Modern Digital Communication Techniques
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Lecture – 18
Characterization of Signals and Systems (Contd.)

Welcome to the lectures on modern digital communication techniques. We are currently discussing characterization of signals and systems and we will explain before that when we talk of systems or when we talk of signals, we are looking for fluctuations. And generally if you look at a signal, you can represent it as weighted sum of sinusoids that is typically the understanding. And generally these kind of representations are well expressed through Fourier transform relationships. And we need some of Fourier transform relationships in writing down a few expressions when we deal with syst signals, on the systems for communication, for digital communications. And therefore, we are looking at a few important relationships and functions which will help us through in this work.

So, in the previous lecture we have discussed about the direct delta function and it is relationship with the signals. So, what we found is that if there is an exponential it is a transform pair is a delta function. So, we move forward and we look at a few more important things today before we get into some more actual communication system.

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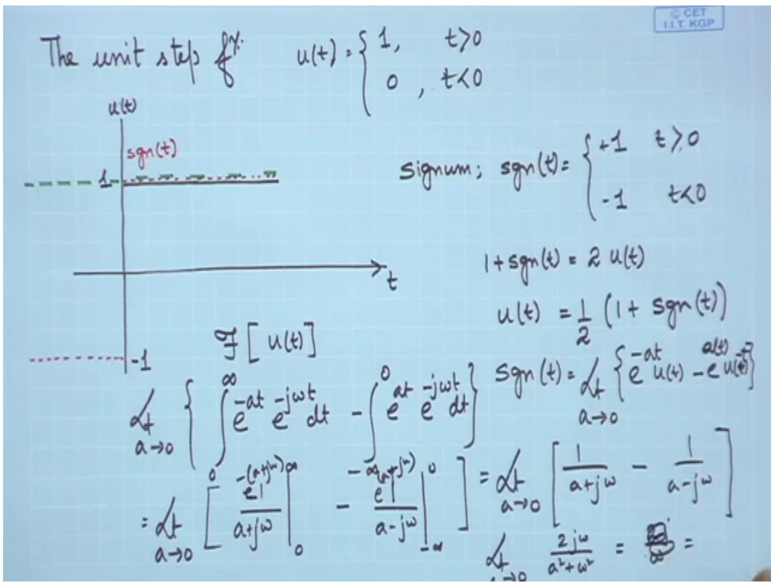

So, we have the unit impulse or the unit step function. We had looked at the impulse in the previous lecture. So, we have the unit step function I will this notation to represent function as u of t is equal to 1 for t greater than 0, and is equal to 0 for t less than 0.

And we would we are interested in the Fourier transform of the unit step function. So, I mean instead of directly going into the unit step function because of the discontinuity at 0 or we do it in a slightly different way. So, if we take a plot of the unit step function, we generally would have the plot going like this. This is t, this is u of t, this is 1. Now we know there is a function called the signum function or the sign function or you can call it sgn of t which is generally defined as 1 for t greater than 0, or plus 1 and minus 1 for t less than 0. And in some cases they put t greater than or equal to 0 and in quite in many cases they would have a 0.

So, I mean this would help in a way that if we look at the signum function, the signum function would be going like this. So, that is sgn of t. And if we take let us say 1 plus sgn of t; that means, I would say 1 plus sgn of t, what we are going to get is, if I am going to add 1 to this. So, this is minus 1. So, if I add 1 to this will be 0 and when I add 1 to this section; that means, I am talking about adding a 1, this way the resulting function would be a 2 times because, this is 0 and here the magnitude is 2. We can say it is 2 times ut or you could rather represent u of t as 1 half of 1 plus sgn of t. This is the general way of representing the ut. So, when we are calculating the Fourier transform.

So, if we are interested in calculating the Fourier transform so; that means, we will take the Fourier transform of u of t we would be getting the integrals, exponential integrals with this. So, to improve the situation we would take an approximation of the signum function and the sgn function could be approximated as the limit a tends to 0 e to the power of minus at u of t minus e to the power of at u of t. So, if we take this then we want to carry out the Fourier transform or we can we will get limit a tends to 0 and integral 0 to infinity because, we have this and u of minus t. 0 to infinity because it is ut e to the power of minus at e to the power of minus j omega t dt minus, minus infinity to 0 the reason being we have u of minus t over here. E to the power of at e to the power of j omega t dt right.

So, if you work this out this will turn out to limit a tends to 0 remains as it is, minus 1 upon a plus j omega. And it has to be evaluated from 0 to infinity minus 1 upon a minus j omega to be evaluated from minus infinity to 0. And this turns out to be limit a tends to 0 1 upon a plus j omega e to the power of sorry, minus 1 by a minus j omega. So, you are going to get e to the power of minus a plus j omega. And, so e to the power of minus a plus j omega, e to the power of a minus j omega there is a minus over here, a minus j omega.

So, if you put the values it turns out to be this and that works out to limit a tends to 0 to j omega upon a squared, plus omega squared which works out to be 2 upon j omega because, a is equal to 0. 2 j upon omega, omega, omega cancels out 2 j upon omega and sorry, this turns out to be. Let us let us work this out you are going to get 2 a and a will cancel out, 2 j omega upon a squared plus omega squared limit a tends to 0, and this will work out to be minus 2 upon j omega.

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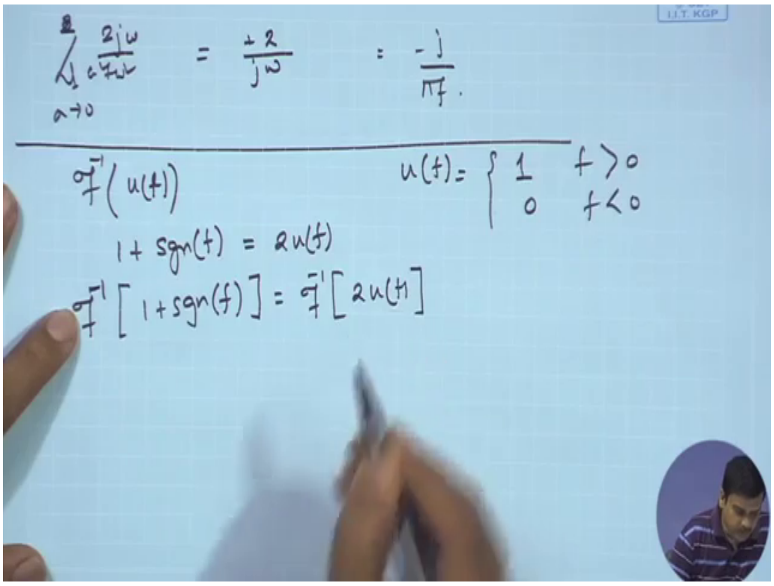

And if you are doing in real frequencies you are going to get minus j by pi f. What we can now do is our, interest our factor of interest is Fourier transform of u of f which is very, very, very important. Because this is what we are going to use and u of f is equal to 1 for f greater than 0 and 0 for f less than 1. So, we are going to use similar technique and as usual 1 plus sgn of f is equal to 2 u of f. And, so we can calculate the Fourier transform of this as will be quite easily evident that, why we are doing this because we want final u of f we can divide this. So, inverse Fourier of 1 plus sgn of f we are interested in Fourier inverse of this.

So, we have did the Fourier transform of this now we are interested in the inverse Fourier worth of this particular 1. So, inverse Fourier of 2 u of f right, this you should be able to calculate as Fourier inverse of one would give you a delta of t.

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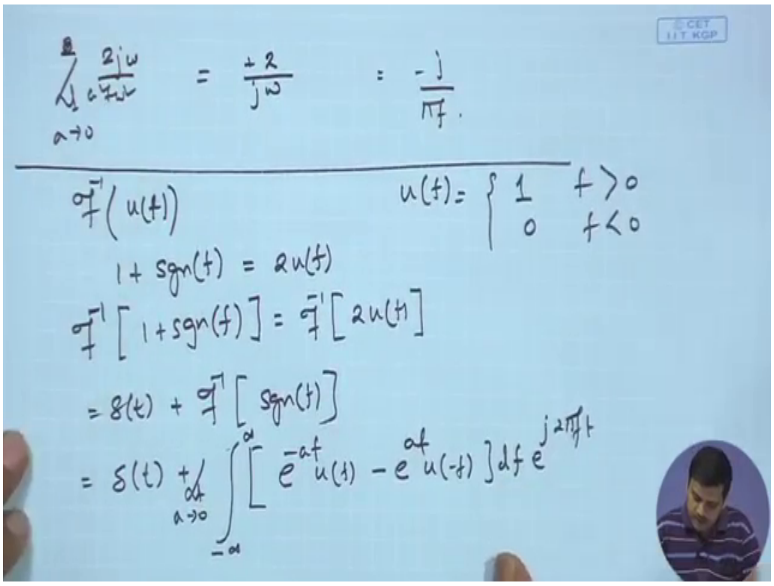

And then you are left with Fourier inverse of sgn of f, which is equal to delta of t plus inverse Fourier limit a tends to 0 minus infinity, infinity let us light the whole thing and then we can split it up e to the power of minus af u of f minus e to the power of af u of minus f of course, df e to the power of j 2 pi ft.

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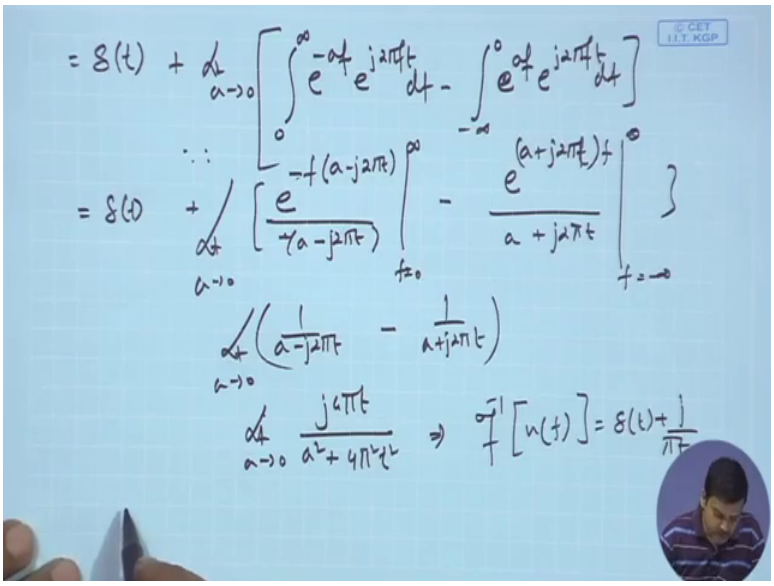

So, this is the whole expression and, we move with this forward in the next step what you will get is delta t plus limit a tends to 0 integral 0 to infinity e to the power of minus af, e to the power of j 2 pi ft, d of f minus integral minus infinity to 0, e to the power of af e to the power of j 2 pi f t df. In a similar way as we did in the previous step. So, you will get I will you need one more step over here and the end result is e to the power of minus f a minus j 2 pi t upon minus of a minus j 2 pi t, integrated from f equals to 0 to infinity minus e to the power of a plus j 2 pi f, 2 pi t of f upon a plus j 2 pi t integrated from f equals to minus infinity to 0 and of course, for all of this limit a tends to 0.

So, if you put infinity here it turns out to be e to the power of minus infinity which is 0. And if you put 0 this is e to the power of 0 which is 1. So, this is on the negative this sign and this sign cancels out. So, you are going to get 1 upon a minus j 2 pi t and here you are going to get 1 upon a plus j 2 pi t and of course, limit a tends to 0. So, if you work this out you are going to get limit a tends to 0 j 4 pi t upon a squared plus 4 pi squared t squared, which turns which implies that inverse Fourier of u of f is equal to delta t plus j by pi t. We have this delta over here.

So, this is the important relationship that we need because if you would recall we started from this and we are calculating a inverse Fourier of a sgn of t. So, we landing up over here sgn Fourier, inverse Fourier of sgn of t turns out to be this function and delta is of course here. So, this is the inverse Fourier of 2 f, if I am doing it right. Yeah this is of 2, f 2 uff right.

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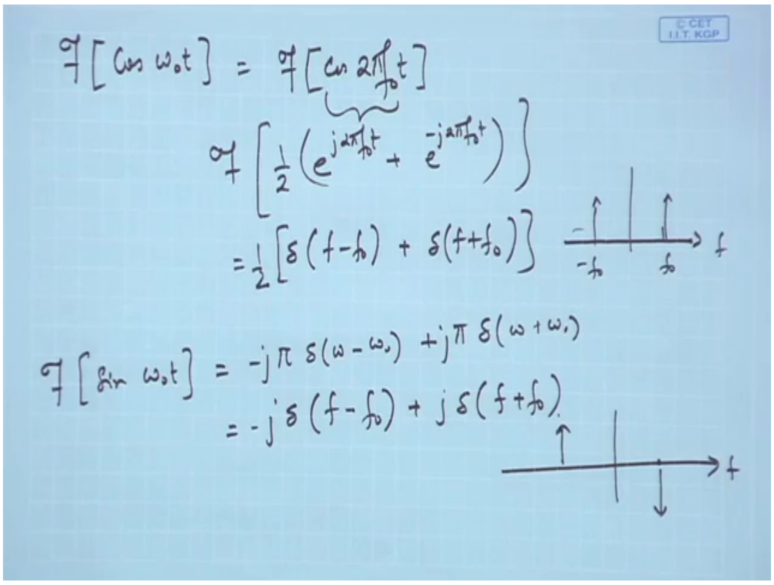

So, we move further and we will calculate a few more things which you may be quite aware of is the Fourier transform of cos omega naught t which is equal to Fourier transform of cos 2 pi f naught t and cos this can be written as, e to the power of j 2 pi f naught t, plus e to the power of minus j 2 pi f naught t, and half in front of it. So, what you have is Fourier transform of this term.

So, we already know the Fourier transform of this and this. So, what we have is half, delta function of f minus f naught plus delta function of f minus, minus f naught. So, this is the result and if we are talking about Fourier transform of sin omega naught t, will get minus j pi delta omega minus omega naught, plus j pi delta omega plus omega naught. Or it can also be stated as j delta function f minus f naught, plus j delta function f plus f naught. And this can be represented diagrammatically in the f axis as a value at f naught, an impulse and at an impulse at minus f naught. And this can be represented as in the f axis at f 0. It will be in the reverse direction and at minus f 0 it will be in this direction.

So, this is how the relationship of the Fourier transforms sin omega t appears and this is how the frequency domain representation of cos omega t appears. Now equipped with these a quick relationships that we have developed we would now dwell into the analysis of a typical communication system.

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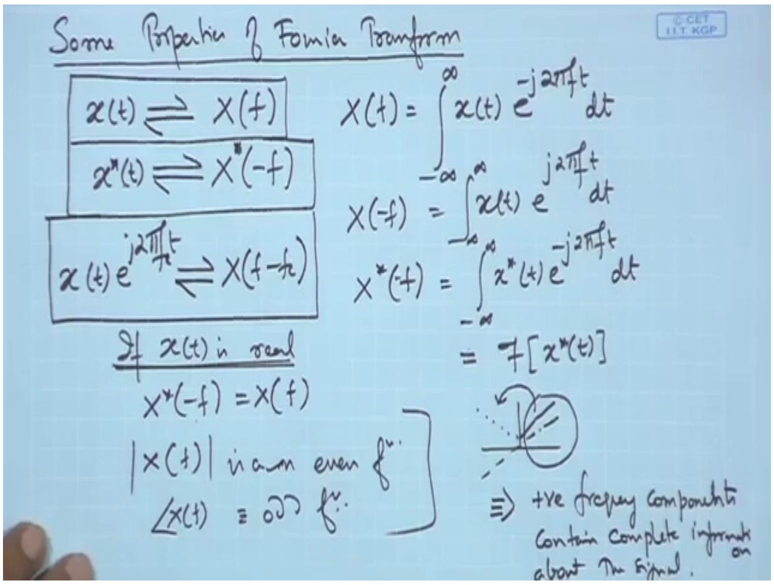

Now when we talk about communication system, well sorry, we have to do a few more things at this point before we get into that. So, let us look at some important properties of Fourier transform again which will be required.

This will again be required in derivation of a few steps next. So, if we are given that x of t has a Fourier transform represented by x of f, where we have x of f is minus infinity to infinity, x of t e to the power of minus 2 pi ft dt. And from this we can establish that x of minus f, I will simply replace f by minus f I am going to get a minus f there, minus infinity to infinity x of t e to the power of j 2 pi ft dt. And then we can take a conjugate x of minus f you are going to get it as minus infinity to infinity, x conjugate t e to the power of minus j 2 pi ft dt.

So, if you look at this you could say this is equal to the Fourier transform of x conjugate of t right. And, so basically what we can say is if this is the relationship then I give you x conjugate of t the Fourier relationship would be x of minus f with a conjugate as established over here. And also we will use the shifting property which is basically the modulation operation, can be presented as x of f minus fc.

So, this is the shifting property which you have encountered earlier in the modulation process where there is a signal and you multiply by some co sinusoid over here. So, the spectrum could appear shifted. Now what we also need along with this is if x of t is real, this is an important thing. So, if x of t is real then we can say that x conjugate minus f is equal to x of f. Look at this if it is real then x conjugate of t is equal to x of t so; that means, x conjugate minus f is equal to Fourier transform of xt so; that means, x conjugate of t also has the same Fourier transform. So that means, x conjugate of minus f is equal to x of f right. And using this relationship, along with these earlier relationships we can say that x of f is a real function, is an even function.

The absolute value of x this is an even function and the angle of x is an odd function right. So, these are the 2 important things that we can use. So, what it basically tells that when it is an even function it means, that we if we have a certain relationship on the positive values we know how to calculate it on the reverse side. And if the angles is an odd function we know that the angle will go like this so; that means, if I have information on this side then I can reconstruct the signal on this side as well.

So, effectively it tells us that the positive frequency components contain complete information about the signal. This is very important that, we do not require the negative side of the signal components. So, at this point when we look at these relationship that we have established, we have got the Fourier relationship of the delta function, that of the unit step function. As well as we have seen the translation and we have also seen the relationship of the Fourier transforms for the case of real signal.

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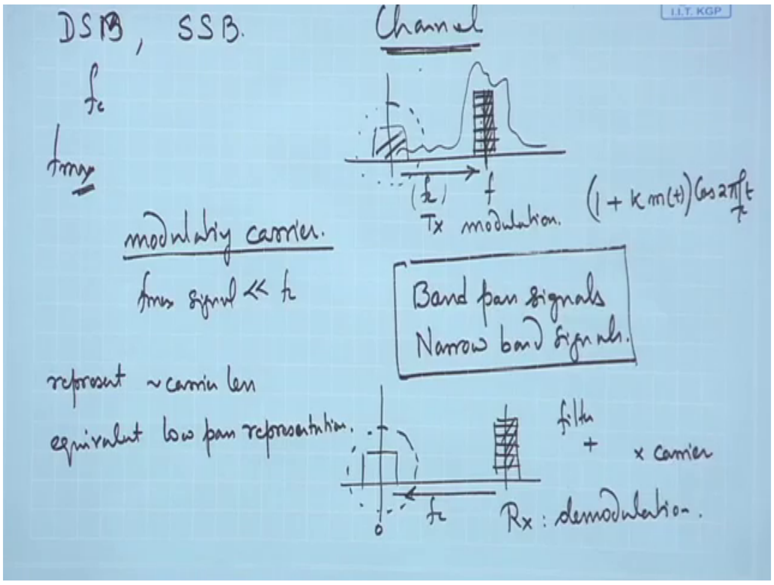

Now these things are important because why are we dealing into these things. The reason is that when we are talking about transmitting of signals, you have studied modulation already in analog communications and you have encountered possibly a double sided modulation, a double sideband or a single sideband right. And generally it is assumed that the channel allows a set of frequencies which is around a certain carrier so; that means, if the channel has a frequency response, which is like this and there is a certain a frequency we would generally design the signals in such a way, that we choose a carrier fc and we allow f max of the modulating signal, So that it is completely contained within the pass band of the channel.

So, what we are effectively trying to say is that, generally channels have bands through which signals go into and when we are modulating signal. So, basically you are modulating carrier and you generally have the assumption that the f max of this signal is much less than the carrier frequency. Or in other words the modulated signal is within a certain range of the carrier frequency. If it is double sideband it is spanning on both the sides if it is a single sideband, it is generally on one side and a little bit signal is available on the other side.

And we have seen previously that if you have the positive a set of frequencies you can reconstruct the entire set of signals. So, all we are trying to say at this point is that we are talking about band pass signals and we are talking about narrow band signals. So, you can say narrow band, band pass signals is what we are interested in. And we would like to establish a few important relationships and the relationships are needed because, when we are doing this transmission, what you are seeing is that you generally have the signal around the center frequency. Around the 0 frequency and you need to up convert through fc.

So, when you need to up convert with the carrier frequency this operation you generally do it at the transmitter and you call it modulation. Similarly at the receiver side what you do is the reverse operation. You would down convert this signal and bring it back to it is original position. So, this operation is usually done at the receiver and it is called demodulation, and this is what you have studied already. So, for example, the modulation you could say that I have my signal 1 plus some k times m of t, times cos 2 pi fct right. You could say that this is my signal and that you are modulating at the receiver you can demodulate by again multiplying it by this carrier and then filtering it and taking only this part.

So, if I multiply with this carrier I am going to get cos squared term. So, you multiply by a carrier at the receiver cos squared term, cos squared can be expanded in terms of 1 plus cos 2 theta, and if you have 1 plus cos 2 theta plus you have a filter, which is going to filter out the cos 4 pi fct terms and you will be left with only the baseband component. So, what we are seeing is that we have told earlier also that this channels could have different bands which could be passing the signals. If that is so, then according to the channel you would choose the different carrier frequencies; that means, you will be up converting the baseband signal to that particular frequency and at the receiver you will be down converting.

However we would this up conversion and down conversion does not really impact this original signal which is of our interest or which is the information bearing signal. It is just used to translate it from baseband to carrier and back to baseband. So, and this is dependent only on the channel and not on this particular signal. This is of course, some dependence depending upon the bandwidth, but it is not connected with the information bearing signal. And we would like to design a communication system which, where this is not playing an important role. So, therefore, we can say that we would like to represent you can say carrier less representation or equivalent low pass representation right.

So, in that case we will be removing the effect of the channel and effect of fc and we will be studying only in the base band. Now in this way we would be removing unnecessary need of a very high frequency. If we are analyzing system let us say by simulation or other purpose then also we would be avoiding the generation of a large number of samples. For example, if you take a few gigahertz of carrier frequency, but then you need to sample it at even a larger frequency going by the nyquist sampling theorem and then we have a huge amount of data to process.

So, what we are going to show you are the equivalent relationships of how the signal should be represented when we look at it in the equivalent pass band. Now why we are saying that we want to bring it back with equivalent pass band because, actually the signal is in a band here. So, this is the band pass signal; however, so the effects of the channel would be in the band pass; however, the signal matters mainly at the base band. So, we would like to represent signaling the base band, as well as we would also like to have the channels representation in base band which will be clear in the next lecture.

So, that we can study the entire thing in base band without affecting the performance of our study and if we know this carrier frequency then we can translate all the studies that we do in base band to the corresponding effects of the pass band. Or in other words what we are going to established a soon is the relationship between the baseband or the low pass equivalent signal, and that of the pass band signal, so that we can do the studies in the equivalent low pass form.

Thank you.