Modern Digital Communication Techniques
Prof. Suvra Sekhar Das
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Lecture - 16
Analog to Digital Conversion (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. In the previous lecture we have seen the process of converting analog signals to digital or sequence to an discrete time analog signals you can say.

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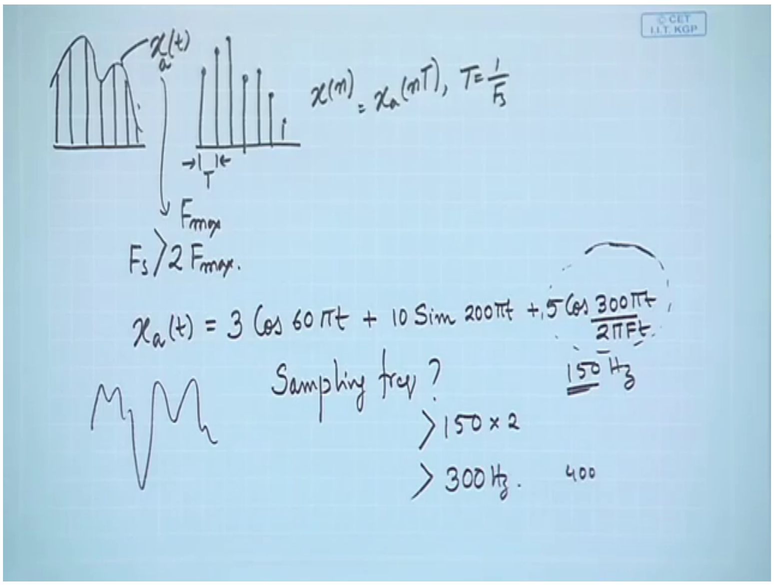

So, in summary what we have seen is if you had an analog signal of this form, you could sample it to certain intervals there is a the first step there is a sampling, and you would if this is represented by x of T need to represent the samples, which come at regular interval as x n of to this x a of T this is x of n, because this these are the signals which happen only at intervals x these are basically signals x a of n T where T is equal to 1 by f s that is the sampling frequency. So, this interval is known as T is what we discussed and we also said that the relationship or how do you provide the sequence which is a unique sequence so that you can reconstruct x T.

So, that is governed by the sampling theorem which says that if this x a of T has the F max has the maximum frequency, then F s must be greater than 2 times F max. So, this is the criteria that we have seen in the previous lecture. So, going by what we have discussed we can take a tiny example to see that how would you choose a sampling frequency because that is the important parameter which one should use when taking a particular analog input, so that you have an appropriate signal out of it. So, suppose we have x a of t is given as 3 cos let us say 60 pi t plus let us say 10 sin of 200 pi t plus let us say 5 cos of 300 pi t. So, if this is the case then what we could ask is suppose this is a combination of signal which comes in. So, that is means the signal is some kind of a signal which is this which is made up of 3 sinusoids of different amplitudes and of 3 different rates.

So, what is the sampling frequency? So, in this case we clearly find that this signal contains the highest frequency, and that highest frequency is a 150 hertz because this is 2 pi 2 F. So, 2 pi F t. So, definitely F is equal to 150 hertz is the highest frequency of this. So, definitely sampling frequency must be greater than 150 hertz sorry is greater than 150 times t2; that means, it must be greater than 300 hertz. So, it does not put a restriction on the sampling frequency on the upper side, it says that the minimum value should be more than 300.

So, as a result what you can do is you can choose numbers which are some 100. So, for example, you could choose in this case a sampling frequency of 350 or 400 maybe 350 or 320 could be good enough.

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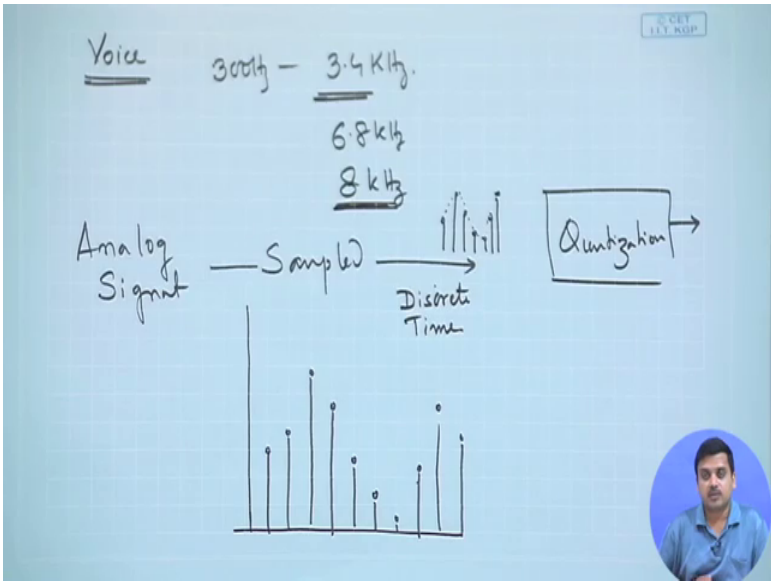

If you take a practical example let us say we have voice. So, if our voice that goes through the telephone line is generally between 300 hertz and 3.4 kilohertz right. So, if this is the case then one would ask that what is the sampling frequency one would choose the answer is what is the maximum frequency content is 3.4 kilohertz. So, therefore, twice the maximum frequency content is 6.8 kilo hertz and what is used in practice is 8 kilohertz. So, 8 kilohertz is a number which is greater than 6.8 of course, nobody stops you from using 6.8 1 which is a very very close to 6.8, but this is a safe number which has certain amount of margin and this is used in most practical cases. So, a voice if you want to sample it with telephone quality signal, I do not know mean the mobile quality signal and you mean a true telephone quality signal, in that case if you are sampling it at 8 kilohertz then you would get a reasonable quality of voice out of this particular signal right.

So, moving down further we have a sample. So, there was analog signal which is which has been sampled and still we have analog signal, but it is only discrete time. Now if you remember in one of the early lectures we had said that if at all one could communicate between the transmitter and receiver or through a channel one could do it through binary interfaces that was one of the statements that we said was duration on. So, what we have with us at this point are still samples of course, those are discrete in time, but still continuous in amplitude.

The next step which is the quantization is the next step which is used to generate sequences which have discrete values of amplitude. That that means, because why do we need. So, if you like to study that what we have over here is a signal which can take any values. So, if you take one of the samples if this is the amplitude axis the signals could be represented it is I am drawing in arbitrary signal and so on.

So, now if we have to represent this through a digital sequence because finally, we need a binary interface right. So, if we need a binary interface we need to represent it through a particular sequence. Now if we have this we have a problem because the what you would define as a digital sequence is what we need to look at.

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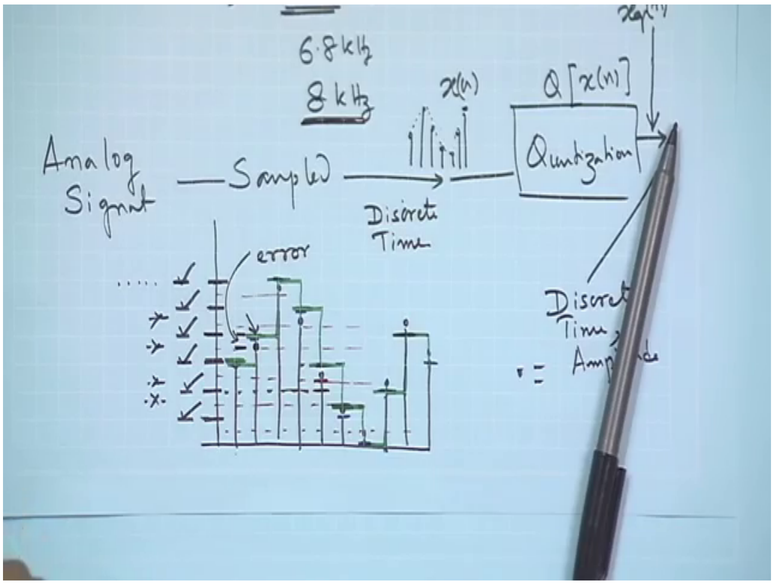

So, to let us get back to a digital signal. So, what is meant by a digital signal? A digital signal you can define it as a sequence of numbers in which each number is represented by finite number of digits that is you are talking about finite precision.

Now, why this is so, why do we have this kind of a constraint? One of the reason is when we are dealing with these digital sequences, we are basically talking about a finite systems because all practical systems have finite in nature. So, if I have a particular value if I do not fix the precision of it then I will require infinite memory or looking at it in a reverse way that generally the numbers which we can process in the digital form and we fix the word length of that number we have a finite precision; that means, we can take only certain values and one example is what we have looked at while doing the Kraft inequality, where we said that each sequence represents a number following within a certain range.

If there are l bits in it from the decimal you have the interval 2 to the power of minus l. So, if it refer back to that particular lecture, we will recall this particular discussion. So, now,. So, once we have the sequence if we have to be presented represent this exactly what we need to convert is to finite precision; that means, we need to have specific intervals. If it is in infinite precision; that means, this intervals are infinitesimally small they are infinitesimally small. So, they cannot be. So, they must be of finite precision. So, finite precision means there are some intervals and what is generally done is if the signal lies within a particular interval then it is accorded a particular value corresponding to that interval.

Now, what one can do is suppose one have decided this intervals one can find the middle point of each of these intervals and if the signal is if the signal has gone above the midpoint, then it would refer to the higher end of that interval and if the signal has remained below the threshold it would refer it to that lower point. So, after conversion a signal might look like something which we are going to draw very soon. So, after conversion the signal might look like and so on. So, this is how the signal might look once you have converted to a sequence which is mapping to a particular level.

So, basically why I have drawn this, I have simply indicated that this value is held for this time. So, one can we construct the analog signal in this way. So, rather actually what we mean is that we have confirmed with this level, we have confirmed to this level we have confirmed to this particular level right.

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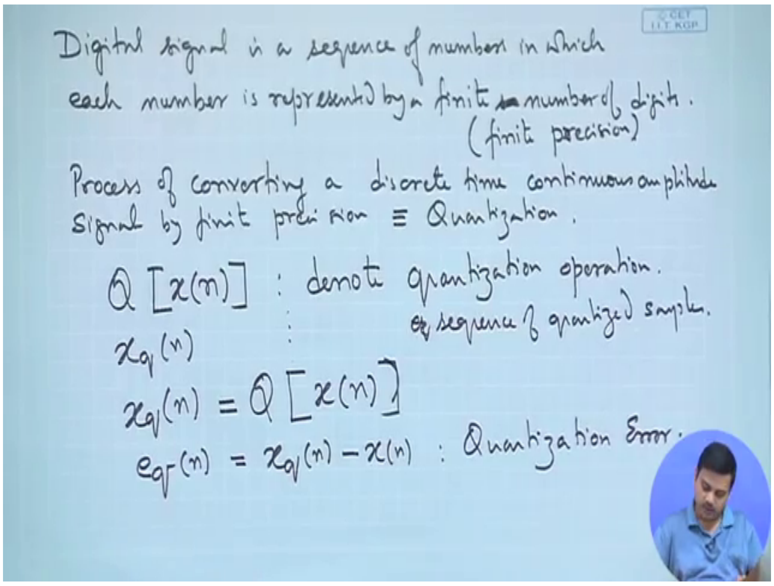

So, that is how it would look like and this particular process is known as quantization or the process of converting a discrete time continuous amplitude signal by finite precision is also represented as quantization. So that means, instead of having continuous values you have quanta which is a size. So, a value can take only one of these levels it cannot take any value here, it cannot take this value it cannot take this value, it cannot take this value it can take only this value or this value or this value or this value or this value.

So, what we have now at the end of quantization is discrete time. So, here we have which is already here as well as we have discrete time and discrete amplitude. Not only that at the end of quantization one should also convert these amplitude values to some particular bitmap that will also see. So, now, you would represent Q of x n if you would recall from the earlier discussion then x n represents the output of the sampler; that means, the sampled sequences. So, let this denote the quantization operation I and x q n denote the quantized or the sequence of quantized samples so; that means, this operation I would represent by Q of x n because at this point we have x n and here we are going to represent this as x q of n. So, this is how does the representation would be like.

So, in other words x q of n is the output of quantization on the sampled sequence x n. So, when you do this, so instead of representing the signal which is here you are actually representing it by this level right. So, there must be an error. So, there is an error. So, there is an error that is introduced. So, this error is known as the quantization error. So, you would write this as e sub q of n which is x, q of n minus x n this we call it as the quantization error. So, with this armed with these things we can proceed and would like to calculate a few things.

So, if we again see this particular picture what we find is that this error. So, if this is one of the levels and this is another level. So, as we said that if we have a threshold at the midpoint.

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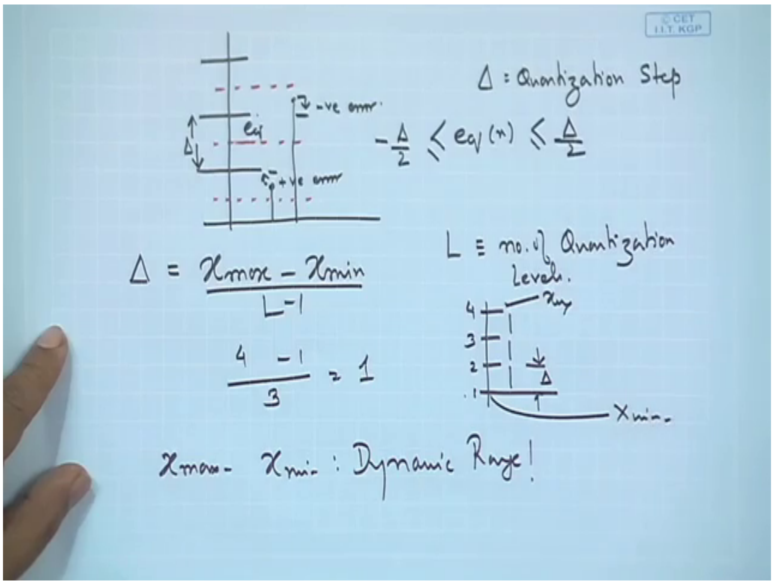

So, if I would draw this picture little bit bigger, let me draw it a little bit bigger. So, let these be quantization thresholds. So, and sorry these be quantization levels and these be quantization thresholds; thresholds means comparison point. So, if a signal is here you would elevate it to this level and if a signal is here you would de grade it to this level. So, this gets converted there and this gets converted there so that means, a signal above this threshold goes up signal below this threshold goes to the limit there and we would say that this interval.

Let this be marked as delta there is a quantization step. So, let us say delta is the quantization step. So, what we can see is that error is only halfway. So, e q, we can say e q of n is less than or equal to delta by 2 or greater than minus delta by 2. So that means, either you are there or you are here and going by our definition we have said quantization error is x q n minus x n. So, you can clearly see over here when x q n is converted to this and x n is here you have a positive value of the error and where x q n is here and x n is there you have a negative value of the error. So, this is negative and this is the positive error all right.

So, since we have this, we can actually now go ahead and do a few calculations and of course, we have this delta interval which is x max minus x min upon L minus 1. So, where L is the number of quantization levels. So, if there are L levels. So, basically this picture level is 1 2 3 4 if there are 4 levels 1 2 3 4. So, you have 1 2 3. So, there are 3 intervals. So, if this is the max value this is the mean value. So, you have 4 minus. So, if this is let us say this is 4 this is one. So, you have 4 minus 1 that is 3 divided by 3. So, this is 4 this is 1 in our example and there are 4 levels of 4 minus 1 that is 3. So, which is equal to 1.

So, in this example delta step is one and you can clearly see the delta step is one in this particular example right and x max minus x min is also known as the dynamic range. So, this is the dynamic range of the signal this is the x max and this is the x min in our particular example. So, what we are interested in is calculating the mean square error.

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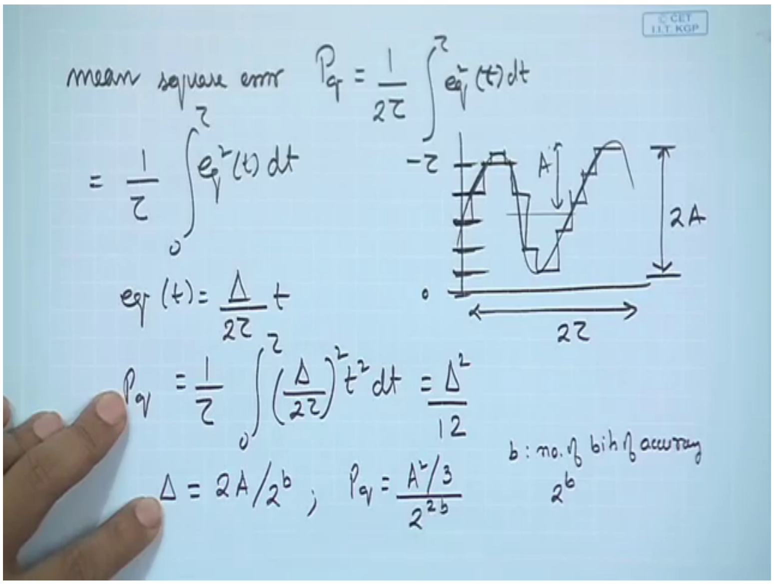

Which is represented by P q which is defined by 1 upon 2 tau integral minus tau to tau e q squared tau t d t; that means, we are evaluating over a certain region of time. So, we are assuming that there is a signal which is proceeding.

So, for example, we have a sinusoid let say, and it is there are certain quantization levels and we are observing it over a certain duration which is 2 tau so that means, this has been quantized in this form let say and so on. So, every time there is an error you are squaring that value and then we have integrated for this interval. So, what you get as a result is 1 over tau integral 0 to tau, because it is symmetric on both the directions e q square t d t. Note I have used over here because this is the range of interval and a range of interval in time and this is t and we will also use the relationship that within this interval this signal is as if flowing in a linear fashion this approximation. So, we are going to do. So, we are going to do this approximation. So, as if the signal is going linear in these intervals.

So, if you have that then you could write e q of t with that approximation in case of sinusoids, now we should remember that all signals can be represented as a means finite duration signals of course, can be represented by sum of sinusoids and of course, periodic signals are definitely possible to represent in that way.

So, if you can represent in some of sinusoids; that means, that means you basically of sinusoids then e q and in each of these you have this linear approximation, you can say delta by 2 t times t is the error then you could calculate P q as 1 upon tau integral 0 to tau that is what we have over here delta by 2 tau squared t squared d t which is equal to delta squared upon 12, and now if you say that this range for sinusoid is 2 A because this is the amplitude right this is the amplitude this is A. So, this is 2 A then you could write the delta is equal to 2 A upon 2 to the power of b, where b is basically the number of bits of accuracy b is number of bits of accuracy required.

So, of course, if you have a b number of bits then 2 to the power of b number of levels are possible. So, basically 2 to the power of b as the number of levels that are possible because there are b bits. So, for example, in our previous examples if you take this to be 0 0 0 1 1 0 1 1; that means, there are 2 bits of accuracy 4 values it can take so, basically 2 to the power of b. So, delta is 2 A upon 2 to the power of b. So, in this case if we use this expression here we can calculate P q to be A squared upon 3 by 2 raise to the power of 2 b.

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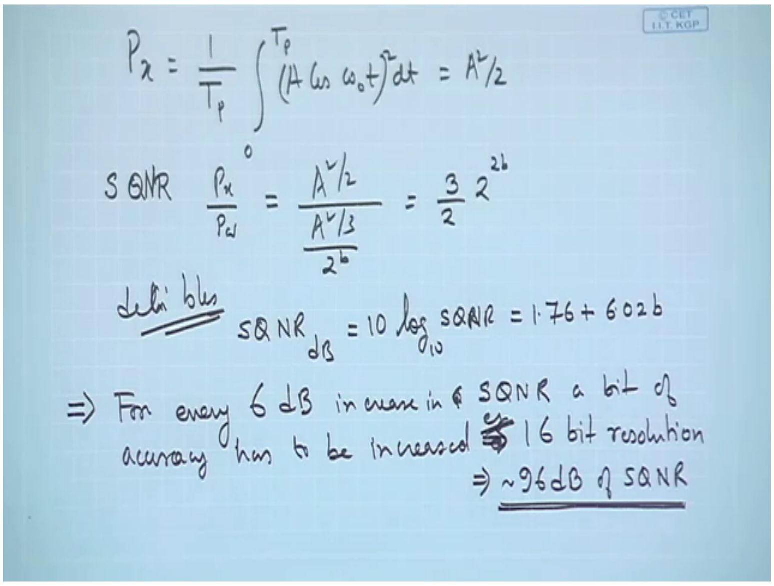

Now, if you proceed with this and considered the average power of the signal. So, P x is equal to 1 by T of p; that means, we are taking one period of the sinusoid 0 to T p A cos omega naught t squared d t this turns out to be A squared upon 2. So, in such a situation what we have is this is the power of the signal and this is what we have got is power of the error or the noise that is developed because of quantization. So, we usually referred to a term known as signal to quantization noise ratio or signal to quantization ratio its some people referred to as SQNR r or SQR. So, I will signal to quantization noise ratio is

P x upon P q and then you can calculate P x upon P q using these two relationship this and this A squared upon 2 divided by A squared by 3. So, you can easily calculate.

So, what you have is 3 upon 2, 2 to the power of 2 b now b is the number of bits of accuracy that we have said. So, if you convert this to decibels, decibels that means, SQNR in dB what you get is ten log base 10 of SQNR that you calculated here with this turns out to be 1.76 plus 6.02 b. So, what this effectively means is that for every 6 dB or for every 6 dB increase SQNR one would require that a bit of accuracy has to be increased.

So, in other words what this implies is that a suppose I have 16 bit accuracy or I have 16 bit resolution, if I have 16 bit resolution example if I have 16 bit resolution this would imply I want to get approximately 96dB of signal to quantization noise ratio right. A factor of 100 that means, if I go for 1 extra bit I want to get a another 6 bit 6 dB increased. So, this means that if I have 16 bit of accuracy, I get nearly 100 dB of signal to quantization noise ratio or in other words it means that we are going to get 100 times signal power over the quantization noise power. So, if we now take the output of these quantized values that we have over here we have to give some notation for representing these amplitudes. So, if there are L levels what we should do if we have let us say suppose we have L levels.

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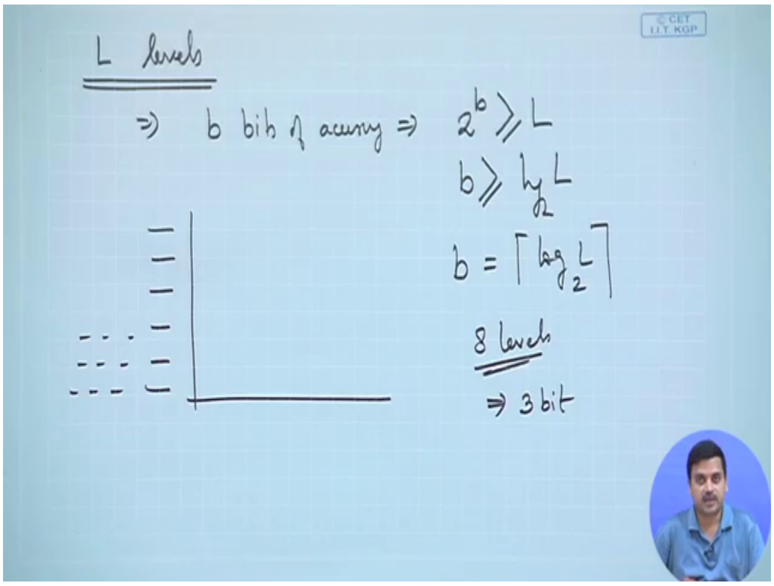

So, if as we have taken there will be L levels. So, this would imply that we should have b bits of accuracy which implies that we should have 2 to the power of b should be greater than or equal to b or should be greater than or equal to L or in other words b must be greater than or equal to log base 2 of L. So, clearly we can say that going by our earlier results we can take b to be equal to this. So that means, each level in our quantization each level is going to get a unique sequence. So, if there are 8 levels that means, we are going to get a 3 bits for representing each level. So, these 3 bits are generally fed into the source encoder which we have already studied. A quick example at this point is if we take speech again and we said that speech is approximately 3.4 kilohertz as the max frequency.

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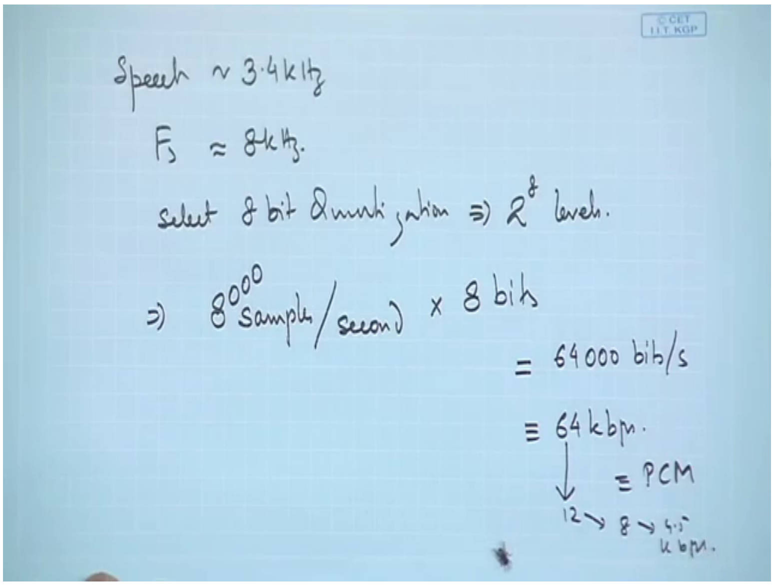

So, sampling frequency we said you select 8 kilohertz. Now if we select 8 bit quantization what does it mean it means you have 2 to the power 8 levels. So, this would imply I am going to get 8 samples per 8000 samples per second at the end of the sampler and every sample is to be encoded with 8 bits. So, together it is going to generate 64000 bits per second or 64 kilobits per second which is generally the pulse coded modulation bit rate. Now usually in voice communication what we do is a we apply a speech encoder in different forms not necessarily in the form that we have discussed and you could bring down this data rate because there is lot of redundancy down to maybe 12 or nearly equal to 4 or maybe 8 or even down to 4.5 kilobits per second with very very advanced encoding schemes.

So, we stop our discussion of a source coding and processing of analog signals. So, we have seen in somebody how to convert analog signals to discrete time signals, from discrete time signals to quantized version that is digital sequence and we have already saying how to take this digital sequence and compress it so that we produce a lower bit rate signal output that can utilize the channel bandwidth most effectively.

Thank you.