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NPTEL ONLINE CERTIFICATON COURSE

Course
On
Analog Communication

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Lecture 09: Fourier Transform (Contd).

Okay so as we have discussed in the previous class that will be now concerned about a signal, which is no periodic in nature.

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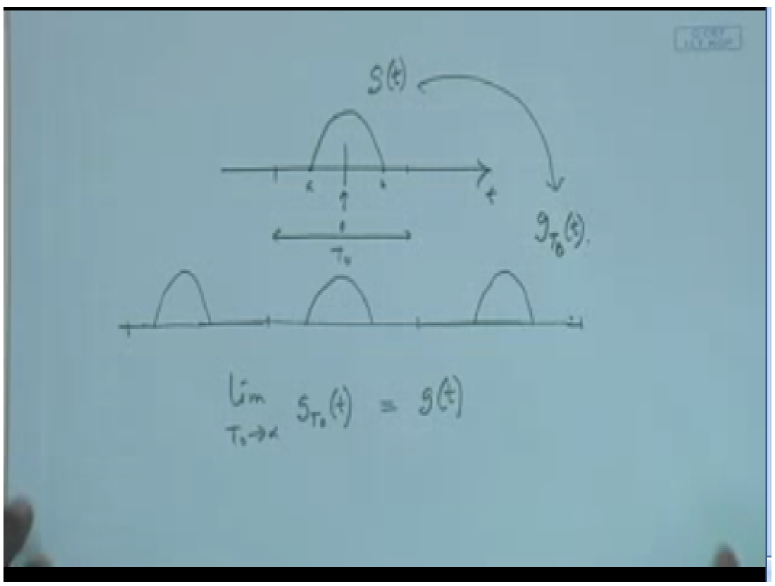

Most of the signals if you see we have defined periodic signals and the fundamental property of a periodic signal is that it must be defined from -∞ to ∞ that means that periodic signal must repeat at some interval let us say T0 which is the fundamental interval and it should go from -∞ +∞ in otherwise that periodicity will not hold good okay so but most of our signals actually start at a particular instance whatever practical signal we can see suppose avoid Y signal generated by me.

It will start at a finer particular time instant that is not a in that effort particular instance so they are finite in time they cannot start actually at +∞ and -∞ and cannot go extend up to +∞ so most of our real life signals are time bounded okay so therefore they have a finite energy because if you go from -∞ to +∞ you will probably means if you wish to evaluate power you will see probably 0 okay.

Because the signals are not stretched up to -∞ and +∞ so our signals are generally energy signal that means it is time bounded it starts at a particular time and ends at a particular time so we should have a strong analysis of those kind of signal otherwise our analysis is not true so whatever Fourier series gives us that is a very strong tool we can agree on that but still that particular thing is not good for real signal.

So we need to have good understanding about the real signal which are finite in time okay so let us try and take an example that is a very simple example signal to define as GT its tarts at some value of P let us say a it ends at some value of TD so this is the time so if this is my present let uss ay 0 times zero so this was my past at which time some value a which is negative in time it has started and it will end at B. so that is the future of the CT.

So this is the signal so its starts at a it ends at B if this is a signal and I wish to still do similar analysis as Fourier has done in his Fourier series analysis right so I still want to see what are the frequency component it has how to define them can we get some spectrum out of heat all those things okay but the problem is whatever Fourier has done that was good for periodic signals so what we will try to do or what Fourier has also done that for a periodic signal only we have some strong result.

So can we apply those result x this so for that we have to forcefully make the signal periodic so let us try to do that let us say beyond this we define some amount of time that is our period okay and the same signal is repeated at every t_0 so basically what happens if this is T_0 my signal was like this after another T_0 there is another period of T_0 where the same signal is repeated and they are also repeats and it gets stretched to -∞ to +∞.

So basically from our signal which was defined from A to B we have created a period which actually takes x account this a and B inside that period and then we start repeating this signal whereas in the original signal there was nothing beyond A and B or beyond even this T_0 but we have Δ deliberately made this signal looks like a periodic signal very nice so this is a newly constructed signal from that original signal we call this as G t_0 T.

So that means from this signal we have transformed x of periodic signals right the good part is by this by means of this clever construction we can always say if I take this limit T_0-->∞ G t_0 t that exactly goes to Gt this is what we were targeting so we were actually trying to see a particular signal can that be represented as a special case of periodic signal okay so we have constructed that particular signal and then we are told okay.

Now if we stretch this period to ∞ okay or this t_0 stretching from -∞ to +∞ then definitely these parts will not appear so they will disappear because the period gets stretched up to -∞ to +∞ so that will exactly look like our GT so this is what we are now targeting so what you are targeting is Δ deliberately we are making a periodic signal out of non periodic or a energy signal which is time bounded so this something you have done now for GT_0 T which is a speedy periodic signal I can do Fourier series analysis.

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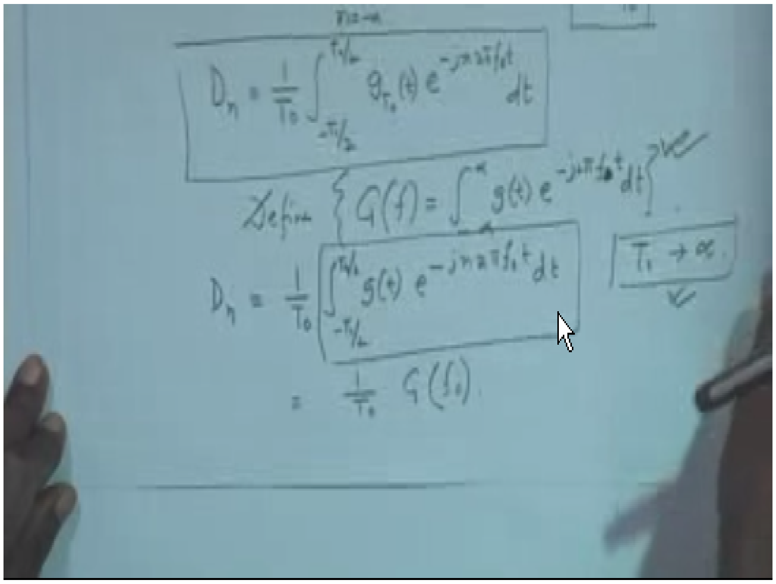

So for GT_0 T which is a periodic signal with period T_0 I can always do a Fourier series analysis for n-∞ to +∞Dn $e^{Jn2\pi\ F0T}$ I can write this way where this F_0=1 / T_0 the period I have constructed the frequency fundamental frequency is 1over that ok so this representation is true I can always do a representation like this.

And after that we have evaluate Dn so Dn is something we already know it should be 1/T_0 and integrate it from -t_0/2 to + t_0/2 over the period g(t0) T which is the main signal and it will power -Jn2π and f_0TDT so this is something we already know right very good okay now let us see what happens to our representation okay so in our representation let us try to do it this way let us define it just arbitrarily defining for our you will see why we are defining this.

So let us define a GF which is nothing but integration from -∞ to +∞ GT original signal it is the power -J2π F_0Tdt is just defining this is our by definition we are saying that because this sorry it should be on the F because this is our integration over T so that T will vanish it will be a finally a function of F we are defining that as a G F function of smallest right so this definition is correct no problem in that as long as this integration can be done.

So if we can if you say that this integration is possible we can always say there will be this will be a function of F because T goes away right so we can write that function as some capital G as some function okay so this destination is correct when this definition is correct what I can write about DN so let us see what is VN so now this DN if you carefully see this DN has1 /t_0 that is alright and in place of GF if I just write F0 okay so if I just write F0 then what will happen I will be getting this GT integration - t0/2 to + t0 /2 and I will have $e^{-Jn2\pi}$ similar things are there just in place of F I have a F 0 right so basically what I can say this particular part if I put T0→∞ okay.

Immediately this limit goes from -∞ to +∞ and as long as I say t0 →∞ my Gnt if you see the previous definition for GT 0 T if I put T0 →∞ GT0T I can write as GT so I can write GT and replace that as GT0T okay so I can if I say that whenever I will put this particular limit I will be always getting this representation as we have G in place of F I will be putting F0 so this is quite obvious as you can see what is G of 0 G of 0 will be just replace over here GF 0 so that should be GT $e^{J2\pi F0T}$ DT okay.

Only problem is we have a GT over here so GT as long as T0 is tending towards ∞ which is this case GT and GTS 0T are similar so I can GT I can replace by GT0 T now if you go over here now this to limit T0→ ∞ means -T0/2 and + T t0/2 goes to - ∞ and + ∞so I can replace these two limit as if going to - ∞ to +∞so therefore DN exactly limits these things okay so that is the duty I could do by this definition and by our definition of GT and GTS0T or the relationship between these two okay.

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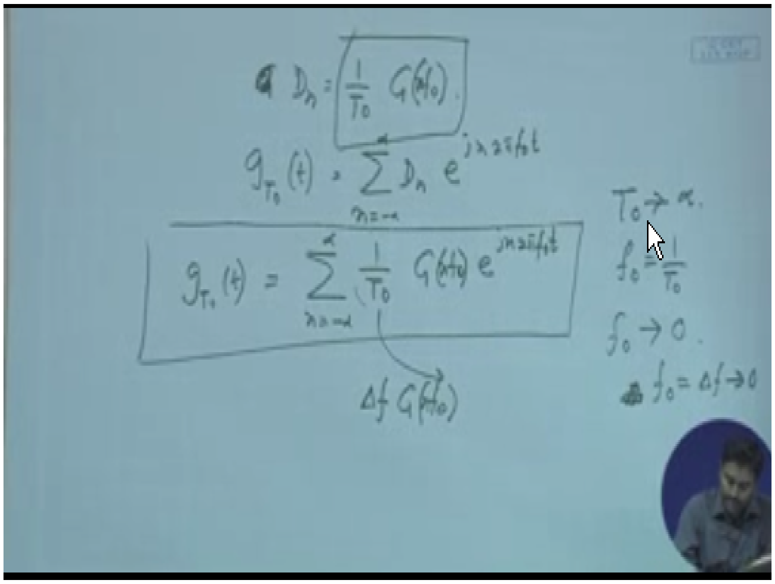

So once I could do this I have got a very nice construction so what is that construction now I can see that what happens to my Dn, Dn becomes 1/T 0which I have already written this is my DN and immediately I can represent my G_{T0}(T) which is the Fourier series expansion so which is ∑n= -∞+ ∞ DN $^{eJn2\pi/f0T}$ and in place of DN if I can replace this so it happens to be n=-∞ + ∞ $1/T_0GF_0$ $e^{Jn2\pi F0}$ E right this is what I get for this particular part okay so far it is all good.

Now let us try to see what that this particular thing means so what is happening in this case we have already assumed otherwise you could not have constructed this DN=T0→ ∞ so whenever T0 goes to ∞ I have a relationship F0 =$1/T_0$ so what happens to F0, F0→ 0 okay so this $1/T_0$ or F_0 I can that becomes infinitesimally small okay so that becomes infinitesimally small instead of writing it as $1/T_0$ I can now write as ΔF where this so I am just replacing F0=ΔF which tends to 0 it is just for my convenience.

Because we are used to CΔF and this is GF_0 or sorry let us see there is things that we have missed here so it is n f_0 which makes the f so it should be written as nf_0 right so that that was a mistake I did okay so because it is if you see the construction GF is written as Gt e $^{J2\pi FT}$ now here in the construction we have e $^{-J2\pi T}$ all of this and n f_0 is there so therefore it must be nf_0 right that's all right so this should be all replaced by nf_0 so ΔF nf_0 must be replaced as nΔF right because f_0 is replaced now by ΔF.

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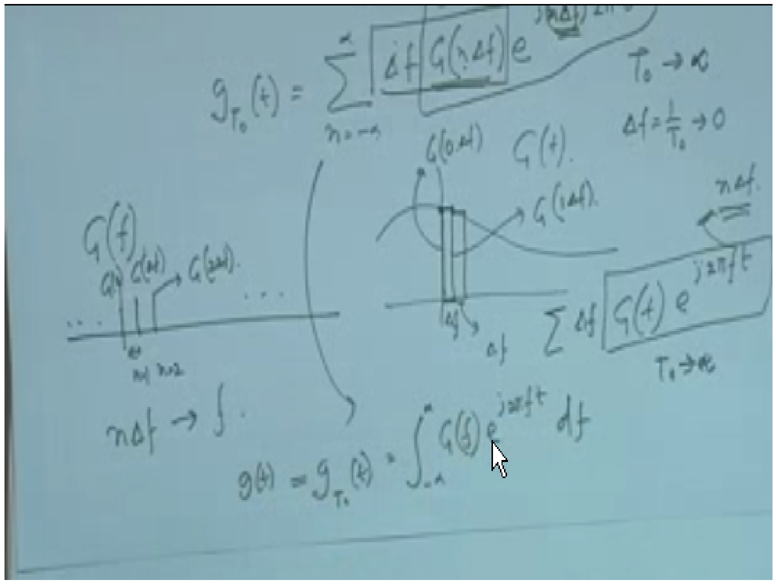

So GT_0T may be constructed GT_0 T becomes $\sum n$ =-∞ to + ∞ $1/T_0$ becomes Δ F G n f_0 becomes Δs x e^{Jnf0} becomes ΔF2πT right so this is the new construction that we have got for GT_0T and we know here T_0→ ∞ and ΔF which is $1/T_0$ goes to 0 okay so this is something we already know now let us try to see what exactly is happening what is this part this was some function GS okay now whenever we start varying this n=- ∞ + ∞ so what is happening whenever n = 0.

So it gives G_0 n =1 it gives at ΔF Whatever value of GF so this must be G on the ΔF so this is G_0 at n = 2 so this is a n=1, n=2 it gives me G2 Δ s and soon and also so on in this direction so basically it is almost like there is a function G F which is a continuous function and whenever we are putting this different values of n we are actually sampling this function at Δ F interval now as long as this Δ F is becoming infinitesimally small.

So what is happening all these samples are coming closer and closer together okay and in the same method what is happening to this particular value so this is nothing but suppose I have a GF let us say this CF looks like this I take suppose n =0 I take this value and I take this box so what will be the area under this box that should be Δ F x G 0 or 0 x Δ F the next part will be again Δ F and what is this value this value is G1x Δ F x Δ F.

So it is almost like this small boxes I am actually taking the area under that okay and I am adding all this one ID once I do ∑ I am actually adding all these things right so as long as my Δ F →0 what is happening this ∑ almost becomes the integration because integration also gives me area under a particular function so if I just now think that my overall function is this particular thing that is gf x e $^{J2\pi FT}$ where F is replaced by n Δ F.

So basically what is happening this particular function I am adding with or I am multiplying with ΔF and I am summing it up whenever we put F as discrete values so it is nothing but this ∑ becomes the integration and my G t0 T just happens to be our integration and because it goes from- ∞ + ∞ whenever we put n = - ∞ what happens this n x F 0 goes to -∞ so if my variable is f that goes to - ∞ and whenever I put n = + ∞ it goes to+ ∞ this Δ F becomes DF and I have this gf it will be power J2π this is replaced by FT.

So basically n Δ F is almost becoming a continuous variable F this is very true we have already stated that if Δ F is small then all the values I am taking almost becoming continuous in time or sort in frequency so basically I was taking initially I had this function which is gf x this I was sampling that function and I was trying to calculate the area under it because I was multiplying by Δ F and taking that box and I was adding all those things.

So this is nothing but integration that we have already understood and what is also happening I am making this Δ F smaller and smaller so those samples are coming closer together and getting very closely packed and actually those samples are almost indicating now a continuous variable on F so whenever I write n ΔF that goes to F okay whether f varies that becomes the integration variable so immediately I get this and because my $T_0 \rightarrow \infty$ already for this construction.

So this can be written as Gt because GTI and GTD ρt becomes equivalent as long as 0 goes to ∞that was our definition of the signal so we made this a periodic signal to be periodic with period t0 and then we have told that this p 0 if we stretch to∞ it becomes a similar a periodic signal this is something we have already stated so as long as T 0 goes to∞ we get this very relationship so what has happened.

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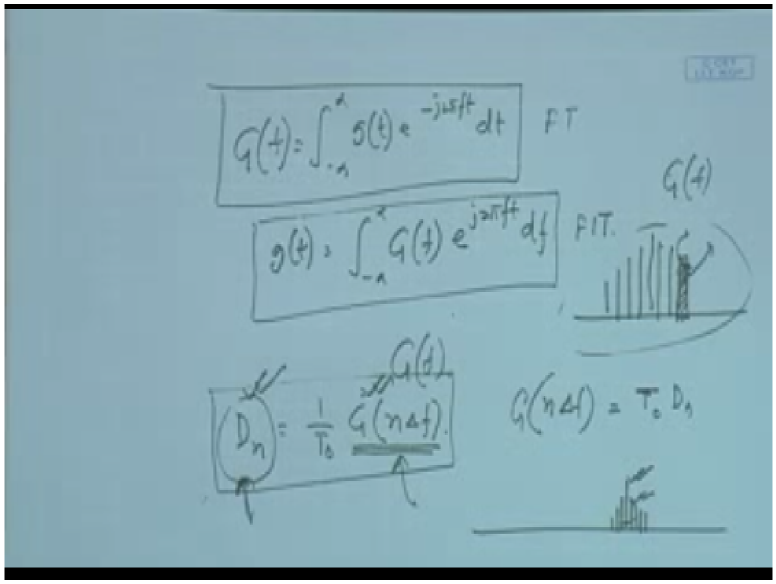

Now we have a two relationship that we have said one was our original statement that gf must be related as this was by definition GT it is the power -J2pi FC DT and now from the construction we have got another relationship where we say GT is nothing but - ∞to + ∞ gf in the power + J2πft DF so this is the famous Fourier transform and Fourier inverse transform theorem you can immediately identify okay.

So if I have a signal GT I can always get a corresponding Fourier transform which is gf and if I have a signal G means if I have Fourier transform I can always do a inverse transform to get my signal back so this is the Fourier transform and inverse transform in this whole process the most important part is this gf which is intermediately constructed so what is this gf if you carefully see the construction this gf okay is actually almost similar to as you have seen it is almost similar to our DN.

We have derived this art here so basically we have already stated that DN must be 1/T_0 x G and LF what was our DN, DN was for every frequency component the corresponding amplitude and phase DN was giving me that now G if as long as Δ F tends to 0 this becomes a continuous variable ok so this end LS s long as Δ F is very small this almost becomes a continuous variable of F so this is nothing but our T0 x DN or I can write this DN equal to this.

So basically what is happening the corresponding DN can be calculated if I know that gf whatever that GF is and I multiply that with that very insa infinitesimally small frequency component Δ F so what is happening suppose I have a particular let us say of Fourier transform representation gf okay so each one of this for a particular value of n are actually representing suppose this is my gf right so that sample value if I multiply that with Δ F so this area okay or let us say this area.

So GF x that Δ F this area as if giving me this TN at that frequency what is the overall coefficient of that frequency component so what is eventually happening as I put this Δ F tends to zero very small and small I am actually getting all frequency component so it is almost becoming continuous earlier in my periodic signal I was only getting those harmonics of 0 to f0 and all those things now for a periodic signal what we have observed that because for that appearing signal we have represented it with respect to a periodic signal.

So if we faithfully represent back the periodic signal t_0 has to be ∞once t0 is going towards ∞ this Δ F is becoming very small so that means it is now almost those frequency components are getting closely packed so we are getting almost all the components all the frequency component in the continuous domain right and everywhere if I multiply by this Δ F that gf whatever I have if I multiply by this Δ F I get the strength of the frequency component similar like this pier right.

So what might happen suppose in a hypothetical experiment I keep on because I have to go means stretch my t0towards ∞ I keep on reducing my sorry increasing by t0 so earlier suppose this was my representation okay so this was my Δ s now what I do I actually increase my t0 so immediately what will happen if I increase my t0 so immediately this will suppose I make t0double so if I make t0 double immediately what will happen this DN value because t0 is doubled DN value will be reduced by half okay.

So what happens the N value will be reduced by half but the samples will be now more so basically instead of this it will be all reduced by half but the samples will be closely packed double close the pack because Δ F is now becoming half of the previous this one so there will be double closely packed and the sample values which is represented as DN which is the integration or we should say which is the area multiplied by Δ F GNF n Δ F ok so that area is represented by these values.

So they will be becoming 1/2 because I have increased my T 0 to double corresponding by a0 has become half so basically sample will be there will be double samples now more closely packed but this strength will be reduced and if I keep on doing this experiment what will happen

I will still get this samples or their values as long as T0 goes to ∞ their relate means individual values will go to 0 so basically what I am getting I am getting at every point in continuous frequency domain.

I have some value but the corresponding value is 0 because T 0goes to ∞ ok but the relative values are still being constructed by this year so however small you make them they have a relative nature so if this is the strongest one and this is little weaker they will still remain the same their individual value will tends to 0 ok but their relative nature which is captured by this GF will still be the same and that characterizes the corresponding signals the relative nature but individual values at a particular frequency is no longer existent is a very typical understand.

There is a typical understanding behind this so probably we need to have some clarification one very simple example which is often used in probability theory probably will have better understanding.

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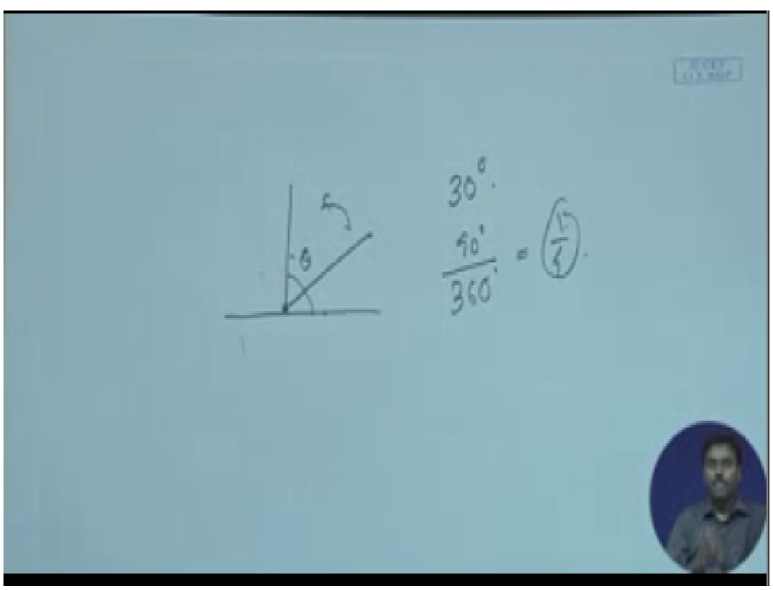

So suppose I have a P voted rod and I give a random force to this rock and this rod can freely rotate around its axis and depending on that random force it will go and stop somewhere okay let us say where it stops so it says it stops over here that angle is called θ now this theta becomes the random variable okay now if I just ask you that can you tell me the probability that it will stop at angle 30 º now how many possible angles are there suppose the force that I will be exerting are almost means the way it is being exerted that whichever angle it will stop.

They are all equally likely so it has an equal probability of stopping of stopping to any angle right so if that is the case every angle is equal probable so they are equally favorable so now I am targeting a particular angle30 degree okay so how many angles in between are there are in finite number of angles because 30.1 is the valid angle 30.01 is a valid annual 001 is validated so there are infinite number of angles so the favorable case for my that30º is only one that it should stop at exactly 30º 30.0000 up to ∞ okay.

But there are infinite number of possible cases so my probability that frequency definition of probability will give me just one upon infinite let zero so stopping at a particular angle is 0okay so it doesn't have any probability of stopping at a particular angle but if I now specify a angle range let us say between 0 to 90º it will stop because things are equally likely now let us try to see it has four quadrant so it has almost 360degreeout of that I want to cover this 90degree right so that must be my probability which is one by four so whenever now I am specifying a range I have a valid probability value whereas if I specify a single value I do not have anything no probability values are recorded same thing happens to this gf.

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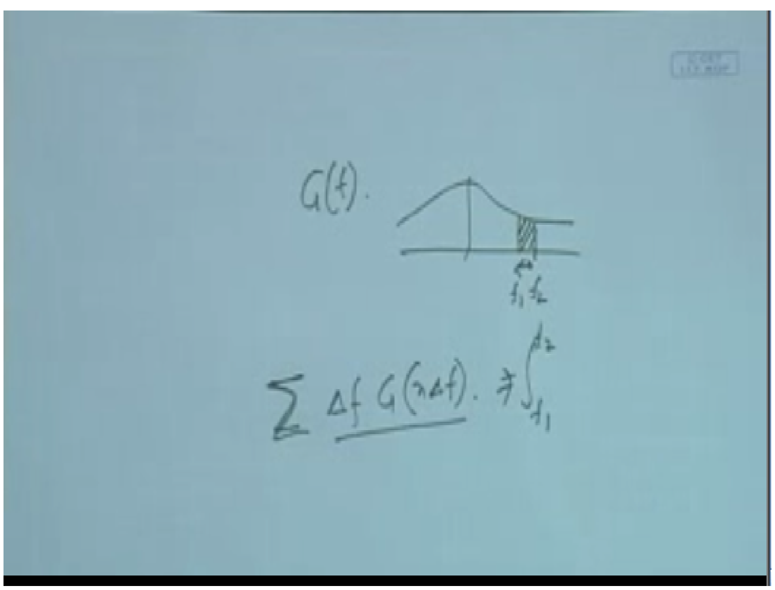

Whenever we are talking about gf I plot a GF individual value I do not get anything but if I specify a range from frequency f1 to f2 it has some value because it will be the integration over that and I will get some value any function if DN is the way DN is defined if you see DN is the length at a particular frequency okay so if I say from this frequency to that frequency in Fourier series term I will be just summing all DN okay.

So from that particular frequency to another frequency I will be summing all this DN whereas in this particular representation if I have to sum DN what I have to do the ∑ becomes integration we have already seen that DN was nothing but that Δ F x so its area actually Δ F x G whatever that n Δ F right so basically and then summing over that n value whichever in value we are targeting so f1 to f2 will have some n value to some n value if I am doing this it is just area and we are summing that it just becomes in a continuous variable that becomes the integration from f1 to f2 I am integrating it.

So basically what is happening whenever we are saying that an individual value that does not exist because it was Δ F →0 this area tends to zero so I do not get a value but if I start integrating from a particular frequency range to another frequency range I get some value okay so that is the beauty of it in the next class we will again what is the implication of this thing.