal to S plus of minus f. And if I would put f greater than 0 so, you want to get S plus of minus f; that means, negative values which is 0.

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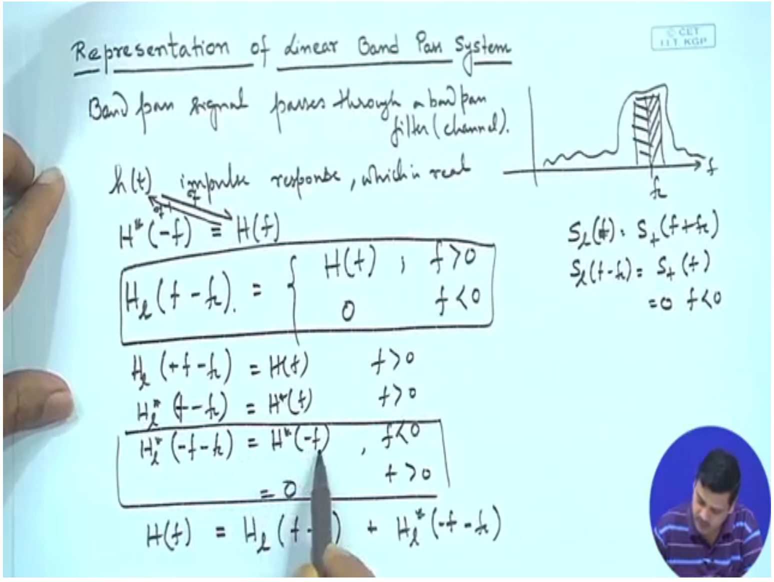


So that means, this thing is nearly 0 for f less than 0 and this thing is nearly 0 for f negatives similarly if we go by the definition of h of f, what you have over here. What we see is that this is H f for f greater than 0, 0 for less than 0 and this quantity is 0 for f greater than 0. So, we could we could say at this is almost equal to 0 for f greater, than 0 this is almost equal to 0 for f less than 0 in a same manner.

Now if we see these things. So, for f less than 0 what will find is only these 2 terms exist right. Whereas, for f greater than 0 this term goes to 0 this term goes to 0 for f greater than 0. So, for f greater than 0 only these 2 terms exist. So, we could have this is half that stays with us as S l f minus f c, this gets multiplied with this h l f minus f c right. And for the other half; that means, for f less than 0 we have S l conjugate minus f minus f c sorry this will be h l conjugate, minus f minus f c. Now if we look at these relationships.

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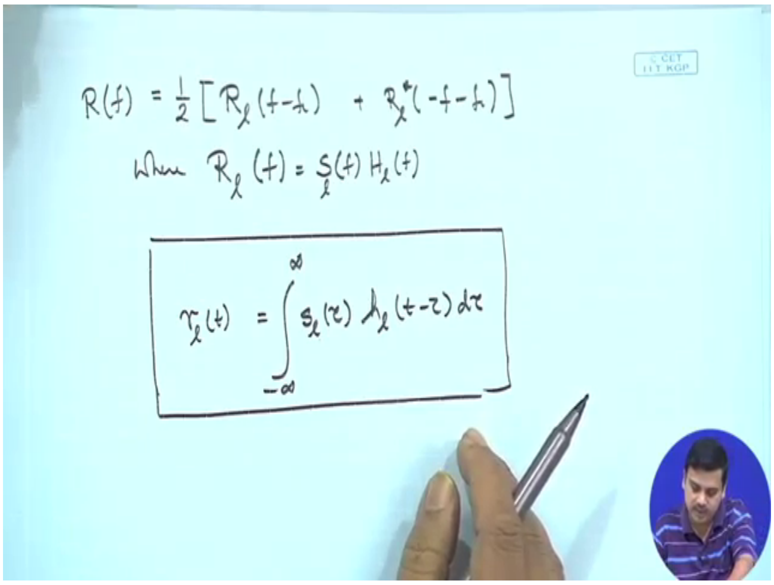


That are available with us, what we could say is that we could write this potentially as half. R l f minus f c plus R l conjugate minus f minus f c right.

So, in other words we could say that where these relationships make sense if only we would define R l of f is equal to S l of f multiplied by H l of f. And therefore, we could see that we could relate this R of f over here, and this is R of f over here to it is low pass equivalent form in the same way and since r of R l of f is related to S l of f and H l of f. In this way we can say that we low pass equivalent signal and the channel could help us in getting the pass band spectrum of this particular signal and since this is a multiplication in the in the Fourier domain in the in the frequency domain. So, what we have is R l of t what we are defined before you could write this as convolution of S l of t and h l of t.

So, this is what you could write as the relationship between the 2 signals; that means, the low pass equivalent in terms of the low pass equivalent signal and the system. So that means, this way we can say that if we take this relationship and if we would take this relationship that we have here with us. Together we could make, together we could say that you can find the outcome of the system which when excited by a passband signal. So, the channel is passband, the passband output is r t and you could relate it through the low pass representation of the signal as well as that of the system.

So that means, once again we have been able to show that given a low pass equivalent representation you can well easily translate to the corresponding passband signals representation between the time domain or between the frequency domain. One could also calculate and relate the energies of the low pass equivalent to it is passband equivalent right. So, we continue in this same mode, but now what we would like to do is we have to look at noise. Noise is very important.

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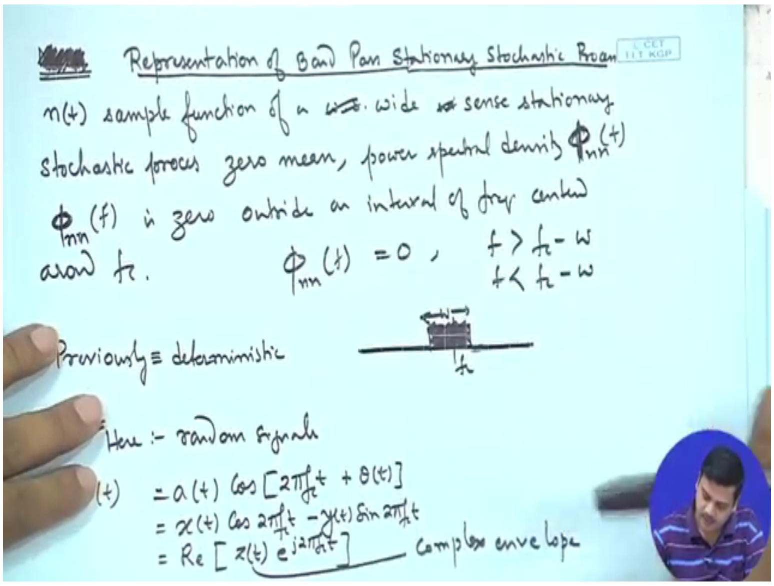


So, we said in the beginning noise is one of the important aspects. So, when you talk about noise of course, what we are going to do immediately is not noise will apply always us to noise. So, you may think of noise as stochastic process or the outcome for stochastic process. And as a stochastic process you can you can describe it in terms of a random variable which takes values as a function of time.

So that means, it is a time evolution of a random process. So, when we said noise we will finally, come to noise and we will see how is it represented and what are it is relationship in the passband with that of the low pass equivalent and how do we take care of this. So, what we will consider is suppose n t that n t be a sample function of a wide sense stationary. So, let me write this down equal of a wide sense stationary stochastic process with. So, what we are writing rather I would not put noise in this particular, we are going to discuss noise, but what going to what we are going to do is with presentation of band pass stationary, stationary stochastic process.

So, once we do this we will find how noise relates to this description and then whatever we derived in this particular thing would be directly applicable to that of noise. So, let n t be the sample function of a wide sense stationary stochastic process. And we also state that this is 0 mean and power spectral density phi nn f. So, phi nn f is given where we are saying that phi nn. So, writing phi nn to represent the power spectral density. What we could do is, there might be a confusion of notation. So, we will put phi nn of f, as the noise is defined in such a way that this is 0 outside and interval of frequency centered around f c. So that means, what we are saying is this phi nn of f is equal to 0 for f greater than f c minus w and for f less than f c minus w. And within this band; that means, if there is a band w so, within this band it has some values right.

What we are not saying that it is defined within this band. It is here elsewhere it is 0 is it centered around this f c. So, we see that there is lot of similarity with what we have done already before, except what we said. What we need to state is previously, what we did previously we did deterministic signals and now that we are handling here we are talking about random signals. So, you may recall in one of the earlier discussions we said that how could you characterize signals. So, the previous section that we just did is about deterministic signals and here since we are dealing with noise or we are talking about stochastic process, which has the senior and randomness we are basically talking about signals which are randomly in nature.

So, we already have encountered some of the characterization of signals which we have started off with.

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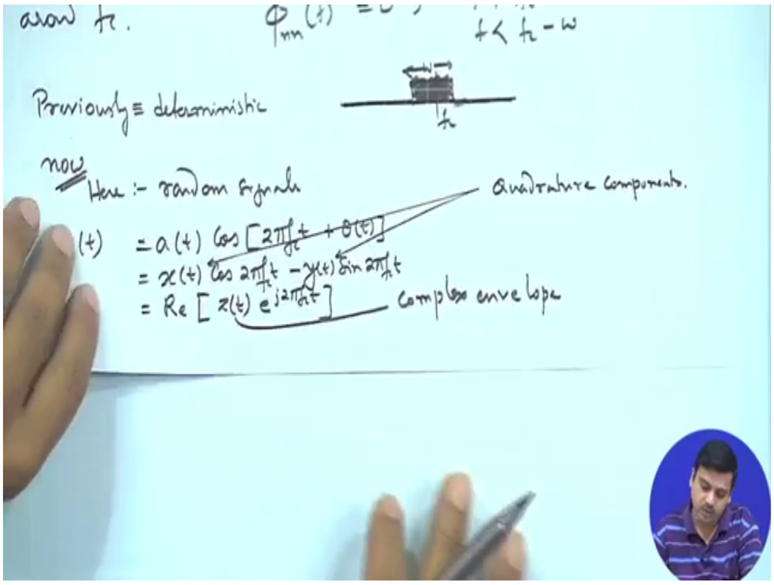


So, in such a case when this is what we have with us we can write, we can write that n t could be represented as at cos 2 pi f c t plus theta t. Now this again holds true which you could write in terms of x t cos 2 pi f c t minus y t sin 2 pi f c t, and again which you could write it as real part of z t e to the power j 2 pi f c t. So, please note here that all that we have said here is, this is a sample function of a signal which is band limited to w centered around f c. And w is much, much smaller than f c, it is a narrow band signal and whatever be the signal, if it is narrow band and centered around f c. You could represent it in a form which is something like this and of course, z t is the complex envelope right of, of this.

So, now what we will deal with is, at this point you have said something. At this point we said that it is a wide sense stationary process right. So, if it is wide sense stationary process. So, we write W.S.S. in short. So, since it is wide sense stationary, we would get this phi xx of tau which is the autocorrelation of the quadrature component, x quadrature component x and y are the quadrature components of n t. So, you could write these 2 is quadrature components. Why we call it quadrature? Because this is on the cosine this is with the sin. So, this is on these 2 sin and cos they are at quadrature ninety degree phase shifted. So, this is you can amplitude of this, is the amplitude of the sin component. So, we call these the quadrature components of noise.

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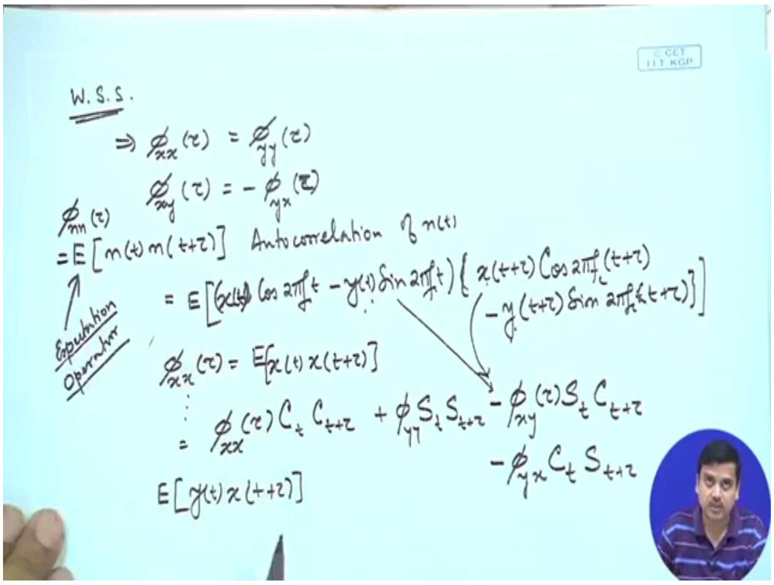


So, phi xx is the correlation of the, of the b component you can say is equal to phi yy of tau and you will also get phi xy of tau is equal to phi minus phi y x of tau. What will do very quickly, is we would like to see whether this is true in very quick steps. And then will use these property to find use these relationships to find some important relationships between the passband and low pass spectrum, power spectral density as well as autocorrelation function of this noise. Now we should be careful at this point that if it is deterministic signal, we are dealing with the low pass representation of the signal and the passband representation of the signal. We are talking about the spectrum of the low pass equivalent as well as we are talking about the spectrum of the passband.

Now if it is noise you do not have a representation of the signal, you do not have a form of representation that it will evolve with time. So, in that case we are rather interested in autocorrelation functions we are interested in the power spectral density to represent or which characterize these processes. So, with the help of these will see how things are. So, will start off with expectation of n t with n t plus tau; that means, we are taking the autocorrelation. So, this is the autocorrelation of n t. This is expectation e is the expectation operator. So, expectation operator it means that you are taking the ensemble mean; that means, over the pdf. So, we have e we would use this expression of it is of this kind of a form and we will be using that form to arrive at the autocorrelation and pass petrol density.

So, we have x cos x of t cos 2 pi f c t plus, minus y of t sin 2 pi f c t, this is n t. And we also have x of t plus tau cos 2 pi f c t plus tau minus, y t plus tau sin 2 pi f c t plus tau. This is what we have with us. And we are going to get an x with x multiplication, x with y multiplication, y with x and y with y. So, when we have the x with x term we are going to have x of t times x of t plus tau, and we will be having along with that the cos of t and cos of t plus tau.

So, when we have this, this expectation operator would work on x t. Because x t, as we see here is the component of noise in the i phase and y of t is the component along the sorry this is along the i this is along the q right. So, when we apply expectation operator on n t it could apply on this and this because this is deterministic part, there is no randomness in this, this complete deterministic with randomness in x of t.

So, what we have done here in this representation if it makes you confused. It is basically translating this x and y which is the baseband or low pass equivalent parts to the pass band by means of this carrier right. Things will be clearer with more later on. So, when we have this expectation operator this will work on x t and x t and not an cost. So, that is So, we will have phi xx of tau is basically e of x t x of t plus tau.

So, similarly we are going to write all the relationships. So, what we will get here is phi xx of tau, I will writing short cos 2 pi f c t and cos of 2 pi f c t plus tau. I will write this in short, and we will get along with that plus phi yy, look at this y and this y we are going to get sin of 2 phi f c t into sin of 2 phi f c t plus tau. And then you want to get e of x and y you are going to get this cross term. So, when you get this cross term you are going to get a minus and in this case also you are going to get a minus. So, we are going to get 2 terms that is phi xy of tau along with sin of t and cos of t plus tau, and we will get another term phi y of x with the cos of t and sin of t plus tau. So, this term x you see there is a t so that means, we are having this term and we are having this term.

So that means, these 2, these 2 terms right. So, these 2 terms make this particular product. So, what we have is expectation of y of t and x of t plus tau is written over here. Similarly this term is a result of this and this. So, that is how this relationship come. At this point we would have to use some of the trigonometric expansion.

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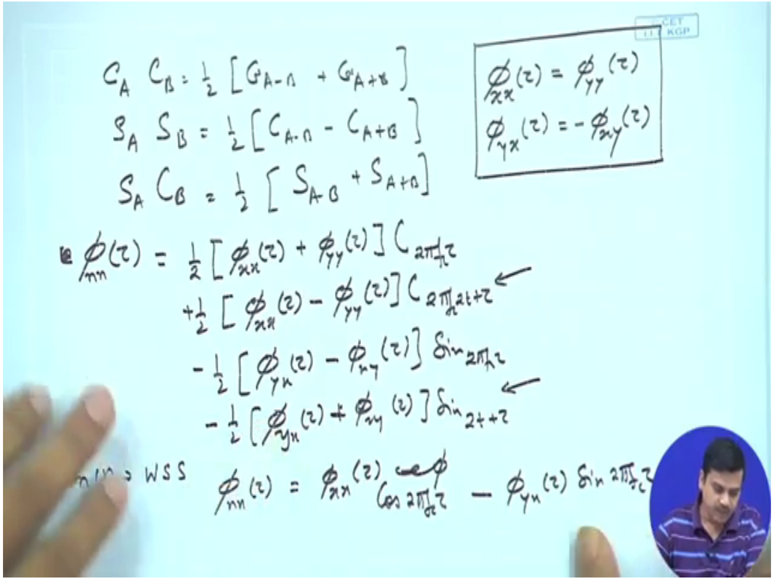


Cos of A times cos of B is half cos A minus B plus cos A plus B. So, I am writing this for cos not another C right. The suffix and sin of A sin of B is half cos A minus B minus cos A plus B, as well as sin A cos B is equal to half sin A minus B plus sin of A plus B.

So, equipped with these things and when we see this particular expression here above. This particular expression is sufficient for us what we can say is cos of A plus B that is here we can expand in terms of cos of A minus B cos of A plus B. So, then you are going to have cos of A minus B. So, where you are going to be left with the cos of tau; that means, 2 phi f c tau and the second term, will be cos of 2 pi f c 2 t plus tau because you are going to have this thing right. So, will be writing these terms accordingly and what you can get is phi of nn of tau that is what we have here, that is what we have here phi of nn of tau is phi nn tau is equal to this you can say this right. So, that particular term what we have before would work out to half phi xx tau plus phi yy tau into cos 2 phi f c tau. And you are going to get half phi xx of tau minus phi yy of tau into cos 2 pi f c 2 t plus tau. Similarly you are going to get other terms phi y x of tau minus phi xy of tau times sin 2 pi f c tau and minus of half phi y x of tau plus phi xy of tau into sin, I would in short, I would like to t plus tau.

We have said that n t is stationary. So, if we say that n t is a stationary then what we mean is that we said wide sense stationary that autocorrelation function of n t is dependent only on the lag time and not dependent on time; that means, if you look at this expression here. This expression should not have any dependence on t, if n t is wide sense stationary the right hand side should not have any dependence on t. For it not to have dependence on t this term and this term which have dependence on t should go to 0 so; that means, we can conclude from this if this has to go to 0 this coefficient should be 0 so; that means, phi xx of tau should be equal to phi yy of tau. And if this coefficient should go to 0 then what we have is phi y x of tau is equal to minus phi xy of tau. And this is what we started off with when we talked about the wide sense stationary.

So, if you look at this phi xx tau equal to phi yy tau phi y x of tau is equal to minus phi xy of tau so, minus sign goes on either side. So, this is a very important result that we have for wide sense stationary narrow band pass process. So, along with this So, if we are going to fit it back what you are going to get is phi nn tau now since these 2 are equal. So, phi yy equals to phi xx this is equal to 2 x. So, we are going to get phi xx of tau this term is going to 0 and this term is going to 0. So, we are left with this. So, you have phi y x is equal to minus phi. So, you are going to have phi xy. So, phi minus x y is equal to phi of xy. So, we can say minus phi sorry we have a phi xx times cos 2 pi f c tau minus, because of this you are going to get phi y x of tau sin 2 pi f c tau.

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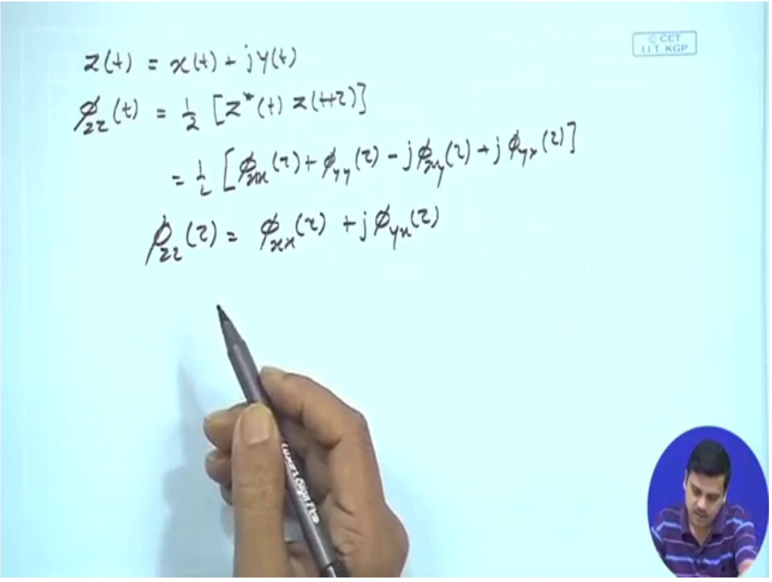


Then we proceed further and we could say, that if we have z of t remember our z of t what we had z of t here. We had z of t there that it is the complex envelope of n t.

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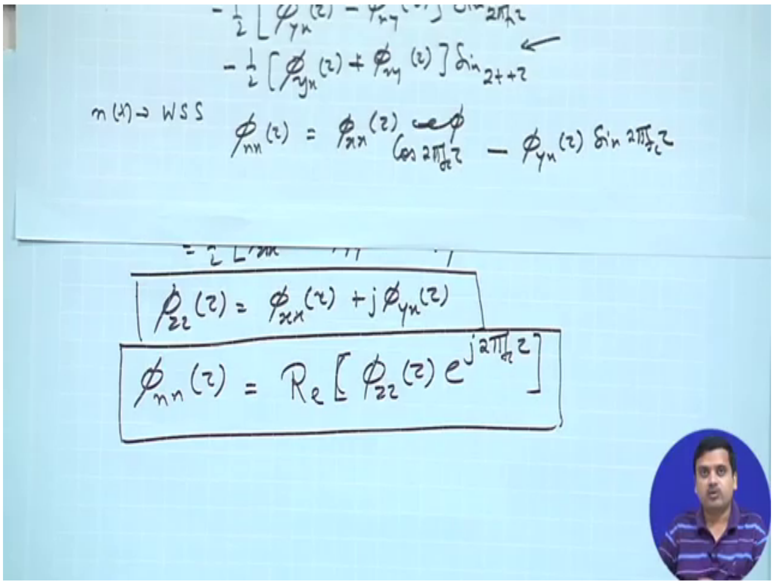


So, definitely z t you could write it as, x t plus j y of t and hence, phi zz of t is equal to half z conjugate of t times z of t plus tau. So, this is the way of defining, and what you would get you could easily go through a few steps you can get this as phi xx of tau plus phi yy of tau minus phi xy of tau plus j phi this you could do a few steps. So, that would mean phi zz tau would be equal to phi xx of tau plus j phi y x of tau.

So, given this relationship one put establish that, let us look at the last relationship that we derived. So, if you look at this relationship that we have derived here, right. Then you could say that phi nn tau is equal to real part of phi zz tau e to the power of j 2 pi f c tau right. So, in other words we are saying so, basically this is what we have yeah. So, if you have phi zz tau is equal to phi xx tau plus this. So, if you do this product you are going to get phi if you are going, to take the real part of this product you are going to get phi xx cos 2 pi f c tau minus phi y x sin 2 pi f c tau.

So, this is the form which is very similar to the relationship that you had when we discussed the baseband equivalent and the low pass equivalent and the passband of a signal. So, what we see in case of stochastic process we basically have the correlation functions related in a similar manner that we have for the signal. And we have a similar thing for the autocorrelation function and it is in phase and quadrature phase components. So, we have discussed a few important properties of baseband signals and passband equivalents. We have also seen the relationship of the autocorrelation function for the narrowband stochastic process. What we need to see now in the next lecture is the spectral relationship of the narrowband stochastic process and it is relationship to noise.

Thank you.