**NPTEL**

**NPTEL ONLINE CERTIFICATION COURSE**

**Course**
**On**
**Analog Communication**

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**Lecture 60: Pulse Coded Modulation**

Okay so as we have already discussed about different kind of sample which is the major tool for doing all amplitude modulation? So what will progress will be our last class what will try to see is how this can be realized in circuit first of all and then from here how do we go towards our digitized version of this okay so that is probably the linkage of analog communication to digital communication so this is with this all we will end out this version.

Okay let us try to see what actually we employee as a circuit for all amplitude model let us say as a natural sample you want to do because we have seen already some advantages of natural sample so what we can do.

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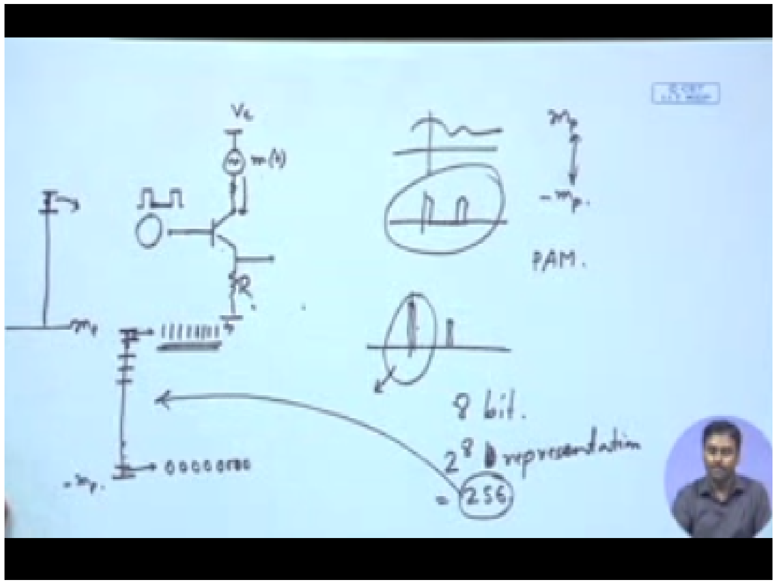


Let us take any switch okay so it can be a consistor switch or it can be a mouse switch whichever switch you have this so basically what you try do is you put your message signal mt of course it is having wires and a resistor over here you also have a resistor over here and over here what you do you actually supply the pulse so this is where you will be supplying pulse okay so whenever the pulse is high that means it is on and that time this will be on and there will be a current flow which will be proportionality because the message signal you are putting over here.

So that will proportionate your message signal so the message signal where is the line is like this the current also will be wearing according to this in nature and the voltage across this or this resistor so we will also have similar variation but whenever it is off nothing will flow through it because of the resistor or that switch will be off and you will get empty so basically what will happen as a output of this resistor what you will see whenever it is on it will follow the signal whenever it is off nothing is getting flowed through it.

Again whenever it is on it is follow the signal and current so that is all natural sample with a very simple circuit we can actually get natural sample so that is also another advantage of natural sample that natural sample can be realized with a very simple switch either a similar switch or a consist or switch can be done very easily just supply the pulse supply the message signal and automat5ically it could be modulated with that switcher okay.

So that is the realization of a pulse amplitude modulation okay what we can also do is pulse volt and pulse position modulation but probably for time savings we will skip those things because of they are not much use and not that important so what we will try to do in this class is we try to see from m how do we go to the digitized version of this okay so let us say I have either a natural sampling okay, but a natural sampling become very small duration okay.

So that means it is almost equilant to that lack of sample because the duration is so small that the variation will not be it will be volute so it almost happens like a lack of sample so if this is the pulse modulation I get now what I do is this particular pulse I have got a amplitude at that particular instance okay. Let us say my message signal it has a range amplitude range from +mp to –mp.

So that is the overall range of amplitude variation immediately I can also say that pulse amplitude because it is unit let us say it is the pulse is art of unique amplitude so whenever we multiply it just takes the message and signals amplitude so basically even the pulse amplitude also will vary in that range so highest pulse amplitude can be +mp and the lowest pulse amplitude can be –mp.

Okay so this is the range or which it varies so I demark that range so from +mp to –mp now what I want to do is I want to digitize this okay, so to digitize this means I want to have a unique representation of each of this samples which can be represented by 2 bit by binary representation 1 and 0 which is easier to encode and easier to transmit we will see the advantages of that also okay.

So I want to digitize it what is the technique that I should employee whenever I digitize let us say I digitize with let us say 8 bit okay so that means each of the sample represent with 8 number of bit with l bit how many representation I can get that is something we know $2^8$ representation okay that is 256 representation so if I do represent every sample with 8 bit then I can only get 256 positive representation.

But within this +mp to –mp the signal is analog so how many representations are there infinite representation every voltage is a positive representation but I only have restricted number of representation so what I do is a very important part where you do from analog communication to digital communication or do employee a method which is called quantization so what does that means quantization means basically this range you take.

And you subdivide this range into multiple sub ranges how many such things we will be taking exactly 256 such ranges because you have to push to 256 representation so you take small sub range and any signal happens to be within this range will be encoded as the middle value and it will have a corresponding binary representation or a 8 bit representation why I have taken 256 because I have with 8 bit 256 distinct representation so if this happens to be within this range entire range can be represented or with a unique binary code let us say all 1 so 8 ones like this is can be represented with all 0 and so on.

Every interval can have a unique representation and what we do whenever we represent it that is all fine that I get a signal which is in between somewhere it can be anywhere so I represent that as the corresponding binary code so this is some kind of modulation again then I am actually modifying the same signal for a better representation okay so or a digital representation so this some kind of modulation and at the other end I have to also demodulate it or have to de-map it decode it.

So for decoding what do I do now I have this unique representation whenever I get this unique representation I will be able to put a corresponding sample value over there but now the problem starts because which sample value I put all these things from this all those sample values in between intermediate sample values are mapped to this unique representation so when I de- map I have no information where exactly it was because I do not have that information.

So what I will do I will actually put this central point as the representation so immediately you can see there is a error because it might be this one and I represent with the central value so basically the amplitude was this much I have represented with this so what might happen.

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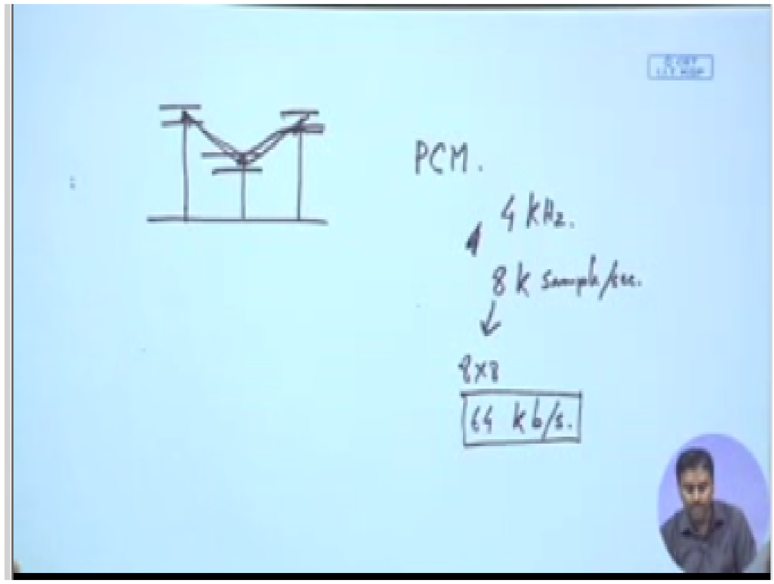


Suppose my message signals where like this the samples are like this and then I encoded with binary so anything falls under this so basically what happen I represent it with the middle value okay and then I joins the middle values whereas actually it should be this tips it should be joined there should be little bit of error due to this quantization this is called quantization error okay so whenever we digitize the signal we will always be having a quantization error and this process of digitization is called pulse cordite modulation.

Because what you are doing is taking a code sorry taking a pulse and you are encoding it into a digital or mapping it into a digital bit stream okay that is why it is called pcm or pulse coded modulation okay why signal use this means even these days also the telephone that we use the landline phone calls we make that goes through this so basically you hear voice signal first of all it is being sampled with because the voice signal.

We assume that it has maximum band width as 4 KHZ so what will be the corresponding might be straight that should be 8 K samples per second each of this samples are represented by 8 bit so overall bit rate will be 8 into 8 so each of those samples are represented by 8bits so 8 into 8 which is 64 kb/sec so that is why the voice is always being transmitted with 64 kbps is the defector industrial standard which has happened that is because of this pcm encoded thing which has 256 representation for quantisation.

Okay so whatever this is we have now seen that there is a quantisation error which is happening due to this process so can we characterize this quantisation so let us try to see if we can do it so let us see my message how it was.

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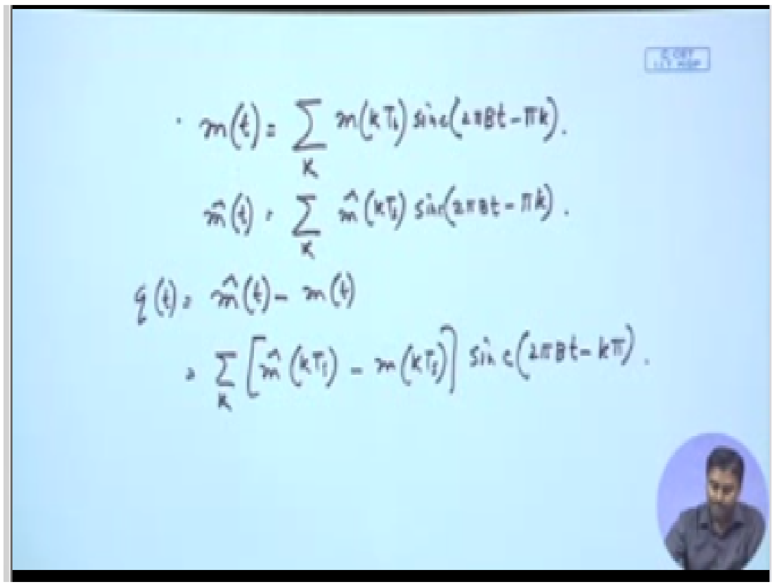


We had this interpolation relationship which tells me that this is actually nothing but samples taken at particular instants and represented by a corresponding sinc function whereas my quantisation 1 will be almost similar but only thing is that instead of taking this or we taking the middle value so let us call that as n^ because I cannot get all these values this can be any arbitrary number that means any arbitrary hall u between –mp to +mp.
Or by taking some all those quantized middle point okay so that let us call that as m^kts and that should again because this r creating a sample at the other end or we actually representing again with the sinc function so that I get my message back. So due to this process there are what is that error so let us say that is the quantisation error that is by this n^(t) –m(t) which is nothing but if I just represent it k ∑ which of those sample error.

So n^ (kts) –m(kts) sinc (2π8t-kπ) so this is my quantisation error so this is the errors signal I need to see how much power so this error signal is almost like an noise sitting over there right so if I just try to see.

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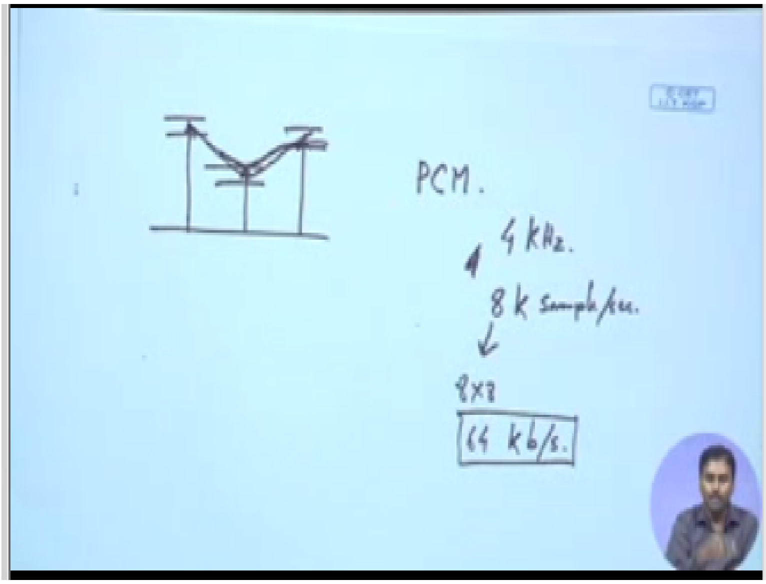


This was my original signal right the above one and this is my after quantisation and demodulation or means decoding this is what I get so the error is actually on noise as if being added on the signal to get my signal right so that can be treated as noise and how do you characterize noise so this is something which can be termed as quantisation okay so this is a particular noise if you see whenever you go from analog communication to digital communication this is a particular noise which is been created at the source.

So for all the noise that we have talked about are either generated at the transmitter circuit but which was very negligible so we have never considered that in the channel of course there were noise and in the receiver circuit so mostly channel and receiver circuit that where the noise were there now this is the noise which is being created at the encoded so whenever your quantising you are potentially creating noise.

So this is source induced noise so your process of transmission is generally creating this noises okay so this noise how do I characterize the noise we need to get the noise power so basically I want to evaluate the noise power.

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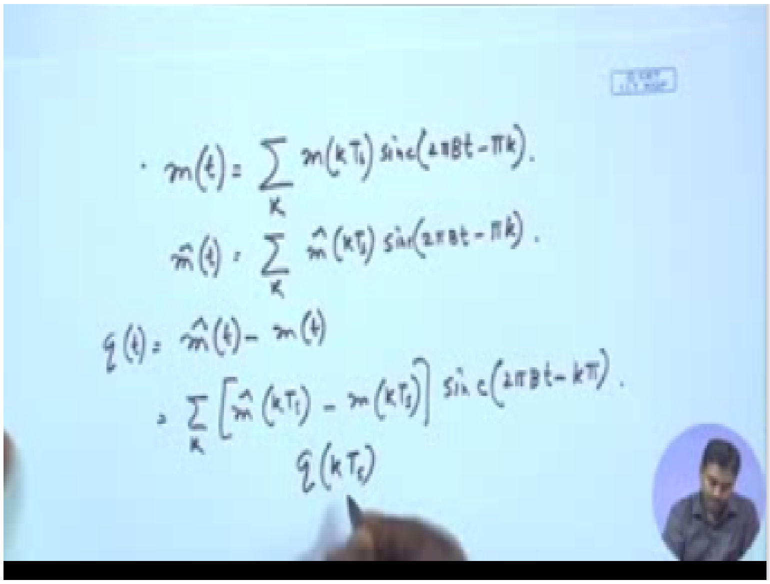


So I $^2$ it and taken the average so in time domain if I just write this it should be this the way we have defined power right this is how we have defined power so this how we should deal evaluating that so if I now put the formula of qt is was ∑k 2(kts) okay so that m^kts-mkts we are terming it as q(kts) okay.

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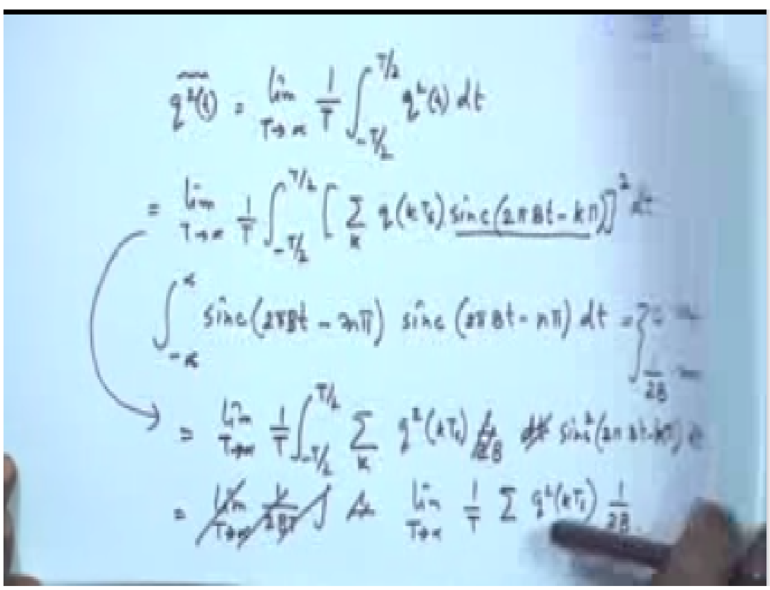

So if we write just q(kts) this is sinc(2π8t-kπ)$^2$ dt now this is how what you can take for your own benefit so what happens we will be able to prove that this sinc functions are mutually orthogonal for different values of k what is that means we can we will be able to show that from –α to +α if we multiply sinc(2π8t-mπ) sinc (2π8t-nπ)dt so that is two signals with different values of m and n they will be orthogonal if m equal to gives me some value and m not equal to gives me 0 so that is what happens.

It is 0 when m is not equal to 0 this is something which we will be able to prove and this is equal to 1/2b=1 okay so these are orthogonal so what will happen this $^{2ed}$ term basically whenever you $^{2}$ it there will be all $^{2}$ terms + 2*this cross term with different values of k like m and n so all those terms will be 0 so only the $^{2}$ terms will be remaining so what we can write from here it is nothing but it tends to α 1/T(+1/T -1/T) so just ∑ $^{2}$ term kts.

And this whenever m=n that is the $^{2}$ term that should be 1/2b okay and of course there should be this means that $sinc^2$ will come then you integrate basically you get 1/2 right so this is what we get of course because we have already done that integration that integration will be getting we will be getting this because t tends to α is there so –t to +t goes to α and then that integration we can evaluate so you get ½ I can write immediately limit t tends to α 1/2bt $sinc^2$ (2π8t-kπ)dt okay I should have written this.

And then the integration I can take inside because this has nothing to do this the summation and this has nothing to this ∑ and this has nothing to do in that integration variable and limit so that they can come out so I can always get 1/ so this is equal to limit it tends to α 1/T tends to ∑ this term goes out $q^2$kt and then I will have this integration because t tends to in pulse so it goes for – integration +integration which happens to be 1/2 okay this is how I can write this and then.

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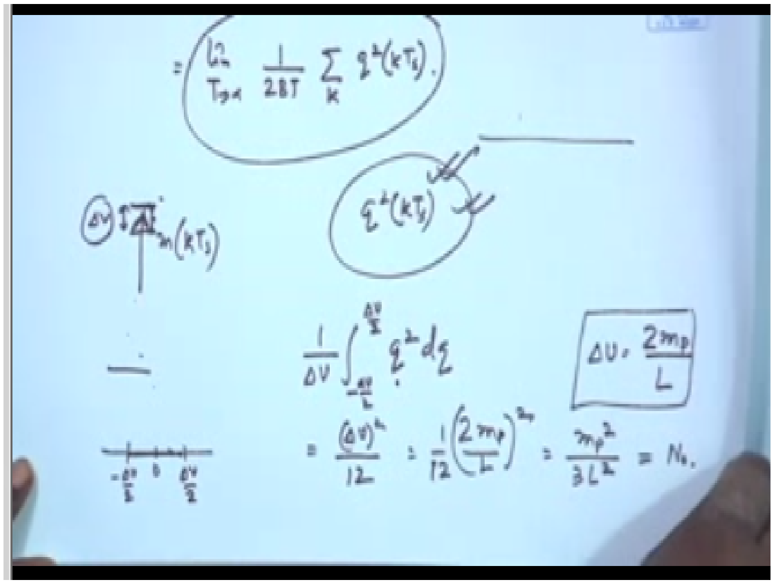

What do I get I get limit t tends to α 1/2bt $\sum_k q^2(kts)$ what is this means so basically I'm taking all those quantisation value to the $^2$ of them let us for our duration of t how many such values I should be getting I will be exactly getting 2b into t because 2b is the sampling rate okay, so power unit second I will be getting 2 piece of samples in t duration I will be getting 2bt because this means actually this means the average value of this quantization.

So $q^2(kts)$ it is the average value of the head okay so whatever those values are at different k I need to take the that means the mean of the $^2$ or the average of that because I am actually summing all of them divided by as many samples are there in t and then t is stretched to α means I take enough number of samples so that it is actual average so basically I just have to do averaging of that should be my quantization noises.

So this is something we have to proven now okay so now let us try to see how do I get this average so let us at this point will be doing two things one is we first assume which is also a valid assumption that this q thing that is of course obvious that will be a random process so basically what is happening it is a distinct time process so it has values at different time instants so this is the time description.

But we also have a assembled description of this one okay so there might be infinite number of such sample function which has different kind of variation and we say that this is a stationary random process as well as regarding okay we are true but people have tested that is true always okay so if that is the case the ensemble average will be the time average.

So we were try to calculate the time average that it must be the ensemble average and now you will make further assumption that those samples actual samples of m is whenever we take m (kts) those samples are informally falling between –mp to +mp so they are uniformly distributed so a sample which are restricted over a particular duration let us say the duration is Δb because the error will be only on that.

It means he is always any sample within this will be approximated by the central value so any error that can actually percolate that will be restricted to the boundary of that particular zone okay let us say that is Δv what is Δv then Δv is equal to overall range is from +mp to –mp so that is 2mp/ as many number of levels I will be creating so let us say that is L for our case our example L was 256 so 2mp / L is Δv.

And what happens within this we are now assuming that the samples are uniformly distributed that is what our assumption is  the samples are uniformly distributed what is the corresponding error because will be any sample we get we will be representing by the middle value okay so the corresponding duration will be from the middle value to whatever it is so let us say if it is uniformly distributed in the picture it should be 1/Δb going from -Δb/2 to +ΔB/2 because the middle point is the central point so what is happening this is 0 this -Δb/2 and this is +Δb/2.

So this is that whole range which is being represented by this okay within this the sample might lie uniformly distributed so that is why 1/Δb is its pediare and then I have to take $q^2$ I want to take the average of that whatever that value of q is the deviation is the error so $q^2$d2 i have to do pdf I have already taken now so this it will evaluate what we will get it will be just it means integrate it and then put Δb so we will be getting $Δb^2/12$ that is the quantization.

Because I wanted to evaluate the mean of the 2 of it so that is what we are doing mean of the √ of it so mean of the √ of it is √ integrate over its range which is the pdf range which is from -Δb/ +ΔB/2 and 1/Δb is its constant pdf so I get this now what is Δb I can put that should be $(2mt/l)^2$ or $mp^2/3L^2$ this is the noise this is called the quantisation noise of a signal okay.

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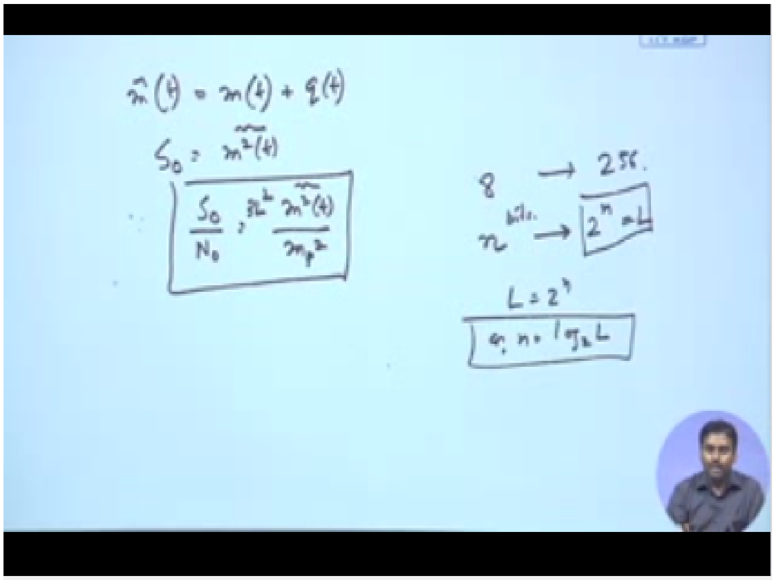


So now suppose I have transmitted a message signal mp along with that I have a quantisation noise which is qt and I get this noise contaminated symbol in ^ because you are actually receiving those m^ and we are connecting them only we are getting this much error this was the error signal so immediately what will be my 0 let us say that is the message signals power so what will be my signal to noise ratio that should be the message signals or divided by the noise.

Which is $mt^2$ into $3L^2$ right so this is how signal to noise to noise ratio is calculated in our ethical this kind of pcm transmitted ratio okay now let us try to characterize it little bit so what is exactly happening is something like this remember we if we had because it is a binary representation so if we have 8bit how many representation you are getting or what was the L value that was actually 256.

So if we have in general n bits or representation will be getting $2^n$ =L at many representation okay alright or we can write that $L = 2^n$ or $n = \log_2 L$ relationship we know all that okay now let us also try to see what is happening to our overall band width okay so we already know that every hedge I can transmit to bit okay so basically I had B that is the overall band width I could vertically transmit to B samples okay.

And overall how much information I am getting so for this b band width because of multiple sampling 2B samples I am taking for that sample weight each of those samples are now represented by n bit so this kind of bits are representing okay so to transmit this how much bandwidth I required this many bits for second so divided by 2 will be the among the bandwidth

I required so b*mt band I require so that becomes by effective transmission model so basically I can write that my this value of n is nothing but.

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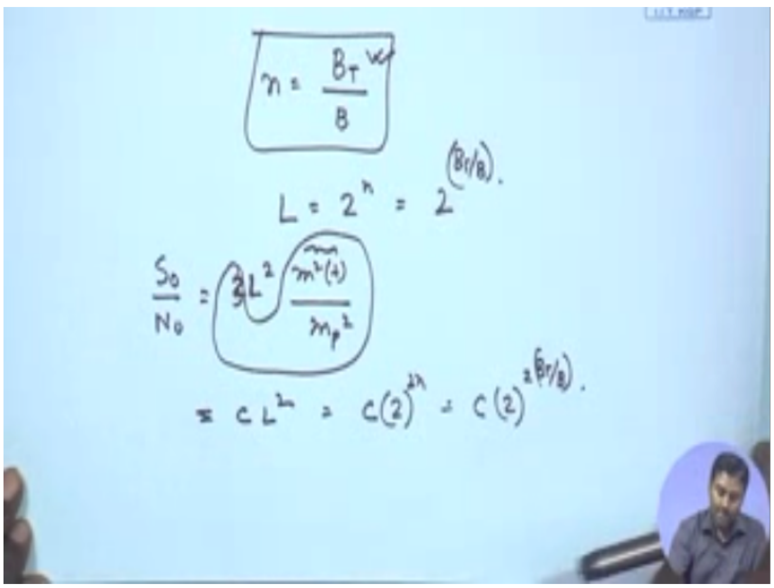


So this is my effective bandwidth model and which could be occupying the channel and this is the base that means actual signal bandwidth okay so this what I require now we want to go back to the relationship between L and m and n so l is $2^n$ or I can write 2 d2 /d3 what happens to my signal to my noise ratio we have already represented signal to noise ratio as $3l^2$ $m^2t$ /$mp^2$ this 3 and these things if you just make them constant.

Suppose signal I put a particular fixed value of power mp carries and remains the same so basically it happens to be just sum $cL^2$ now all we represent by this we can see L means to be important $c(2)^n$ of  and or by $c(2)^n$ so something we get that is the significant noise ratio now if I represent. The signal to noise ratio in db what I have to do I will have take 10 log signal noise ratio in db.

So or we can write 10 log 10 c*n $2^n$ okay or I can write log because it is log is to be distributed so log10c +c the constant and $\log_{10}$ n2 right a log n2 is just 3 so this happens to be $\log_{10}$ 2 into 10 that is 3 this particular part I can write as α log $_{10}$ c because it is a constant into 10 that is α and here we can write 10 log $_{10}$ 2 that is 3* 2 that is 6 and then I get my signal to noise ratio.
In db as this okay and can be further written as α into 6 n is I have already seen that n is dt /b and I have got as wonderful thing over here probably realized what has happened you can see that has I increase my Bt what it does that means I am actually taking more sample so suppose I go increase by n from 8 to 9 immediately what will be the band with increment so here the band with is if it is 8 it should be for noise channel it should be 32kbps or 32 hertz the kilo hertz okay.

So if we just calculate the way I have calculated okay 64Kbps that should be because every hertz gives you one bit so 32 kilo hertz so that is the band if from their I am going to 9 I get 36 kilo hertz so only four kilo hertz of increment of the band with but signal to noise ratio what is happening because n is going from 8 to 9 I get a 6 increment.

So what dripped increment of signal noise ratio so basically here you can see light fm I increase my band I get a better noise quality or netter noise cancellation quality or signal to noise ratio but the advantage is in fm we have proven that if you double the band width then you get a 60 B increment or what dripped your signal to noise ratio.

Whereas here you just increase by 4 kilo hertz that is not doubling the band width but you still get that kind of benefit so basically this particular thing as a huge potential towards the noise cancellation you might be asking okay we have just taken the source noise what is happening to the channel noise so of course that is true there will be channel noise also over here but that is where the digital communication gets advantage.

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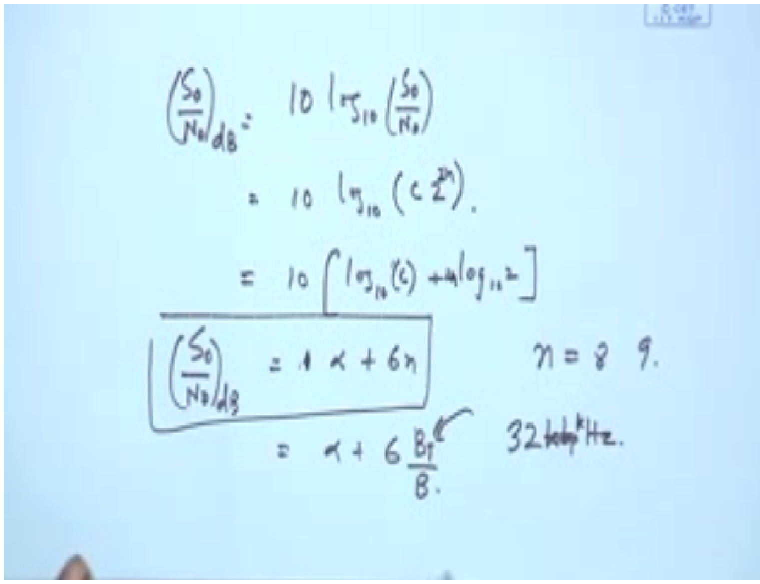


So what happens in digital communication whenever we are communicating the digital signal so let say I communicate like this it is all 1 and 0 I know that might be 1 it might be single so that is what digital communication comes into the picture suppose this signal goes through the channel which are low pass characteristics it has an attenuation plus noise gets added so due to low pass characteristics it will remain like this.

And then on top of that there will be some noise added and the signal will be look like this so all those nice patterns of 1 and 0 actually goes away but the good part is suppose you from here will start integrating it what will happen because of noises sitting on the top of this and the noise mean is actually 0.

Because generally al the noises that we have seen so far these are all 0 mean noise signal with some standard deviation so to because you are integrating what is need is just this bit all unlike communication every location what is the value that is not important is just we have to decide within this duration is it 1 or 0 now for digital communication so do you instant interacting so these are the things which you learning in the digital communication I am just giving you example so that you can understand.

So once we integrate the noise effects gets cancelled, because noise will be 0 mean so if it is + or – once you integrate the noise effect will be cancelled and because of the signal even though it is low pass felted and acquired still integrate and then you can put a threes showed it is bigger than that threes showed you declared it to be one is less than that threes should you declared it be 0,so that way you will have a very cleaner discussion of 0 and 1 and what you can also do you can actually regenerate in between so before the signal degrades to a limit or to a value from you cannot really again reconstruct the signal.

So what you do in between you put regenerated which tells these things tell you that okay, this was 1,this was 0 reshape the pulse so again reconstruct the pulses because he knows once it is 0 then it would look like this once it is 1 it would look like this and again retransmitted so basically he mixed the pulse again very nice and transmitted so that you can come back the noise audition and all other effects of channel so that is why digital communication is generally prone.

Sorry which it is much more effective in terms of noise cancellation and all other things and we can if we employ and there are other techniques also to basically if a bit is getting hernias you have other techniques to basically checked whether it is hernias or not you can correct it in all those things so employ all those things you can almost negate the noise in the channel or that is server okay.

Then the quantization noise becomes the more significant one and in quantization noise we have now shown at we have a huge benefit compared to all other modulation scheme and that is the sole reason why all scoured modulation started becoming very popular okay, compared to analog point so I think we have almost discussed en half analog communication almost came to the door step or the door way or digital communication so now onwards probably you can start discussing about.

How digital communication becomes effective and what are the techniques employed how do you to noise analysis what are the signal analysis that you employ digital communication those things can be expired for the so with that I will probably and the scores and means from this onward you can take it forward towards more advance communication techniques okay thank you.