Modern Digital Communication Techniques
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Lecture – 20
Characterization of Signals and Systems (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. Yesterday or in the previous lectures what we have seen is that how to represent Bandpass signal with equivalent low pass form.

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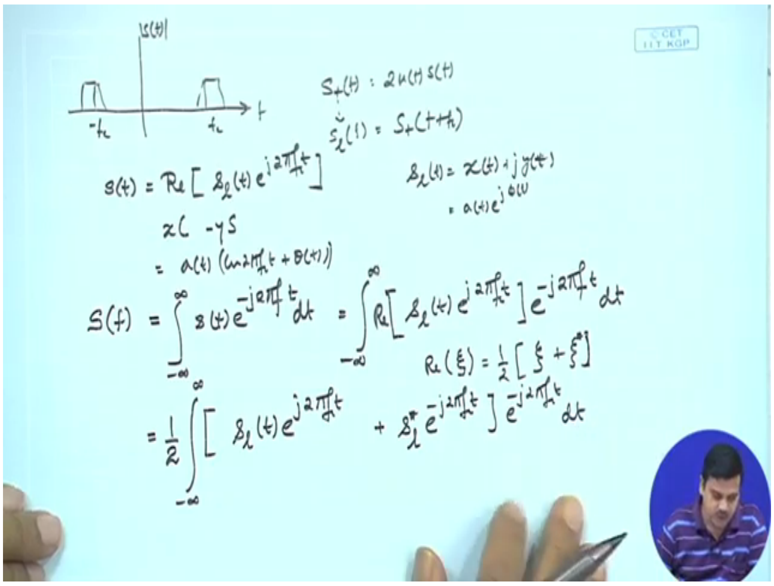

So, just to summarize we had taken a signal whose frequency coverage was represented by figure something like this right and we said we want to represent the low pass equivalent form.

So, what we did is because of symmetricity we said that you could take only the positive set of frequencies and that was given by 2 u of f time S of f right and then we converted this to its equivalent low pass by simply doing a frequency translation and then we got the relationship. And in summary what we had established is that S of t that is the real signal in Passband you could represent it by real part of S l t e to the power of j 2 pi fct.

This was one of the forms and naturally this could extend to if S l t is represented as you write S l t equals to x of t plus jy of t if you would represented in this way then one would get x cos minus y sin in short I am writing this and one could also represent S l t as at e to the power of j theta t and in this case you would represent this by at cos 2 pi fct plus theta t right this is what we have discussed in the previous lecture.

So, now after having discussed these things we would like to look at a few properties of this or a few more relationships relating the pass band spectrum to the to the low pass equivalent spectrum. So, for that what we do is we write S of f is equal to minus infinity to infinity S t e to the power of minus j 2 pi ft dt. So, this is the Fourier transform of S of t and this one we easily replace S t by real part that is what we have here S l of t e to the power of j 2 pi fct; that means, S s of t is replaced by this part e to the power of minus j 2 pi ft dt.

So, if I have let us say real part of something. So, if I have real part of let us say zeta i could represent it as zeta plus zeta conjugate and half of that. So, if you take this you get the real part accordingly we will do the same for this. So, what we do if we put a half straight ahead then an integral remains as it is and this translates to S l of t e to the power of j 2 pi fct plus S l conjugate e to the power of minus j 2 pi fct and e to the power of minus j 2 pi fct dt.

So, this is the representation and now we will use a few of the Fourier transform properties that we have described in the previous lecture. So, now, you will see how these properties come handy.

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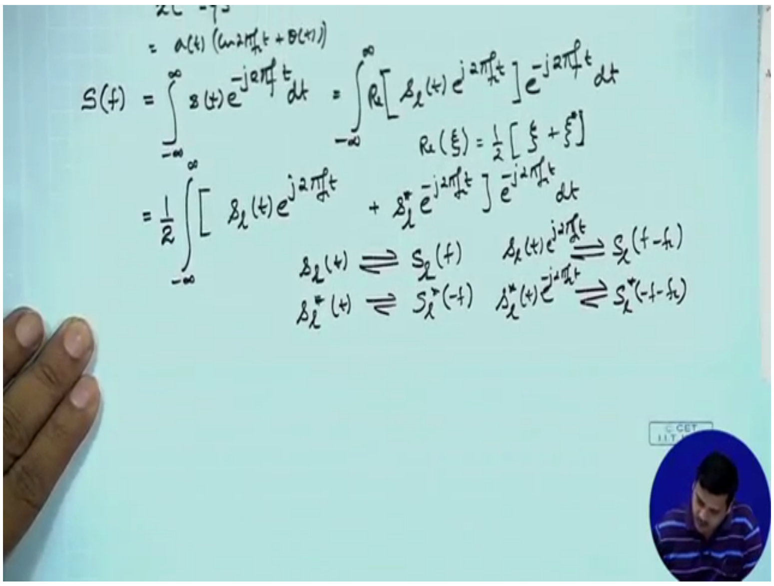

So, we know that if S l of t has a Fourier transform pair of S l of f in that case you could write S l of t conjugate has a Fourier pair S l conjugate of minus f and then we could also state that S l conjugate S l of t e to the power of j 2 pi fct would have the pair S l of f minus fc and similarly S l conjugate t e to the power of minus j 2 pi fct would have S l minus f minus fc conjugate.

So, this is the relationship that we are going to use. So, then we could write that from whatever we have here.

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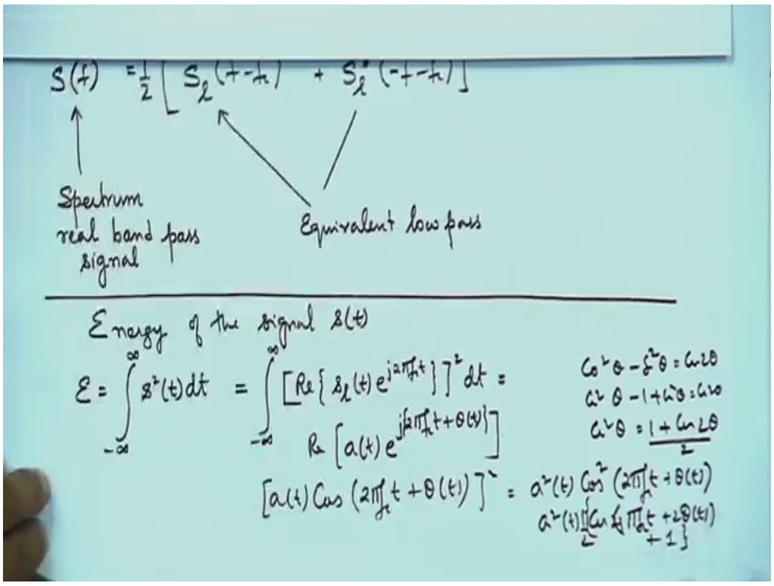

In the last expression is equal 2 half. So, if you should see the first it is a Fourier transform of S l t e to the power j 2 pi fct and if you say S l is the Fourier transform of S l t S lf is a Fourier transform of is the inverse of S l t then we could use this relationship S l f minus fc and if you look at this.

This is the conjugate of this and hence the conjugate would turn out to be S l conjugate minus f minus fc. So, what we see through this particular relationship is that the spectrum that we have on the left the spectrum of the real band pass signal can be represented in terms of its equivalent low pass signal.

So that means, the spectrum in the pass band can be related to the spectrum of the equivalent low pass where we had already stated that S l t is the representation of the equivalent low pass of S t and we have gone to S of f and we have related S of f we place it this way it will be clear S t, S l t, S f. Now what we want is a Fourier relationship of this and S t we have represented in terms of S lt.

So, from S t we have gone to its low pass equivalent and then we could connect a spectrum to its low pass equivalent. So, thereby again we are saying that at this point that if we have the low pass equivalent then it could relate to the past band. So, we can get the past band from the low pass equivalent. So, this is what we set of our discussion with initially that we do not need the career representation we can do with the equivalent low pass and we have seen second step whereby we could connect the pass band spectrum as well to the low pass spectrum of that particular signal.

. So, after having covered this we would like to calculate the energy of the signal S of t there again we are interested in seeing; what is the relationship between the energy of S t and that of S lt. So, we set forth in calculating the energy as e is equal to integrate minus infinity to infinity S squared of t dt and that you could do in many ways you could again go by writing real part of S l t e to the power j 2 pi fct squared dt.

Real part would be S l cos cos component of this with the x and y of course, so, you could also write this as if you recall the other representation that you had here we had this kind of relationship.

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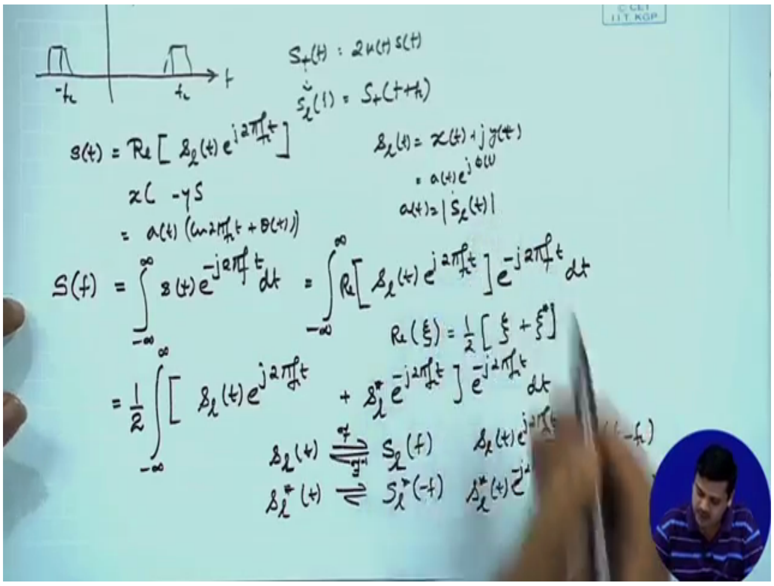

So, where at is equal to mod of S l t. So, if you do this then what you going to get going get the relationship as sorry you going to get this representation.

So, you can get it as at e to the power of j 2 pi fct plus theta t in the inside and you want to take the real part of it. So, we will take real part of it you going to get coz at cos of 2 pi fct plus theta t this; what you get inside this box bracket and there is a squaring of it. So, you going to get a squared of t and cos squared of 2 pi fct plus theta t.

Now, this cos squared term you could replace in terms of cos; that means, you going to have cos squared theta minus sin squared theta equals to cos theta. So, from that you could get cos squared theta minus one plus cos squared theta equals to cos theta. So, which basically gives you that cos squared theta is equal to one plus one would go this side cos of 2 theta upon 2 because you have a 2 over there.

So, you going to get a squared of t half cos of 2 multiplied by whole thing that is 4 pi fct plus of 2 theta t. So, this is what you going to get inside, so sorry you; going to get one plus one plus of this. So, I could put chain bracket there plus 1 and 1, so, now, the representation that is there in front of us we have one with a square of t.

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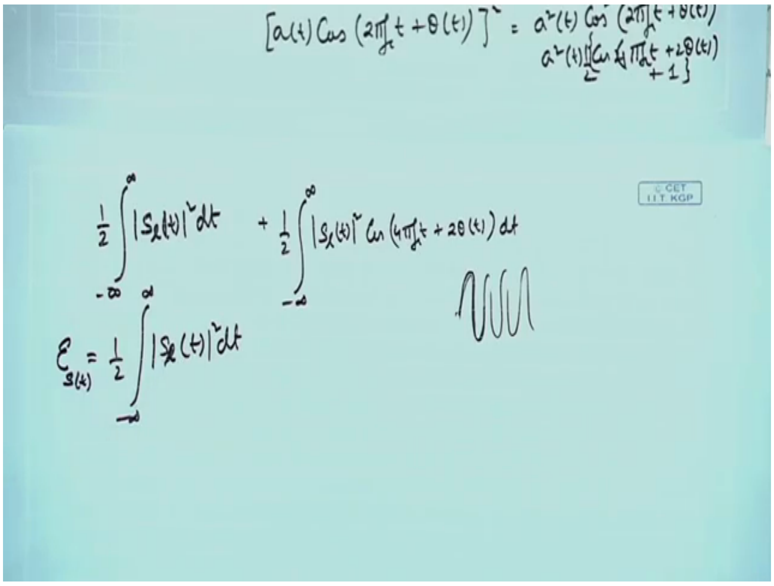

So, basically and there is a half over here. So, you want to get half integral minus infinity to infinity and a of t is basically S l of t.

So, you could simply write mod of S l t squared dt plus of course, you have the second part of it which you can minus infinity to infinity mod of S l t squared times cos 4 pi fct plus 2 theta t dt. So, now, definitely this S l t is the low pass representation this is a carrier frequency and we have said that the bandwidth of this is much much smaller than fc that goes by here.

So, its coverage is much less. So, that when you get back to the baseband you have like this. So, this is much much less than fc going by that this fluctuates very slowly compared to this and cos 2 4 pi fct is basically oscillations like this and if you integrate it it turns out to be 0. So, from minus infinity to plus infinity because of in one cycle you are going to integrate to 0 and there are so many of them. So, this could be approximated to half and if you look at this, this is the energy of the low pass equivalent signal.

So, all that we have at this point is that the energy in S of t is represented in terms of the low pass equivalent. That means, again what we are saying is that we can calculate the energy of the a pass band signal given the low pass equivalent representation we have already seen that you could connect the spectrum and now we have seen that you could also calculate the energy as well.

So, the moving ahead further we are now in we are now in a position where we say that we can relate the low pass equivalent of the signal to its pass band and any signal that we have passes through a band pass channel. So, we also need to represent the channel in its low pass equivalent form we have only describe that the channel is or the signal is conditioned in such a way that this picture we have drawn before this is the f and this is let us say the channel gain.

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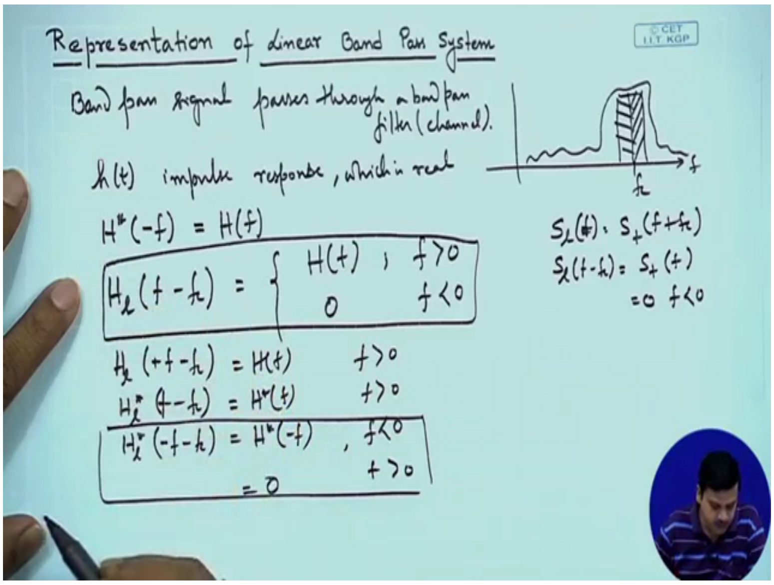

So, channel could have a gain like this and you would like to restrict your signal to this particular band within the channel and the centre of it is fc generally we sends signals in this form. So, since the signal is going to the pass band or in other word we have the pass band signal and we have bringing it down to its low pass equivalent representation. So, if we have to analyze a system we should also find the equivalent representation of the channel in its low pass equivalent form.

So, if we do this we would be we can say that let us consider a channel which is Bandpass channel. So, in other words we have to first say that the Bandpass signal passes through a Bandpass filter we can say channel right and we would consider let us say H of t as the channel impulse response you can also think of this as the impulse response of a band pass filter.

You can also think it the channel is a filter which we have partially described before because when we launched some signals into the channel some of the signals will get at inverted some of the signals will pass through; that means, the channel is acting as a filter. So, in case of linear models we can think of a linear time invariant system we have described in one of the very initial lectures.

So, now we are considering the channel equivalent to a filters and that has an impulse response H t which is real remember we have said the signal S t is real the real single S t is in the pass band because in the pass band you do not have any imaginary signal you are having real signal which passes through real channel. So, H t is real and if H t is real by Fourier relationships we would have H conjugate of minus f is equal to H of f.

This relationship we had proved earlier when we looking at properties of Fourier transform or Fourier transform pairs. So, again you see that Fourier transform plays a very important role in the analysis of these signals and in some sense you would argue that we are still in the analog domain the statement is very correct, but what will be doing is will be impinging digital signal at some point on to these.

So, we need an accurate representation of this without doing an accurate representation will not be able to harmonize or connect each part of the input 2 consecutive parts and finally, reach the output. So, to do that; we started off with the signals, so, some of these things might be very similar to your analog communications, but finally, this would help us in setting up the basic expressions for digital communications. So, all we are doing at this point is harmonizing the expressions. So, that we use a uniform notation throughout.

So, we have this relationship and what we would do is we would define channel where we would say that H l of f minus fc is equal to H of f for f greater than 0 and is equal to 0 for f less than 0. So, this is again in the form that we have defined earlier we had said that S l is equal to S l of f is equal to S plus of f plus fc if you would remember this and definitely S l of f minus fc is equal to S plus of f right which is equal to 0 for f less than 0.

So, this is exactly what we have over here and this had happened because it is real which had a symmetric frequency domain representation and for asymmetric frequency domain representation we can take only the positive frequencies. And by frequency translation you can have the low pass equal representation and a similar thing were happening is happening over here only thing is that we are defining we are beginning with this definition.

There we started with real time signal here we are saying that let us define it in this way and in a few steps you could also have for example, if you would say we want actually go through the steps f minus fc we have already written equals to H of f for f greater than 0 and you could say H l conjugate f minus fc is equal to H conjugate of f still for f greater than 0 and then you could say H l conjugate minus f minus fc is equal to H conjugate of minus f minus f greater than 0 would mean f less than 0.

So, then we have this thing for f less than 0 which is equal to 0 similarly for f greater than 0. So, this is these are the 2 steps by which we define the particular channel.

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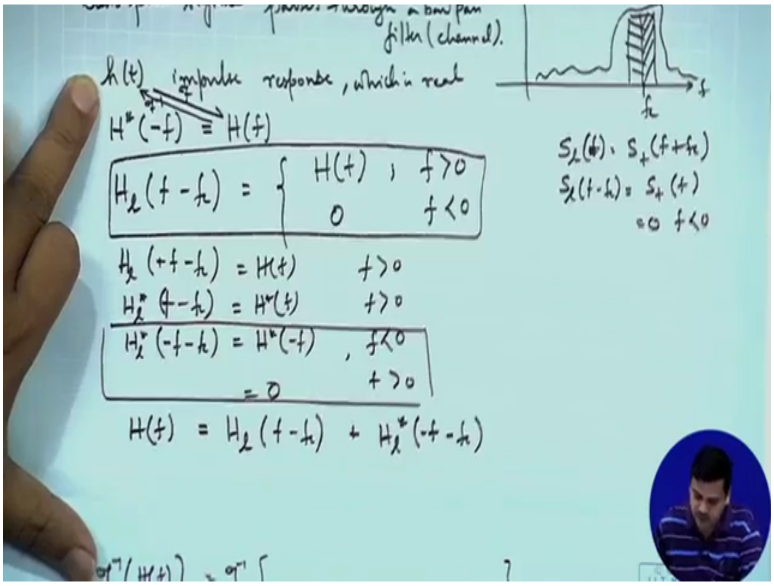

And going by this you could say that H of f you could write it as H l f minus fc which is for f greater than 0 plus H l conjugate minus f minus fc for f less than 0 as is clear evident from this.

So, once we have written it in this form we would be interested in relating this to H of t. So, to get H of t you going to take the inverse Fourier of H of f and hence inverse Fourier of the right hand side as well.

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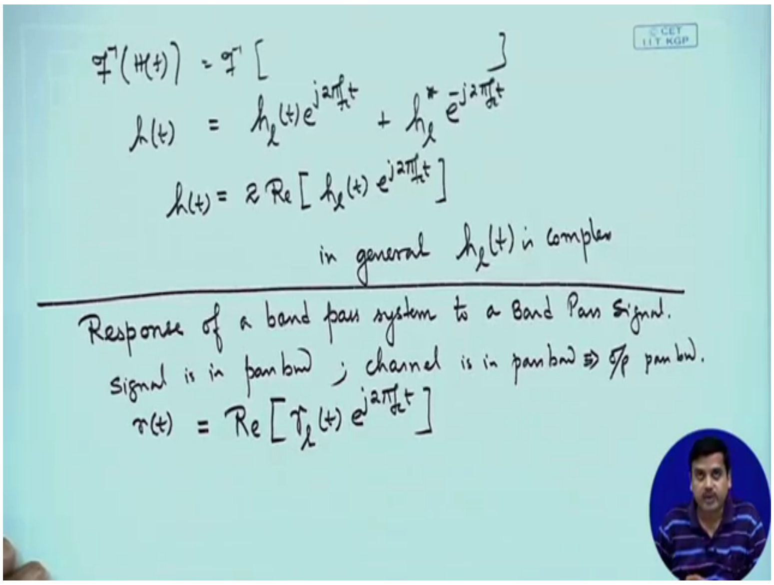

So, this inverse Fourier of this would be represented by H t of course, we had said H t is impulse response and if you take the Fourier transform Fourier relationship reverse direction is the inverse Fourier relationship right.

So, that is what we have H of t is equal to, so H l of f minus fc would result in H l of t e to the power of j 2 pi fct going by our earlier derivations if you would recall the derivations that we did a few steps back over here right we had use these relationships here. So, using the same relationships that we have represented here we can actually carry forward here and we can say that this is here and similarly this one will be H l conjugate e to the power of minus j 2 pi fct and hence it is clear that you could write this as 2 times the real part of H l of t e to the power of j 2 pi fct.

So, where you again see that the relationship looks very similar except that there is a 2 when of course, this is to normalize the power that we have gotten. So, we are again back to same formal representation that we have done for signal and now we have seen the relationship that would exist between the pass band channel coefficients and that of its equivalent low pass representation.

So, since we have got these relationship of course, just at this point we would like to recall that similar to what we did earlier in general we should say H l of t is complex we have not said about H l of t we said H t is real we have not said H l of t would be real.

So, in general it is a complex and when you handle complex signals you will get to see that how do we get H l as complex so that we going to proceed very soon.

So, now, since we have seen signal we have seen system what do we need now we have a signal with us we have representation we have representation of the system. So, the next very important thing that we required is the response of a band pass system to a band pass signal so that means, we need to connect these 2 and we have to show that how these could relate in terms of the low pass equivalent representation.

So, let us proceed with this. So, we would say that the output now since at this point I had like to say that since we know that the relationship between the pass band. And the equivalent low pass is of this form it has this kind of a structure right in both the cases we have seen what we have with us is a signal in the pass band and a channel in the pass band. So, the output will also be in the pass band right.

So, we can say that if signal is in pass band channel is in pass band definitely the output is also in the past band right this is for sure and. So, now, what we have with us is the signal which is coming out of the channel is pass band. So, once again reminding you pass band signal getting into the channel which is pass band of course, reminding you here.

So, definitely the signal that comes out will also be in this band right except if the channel is not translating the frequency if the channel translates the frequency what does it mean the channel translating the frequency means it is doing some kind of a modulation. So, we are not looking at those kind of channels we are just looking at a linear channels which are not play around with the frequency components.

So, now we have a pass band output. So, we can say that the output r of t which is in the pass band would like to relate it to the low pass equivalent and this will be handy for us. So, we will say we could write this as real part of let rlt represent the complex envelope of the low pass signal e to the power of j 2 pi fct going by the earlier things we can say this kind of a relationship should definitely hold between the pass band and the low pass signal and then we should say that how do we get r of t.

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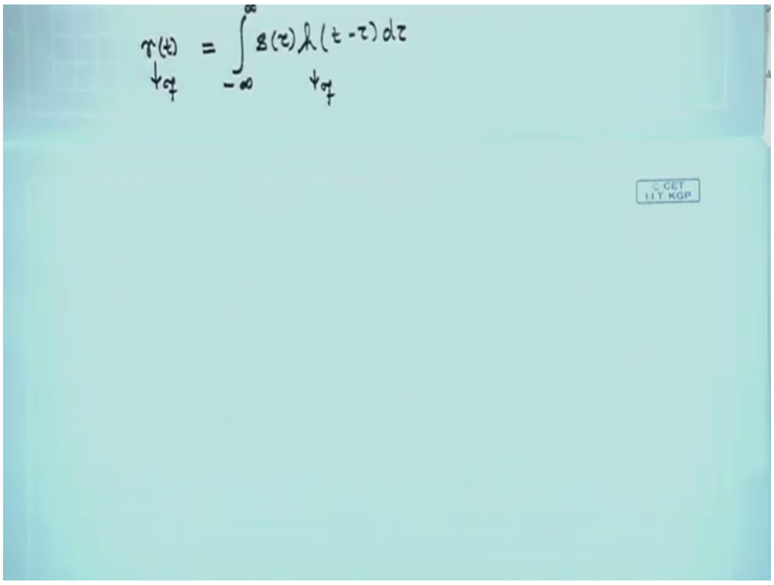

Since r of t is the output of a filter whose impulse response is given by H t and the signal going into is S t. So, definitely the output is the convolution of S t and H t and that we would write as integral minus infinity to infinity S of tau H of t minus tau d tau. So, at any instant t we have the convolution output of the signal along with the channel impulse response and if you take the Fourier transform of this; that means, we are going into the frequency domain to do this analysis right.

We will be always going into the frequency domain back to time domain because you want to find all kinds of relationship and often some operations in the frequency domain turn out to be easier.

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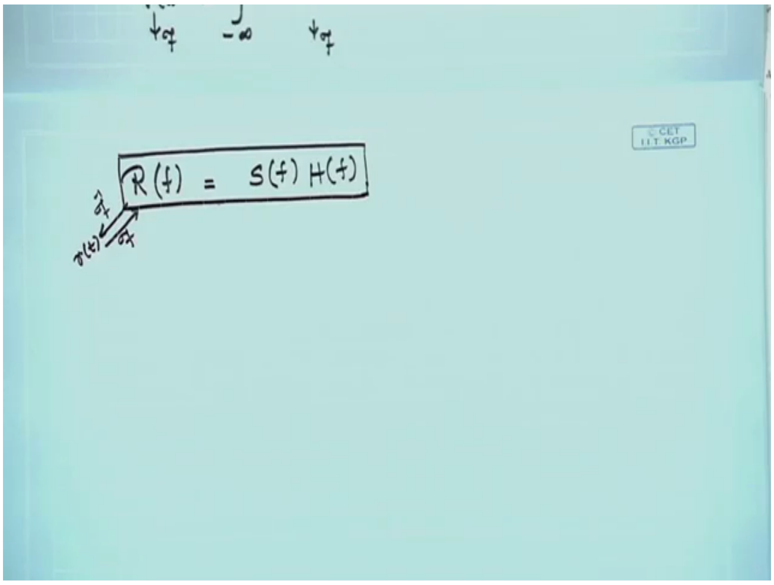

So, what we have is we to be do a frequency domain we will have R of f where R of f is the Fourier transform of r or that is the relationship R of f is equal to S of f because is a convolution in the frequency domain it will be a multiplication right.

So, we have this relationship in the frequency domain right. So, in this particular lecture will stop over here in the next lectures we would like to continue with the derivation of the time domain corresponding to these expressions that we have here. So, that we can relate rt to rl of t and we have this functional relationship.

So, we stop this particular lecture at this point we continue with this in the next lecture.

Thank you.