Modern Digital Communication Techniques
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Lecture – 17
Characterization of Signals and Systems

Welcome to the course on modern digital communication techniques. So, till now we have discussed source coding. And we have seen how the signals which could be analog could be converted to digital sequence. And once we have the digital sequence then we can do different kinds of encoding scheme and one of the objectives would be to reduce the number of bit is that could be used to represent the source. So now, you can carry on further and discuss a many more details of source coding. There is a huge amount of literature available on source coding both are analog as well as digital and there are different characteristics based source coding schemes, which we will not cover in this particular course because of the time constraint and because we have to also cover other things.

Our main interest was to show you or to make you aware of the things associated with source coding. So, that you can understand it is important with respect to communication system design. So, here onwards any further advanced things you should be able to follow based on whatever platform we have created in this particular course in the previous lectures. So, now, if we now is the time that we have to get into a design of communication systems.

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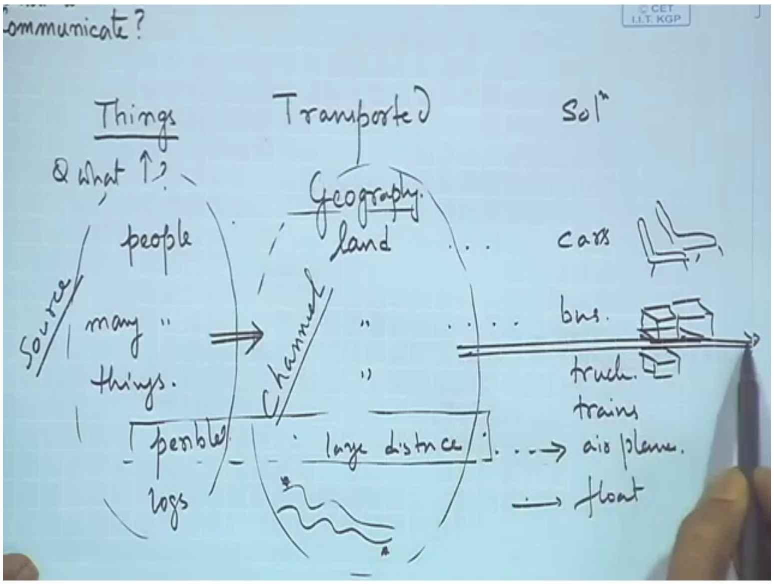

So, we are still preparing for seeing a complete communication system and at this point we would like to ask the question that, what. So, we are basically trying to communicate. So, when we would like to write down this word that we would like to communicate then the question that we would like to raise is what you would communicate right, is the basic question.

So, given our previous understanding we can take a typical example like let us take an analogy instead of directly getting into the details of communication theory. If we think a communication is like sending of information that is what we have discussed before. So, we can compare similar to transport system which transports things from one place to another right. So, if we take a few example of things that gets transported. So, we want to transport things to be transported from one end to another end. So, we have to design a transportation system. So, that is like pure hard hardware things and pure big bulk things, but it is somewhat analogous to communication.

So, let us see what it is and how does it sound the bell towards the design of communication systems were the basic questions that one should ask. So, when we are transporting things let us say you could ask what things. So, question could be what are the things right that you should generally ask because accordingly your solution is going to depend on that. So, suppose we are saying that we want to transport a logs of woods that could be one situation or we could say that we want to transport people that could be another situation; we say that we want to transport. Let us say fruit is then someone could say that we could transport let us say something is made of steel right is a few different things and then you say. So, what if you are asking this question?

So, the next important question that we should ask towards designing a solution is that this particular thing which you have identified, which could be varied transport across what? Let us say we are we are talking about people. So, to be transported across a particular we would ask the geography, right? If you say geography we could say that it is transported over land and then we are seeking the solution, we said well you can go for cars. The next question could be well, if you are transporting people how many people? So, your answer could be many people. It is still land your solution could be a bus right.

Then we could move and say that well we now want to transport things over land. So, we could think of a truck, right. We could think of a good strain right. And then we could think of things which are perishable right, and over large distance. Then we could think of airplanes right we could thinks of logs of wood and then we find that there is a river which is connecting to lands where between which it should flow we could say that let them simply float right.

Now, if you look at this there are certain hidden things that we are discussing behind this, this scenario that we have taken it. All that may appear not directly connected to communications, but there are certain insights which we can draw from this. Certain insights like we are trying to characterize the things based on these descriptions and we have seen that if we think of transported across a particular geography we are trying to characterize the link or we are basically characterizing this as the channel right.

So, and here is the source right, what is to be transported. The solutions would depend upon an understanding of the source or what is to be transmitted and or what is to be communicated or transported and over which it has to be communicated right. So, with this we are trying to draw an analogy that you are seeing that these solutions are customized or in some cases you could also use the word optimized to these situations right. So, for instance when we talk about this perishable goods this particular case over long distance we are saying that we need to transfer things not people. So, that hints that well you do not need seeds. So, you could stack them up and when you say perishable you mean to say that there is a time constraint. So, there is a delay and if you look at these solutions, these solutions they involve a significant amount of time, when they will transport across large distance.

So, you think of as a different solution. You could say that why this solution was not used over here, the answer is very, very simple that if I am going to use airplanes for transporting people across shorter distances, it would be very costly affair and in some cases let us say where the distance is fifty kilometers or hundred kilometers, it might turn out that it is in fact, worse than using a car. So, what I am trying to hint at is that, you need to understand what is to be sent right. And you also need to understand over what things need to be sent. So, these are the 2 important things that we need to understand before we think of solutions.

So, when you think of solutions then comes the design of these things and as well as design of the whole system. So, when we say this not only the bus, the bus communication system is also involved in this. So, we will slowly delve into that. So now, if we are asking a question related to communications right.

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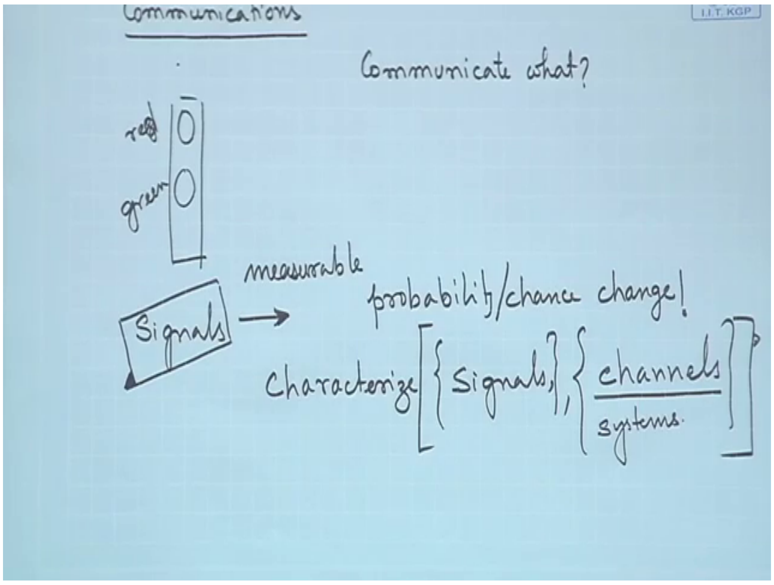

So, if you are asking the questions relating to communication, again I would like to ask what is to be communicated? Similar thing, what is to be communicated? Communicate what? Right.

So, when you can say information that is pretty right answer, but then how do we say that well what do you mean by information? That is a important question and then we would have to go back to how things were handled a years back. So, earlier or let us say we take a direct example and we say that suppose I am seeing a light source. A light and I say I want to convey the information about this particular source. So, let us say it has a particular color. Once that information is sent it is sent forever right. Now how important is it to send information regarding this light source. The answer could be that this is important if there is change of content in that source for example, I could say that the light or the source that we are seeing from far changes between 2 colors, let us say red and green right.

So, then it could make sense to send across to the other side that what is the color of the source right. So, what I am trying to hint at and what we discussed earlier is that if there is a chance or there is a probability that we are measuring something so; that means, of course, we there is a measurable right; that means, you can measure it by some mechanism and there is a probability associated or a chance of change. If this is available then there make sense to communicate right. And generally we can say that there are signals which have these kind of property. So, they are measurable and they do change and there is a chance of changing.

So, if it does not change with time then there is hardly a point in sending some information about it. So, over a very, very large amount of time the amount of information you send is negligibly small. This is something that we should be thinking in that particular direction and when we think in this direction, we say that well we have certain signals right, which are not constant and we have to send this across. So, we have got part of the initial answer that what we have to send. So, basically the things that we are going to talking about is an information in the form of signals right.

So, then as we did here our basic question would be that we have to characterize these signals right. So, without characterizing the signal we would not be able to send it across a particular geography or a particular channel; that means, whether there is a delay constraint whether do they have a particular structure like it is people I mean definitely you need the carrier to have seeds right whether if it is like things they could be stacked up in boxes right. And then you could ask for how many such boxes could fit in, or how many such chairs could be arranged. So, that. So, many people could be transported right. So, the questions or the solutions would depend on characterization of this.

So, what we are trying to say is that we need to characterize signals right and now if we think of this as the system or the channel. So, we need to characterize signals and I would say channels, generally this is referred to as systems the reason is we put something input into this and we get something out of this right. We send a certain signal into the channel, we get a certain signal out we send people into, this system and it produces people at a different place right. We put things into it after a certain time they get delivered and there could be certain changes to it.

So, when we think of communication system we need to characterize signals, and we need to characterize systems So that we can design a communication system. Without being able to characterize it well and a proper solution is not feasible for these things.

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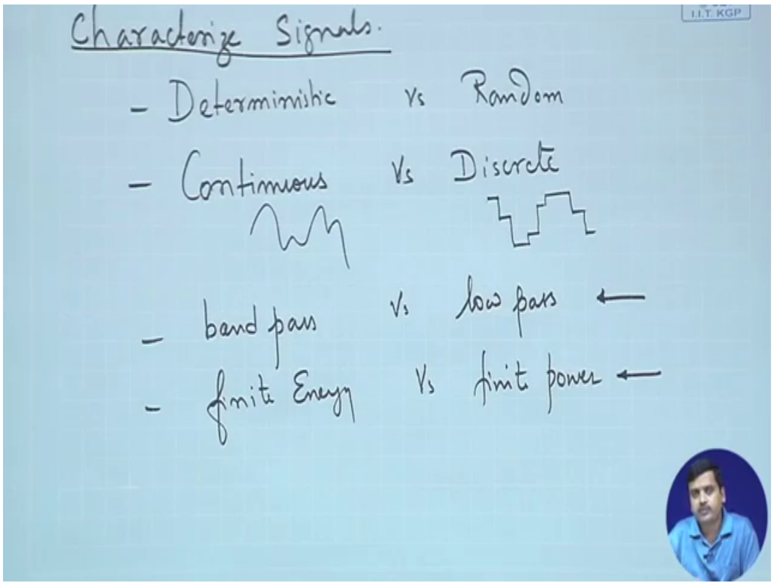

So, when we look at or when we ask the particular question of a characterizing signals and systems we could ask the question that how could you characterize, how would you characterize signals? Let us say, and there could be systems as well right.

So, there are different ways of doing it and you could say that well when I look into a signal, I could say that, I could characterize the signals based on whether they are deterministic or versus whether they are random right. We will slowly get into see what you mean by random we of course, seen something. So, for random we can say that you cannot predict a priori. What will be the outcome in the next trial right or what will be the value of the random variable in the next instant of time if it is a stochastic process, or in the next experimental event. So, it could be deterministic.

Deterministics are the ones which are completely a described by certain expressions given in initial condition. So, if you are given in initial condition and you have the relationships to exactly determine what is going to happen at a certain duration of time. So, this is a deterministic you can characterize signals based on these things. So, that is one way of doing it you could say that well I could like to characterize signals as being continuous versus discrete right. We had seen examples of continuous, and we had seen discrete right. It takes discrete values you could say I could characterize signals based on band pass versus low pass and this is what we are going to see very soon. You could say I would like to characterize based on whether they are energy signals finite energy versus whether there is finite power and so on and so forth, you could you could go on characterizing them.

Now so, what if you if you characterize these how does it help us in a designing communication systems. So, well what we are going to do is we would like to represent the signals within a certain framework or within a certain structure. So, that we can use that particular structure in finding out the interaction the signals have with systems. So, that we can calculate the output when the signals get launched into the into the system and, when we get the output then we can see what kind of difference the signal has from the original form, and our job would be reconstruct the signal back to it is original form.

So, based on this we would like to analyze these particular a signals that we are going to encounter in our day to day communication experience. So, before we move on to describe the actual communication system we would like to just state a few specific signals which are very important and which may appear at certain points during the analysis of our communication system.

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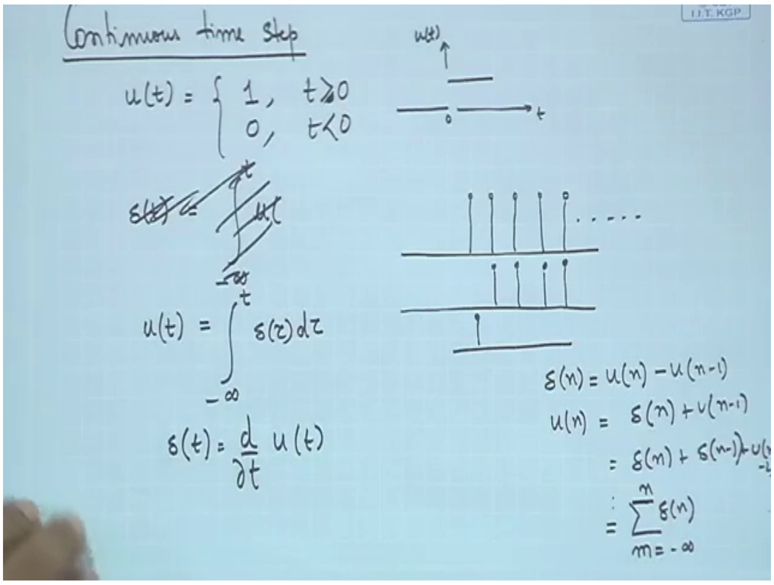

So, one is the continuous time step function. There could be a various definitions, but this is the definition that will follow by, which will be defined as u of t is equal to 1 for t greater than 0, and equals to 0 for t which is greater than or equal to 0, and it is discontinuous at 0 so; that means, you do not know what is the value. So, at 0 it is 0 just at greater than 0 it is 1. So, this is the unit step function if this is 0 and this is the time axis right. And then there is another important function which is the delta function.

So, if you have the delta function a delta function is defined in terms of the unit step function as the integral of minus infinity to t u of, sorry we would define delta function in a in a way that u of t is equal to minus infinity to t delta tau d tau. Now how do we get this to look at this? I would like you to recall the relationship of the discrete time functions which are much easier to visualize. So, if this is the unit step function the delta function would be, a difference of the unit step function drawn over here and the unit step function drawn in this line, So that the difference would result in this delta function right. And you could represent the delta of n is equal to u of n minus u of n minus 1 or you could say that u of n is equal to delta n plus del u of n minus 1. Which you could write as u delta n, So basically I could write delta n plus u of n minus 1, which is equal to delta n again the recursive definition delta n minus 1 plus u of n minus 2 and So on, which you could define in a recursive manner u of n is equal to summation of m equals to minus infinity to n delta n.

So, this gives a hint towards this kind of relationship. And you would also have delta t is equal to the derivative of u of t right. So, these are some important relationships which we will just like to revise and if we see that how you could get to this or you can make an approximation of u of t. So, we will rather right here.

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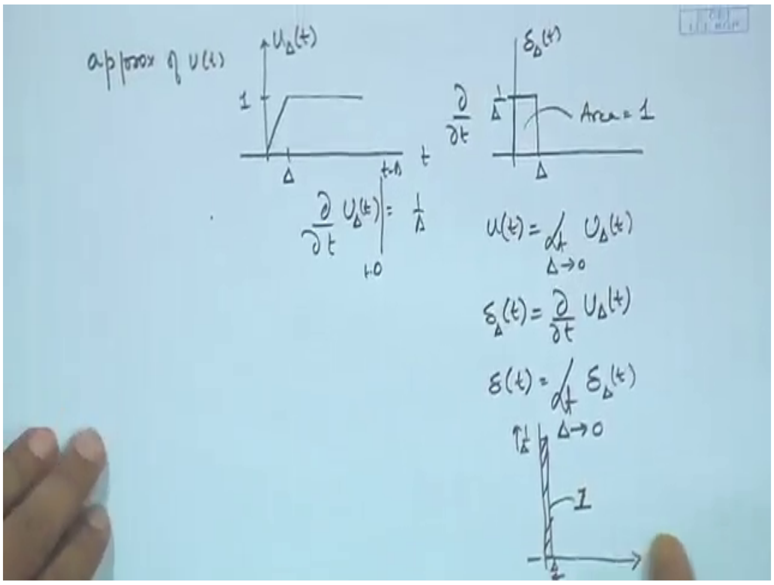

So, you can think of an approximation of u of t by this function, which is t. And this value is one let us say there is some continuity over a period delta and then it takes this value right.

So, if you take the derivative of this particular function from 0 to delta. There is a constant slope, and that slope is basically 1 upon delta, this is 1 upon delta. So, this value you are going to get 1 upon delta and the rest of it is 0 right. So, you will get this constant slope of 1 upon delta. So, if you take the area of this with link limit delta tends to 0, you will get a value of one 1 upon delta multiplied by delta so; that means, the area has been restricted to 1 and we could say that. So, at this point we could write d dt of u delta t evaluated at a t between 0 to t equals to delta. What you are going to get is 1 upon delta in this in this whole period. And we are basically approximating this is what we are writing as u delta of t.

So, in other words we have written that u of t is equal to limit delta tends to 0 u delta of t. And we also know that this area is equal to 1 irrespective of the value of delta. And then we could write delta t as the derivative of u delta t and this we could write the suffix delta and then we could say delta t is the limit capital delta goes to 0, delta sub capital delta t; that means, we are going to make delta equals to 0 this becomes narrow and, narrow and narrow. So, this becomes very, very narrow and the height becomes very high, and this is delta, delta t, basically delta right.

So, as delta becomes small this becomes higher, but the total area still remains the same value of 1 right. This way you could find the relationship between delta t and mu t as well right. So, at this point we move forward and before you proceed with a few more expressions, I would like to say that there is another important thing which we should consider. It is like when we are analyzing systems as we have said that we have the channel you have seen in this picture that signals would go into the channel and they would come out, and this is what we have described before as well.

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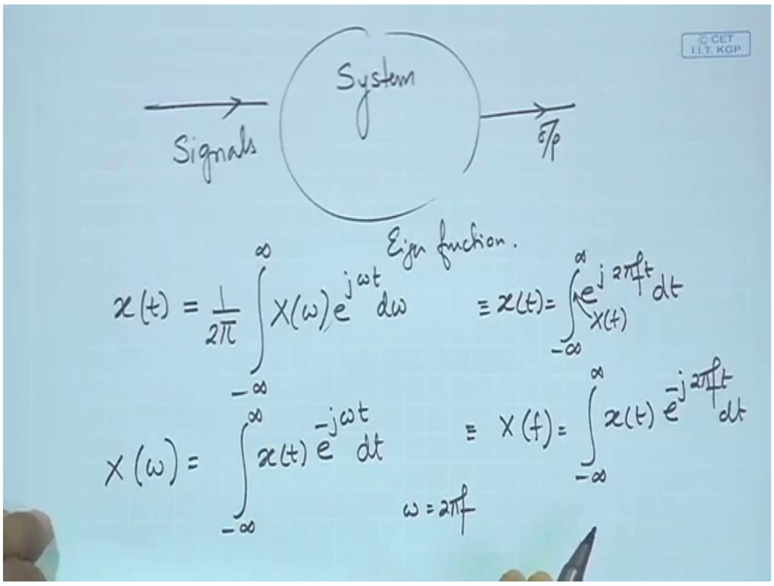

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega \equiv x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\,dt$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt \equiv X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt$$

$$\omega = 2\pi f$$

So, when you have such a system which is going to be impinged by signals, and you are going to get an output then the question arises, that how do you study such systems? And So that you can get a easy or a meaningful representation. So, generally these systems are studied in terms of their Eigen functions right. Generally exponentials are a set of Eigen functions for systems. And the advantage of doing so is that, if you launch an exponential into the system, what you are likely to get of a system, whose Eigen functions are exponential is again another exponential with some coefficient and the coefficient would be the Eigen value.

So, what we are trying to say is that you have you are launching an exponential into the system getting an exponential out and what is changed is the coefficient of this particular exponential. So, so that means, your study could be restricted to the coefficients of this exponential rather than the exponential itself. So, with that motivation we would like to deal with the Fourier analysis because, when you do Fourier analysis you are dealing with the representation of signals or systems in terms of exponentials and there coefficients. So, through that we are going to study such systems.

So, let us revisit or use the definitions X of t through the Fourier relationships is equal to 1 upon 2 pi minus infinity to infinity. X of omega e to the power of j omega t d omega, where this is the time domain signal, this is the angular frequency domain signal and X omega is the Fourier transform of X of t right. And X of t is the inverse Fourier transform of X omega sometimes this is called the synthesis equation and the reverse is called the analysis equation. X of omega is equal to minus infinity to infinity X of t e to the power of minus j omega t dt. And in terms of continuous frequency f you could write this as X of t is equal to integral minus infinity to infinity e to the power of j 2 pi f t dt. And here also we have X of f is equal to minus infinity to infinity, here I made a mistake, here there should be X of f and here it should be X of t e to the power of minus j 2 pi f t dt X of f yeah, same thing over here right.

So, with this and of course, the relationship omega equals to 2 pi f. So, with these relationships we should be able to study the communication systems. So, moving ahead further.

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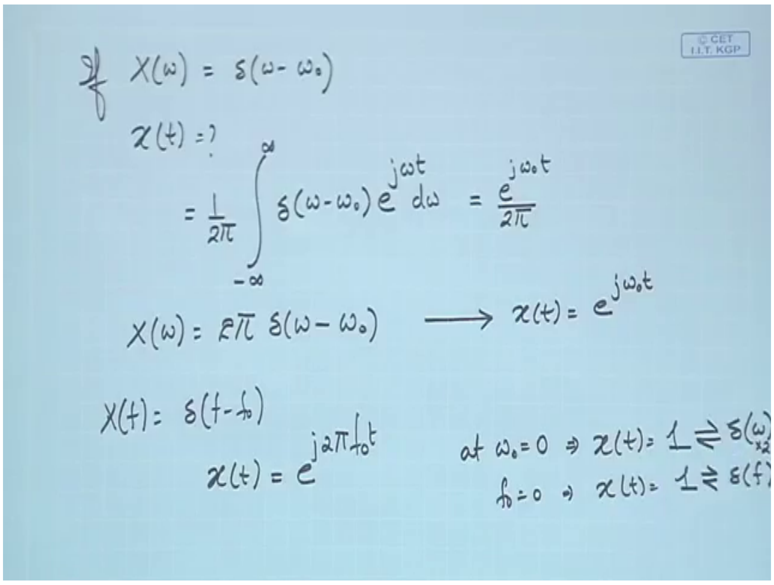

We would like to see that if X of omega is equal to delta of omega minus omega naught because this is another special function. As we said that before we get into the study of communication systems we will revise or tell you a few of the important functions which will appear again and again. So that let us have these expressions handy with us which we will be referring to whenever necessary.

So, we define the important function delta which you have just described and we would like to see that in the frequency domain it is the delta function. So, what is X of t? Is the question right. So, X of t is equal to going by the definition 1 upon 2 pi minus infinity to infinity X of omega. So, we write delta omega minus omega naught e to the power of j omega t d omega, and because it is the delta function the job of the delta function is to collapse this entire function at omega 0. So, when you have at omega 0 that is the only value this function can ever take, in this whole integral.

So, what you are left with is e to the power of j omega 0 t upon 2 pi. So, in other words we could say that if X omega is equal to 2 pi delta omega minus omega naught, then we could say that X of t is equal to e to the power of j omega naught t. Similarly we could also have if X of f is equal to delta f minus f naught we have already said that f a omega is equal to 2 pi f. So, if this is the case then we have X of t is equal to e to the power of j 2 pi f naught t. Or in other words we could say that if this is if the if the X of t is e to the power of j 2 pi f naught t then the Fourier transform of this is delta f minus f naught. And at omega naught equals to 0, if I say omega naught is equal to 0 what do we get is X of t is equal to 1. Similarly this as well, as f would implies X of t is equal to 1.

And what we have on the other hand is delta omega and what we have over here is delta of f. So, of course, we have this 2 pi associated with this. So, if there is a 1 on this side we have a 2 pi associated here right. So, these are some of the important relationships that we will be using in the future, setting up of the expressions that involve signals and as well as systems which we will do encounter in the future lectures.

Thank you.