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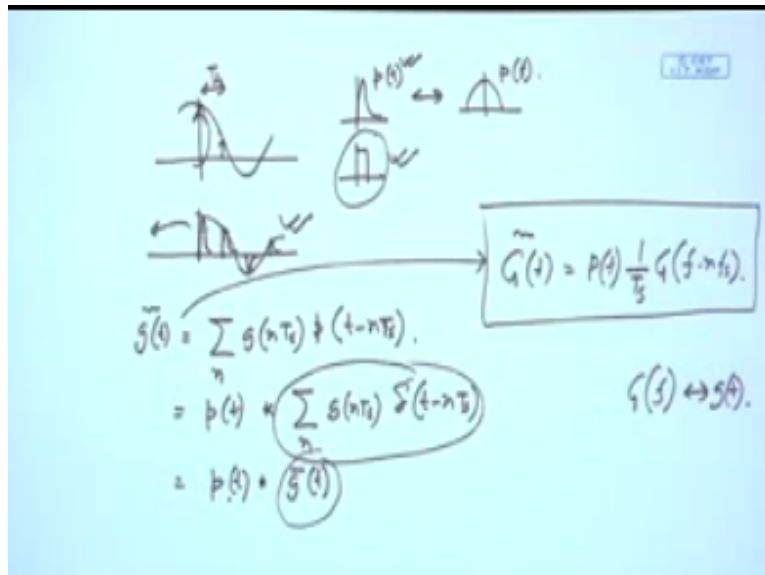
Course  
On  
Analog Communications

by  
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Lecture 59: Flat Top Vs Natural Sampling

Okay so in the previous class we have talked about few fundamental aspect of communication, now e are into pulsed modulation, so specially pulse aptitude modulation, so let us try to see if we try to employe a practical pulse modulation okay.

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So let say I have a signal it looks like this okay, I want to do a pulse amplitude modulation but remember the pulse is longer impulse like one, we have taken impulse strength for proving our Nyquist theorem, it is unlike that. So let say the pulse might have some shape and we are calling this as  $P_q$  correspondingly we will also have a Fourier transform of this let say, it might be band

limited, it might not be band limited so that depends on the pulse another shape of the pulse okay.

So let say we have we know this there is some pulse and we know the corresponding frequency response of that pulse so we are trying to make this analysis generic, so that for any pulse this will be true. So we can have pulse like this of finite duration and this will just the special case of that Pt. we will try to see these things later but right now we are saying any pulse any pt which as the corresponding furrier transform which is Pf.

So basically what we are doing we are again sampling it but now and the sampling interiors are again related to the Nyquist interval and all those things okay. But now what is happening this a particular part will be multiplied by this kind of pulse similar to  $P_q$  then this one, something like this and so on. So again the tip of the pulse basically constructive okay so this we are trying to do.

So let us try to see whenever we do this modulation what exactly we are getting so let us say we call this particular modulated pulse train we call this has  $g\delta(t)$  okay so what is this we are just taking a sample at every instance separated by  $T_s$  so we are taking samples and multiplying that whatever instance that what every the strength of that sample with the pulse okay so basically our this thing will be represented has nothing but this  $g(nT_s)$  e whatever the pulse is  $t - nT_s$  right so that is actually our this particular signal.

So basically what is happening every time at a particular instance suppose  $n = 0$  so that instance I get this amplitude that multiplied by this  $p$  I get this pulse bigger pulse prob next instance at  $n = 1$  so I get  $gT_s$  and then multiplied by pulse must be shifted by  $T_s$  amount so  $n = 1$  so it should be shifted by  $T_s$  amount and again multiplication so this is exactly what is happening whenever I means represent this pulse modulation okay so this is basically my over all pulse modulated signal.

Now  $p$  I can always write has pulse and the convolute it with  $\delta$  because I know any signal convoluted with  $\delta$  remains the same so I can definitely write this as  $P_t$  convoluted  $\sum_n g_n T_s$  that has nothing to do with the  $t$  so therefore this is just a constant thing and this  $\sum$  and the convolution can be means basically exchanged so weather I write it inside the convolution or outside the convolution it is all the same.

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O basically  $\delta t - nTs$  where ever the  $\delta$  position is  $p$  will convoluted with that and create a pulse at that position corresponding the  $gnTs$  will be multiplied and get the same thing so I can have a representation like this what we can see what is this part we have already studied this is basically the impulse sample version so earlier we have called this has  $g^t$  okay this is the impulse sampled version of the signal okay so I can write this  $g^t$  which is actually means modulated with practical pulse which is  $P_t$  which has a frequency response of  $p_f$  so I get this okay.

No problem in that now let us try to see what will be the corresponding frequency response okay or if I do Ferrier transform so let us say I call this as  $G^f$  the Ferrier transform of this one so that must be ferrite transform of this one now it is convoluted in time domain so in frequency domain they must be multiplied so what was the fierier transform of this was  $1/Ts$  we have already proven and  $G(f-nfs)$  okay.

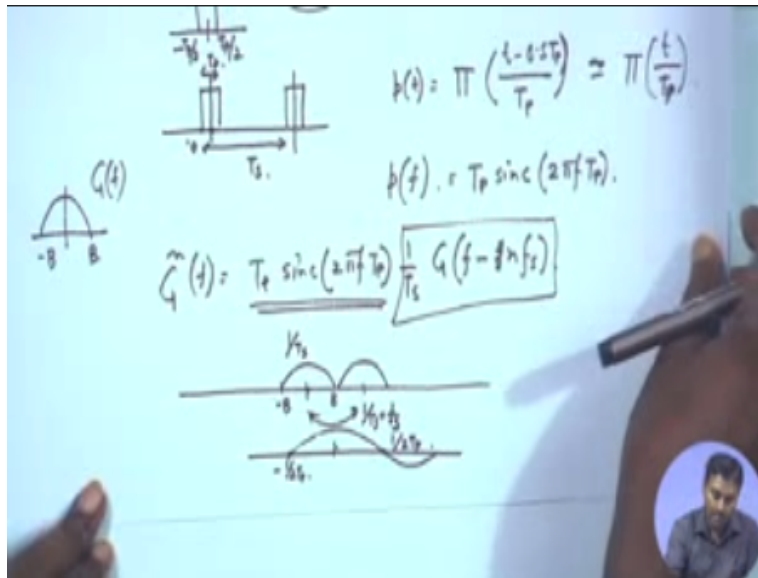
So this has been already proven we have already proven in that impulse means if you take impulse train and then multiply the signal wit that or modulate the signal with that we get tjisi frequency response where  $G$  is actually this  $G_f$  is the fierier transform of  $g_t$  okay, s o this is something we know already that is a band limited signal and all those things are there and our FS is also following that inquest criteria okay.

So this is this part  $G^T$  that is the ferrous transform of that  $P_t$  I know  $P_f$  so that should be my ferrous transform so this is the overall ferrous transform okay so this is something we have now understood that if I basically try to modulate my signal with a realistic pulse then the corresponding Ferrier transform will be this Ferrier transform which was the original Ferrier transform if I do it with a impulse okay.

Which with which we have prove in the inquest sampling and then we could show the interpolation that just by the low pass filter we can extract that signal out now we are seeing that if we have means no ideally or non impulse train then the overall fierier transform has to be multiplied by the Ferrier transform of that pulse okay.

So let us try to employee these thing for a particles scenario let us say of course impulse I cannot change but I can always generate.

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This kind of pulse which is of finite duration let us say the duration of this pulse is  $T_p$  okay where of course  $T_p$  must be less than  $T_s$  okay, wherever I am constructing a pulse train so basically this is the pulse the next pulse so this between these two pulse the difference is  $T_s$  which is the sampling interval and this is  $T_p$  okay so then what is the corresponding  $P_t$  that is actually a box function.

So this is a box function of duration  $T_p$  so duration  $T_p$  and because this I take as 0, so basically function is must be the center point so there is a  $0.5T_p$  shift so I can write this as  $T - 0.5P_t$  okay, so this is my  $p_t$  right so that happens to be  $\pi P_t$  immediately what I can do for box function I can always generate a  $p_f$  corresponding  $p_f$  is something we can always do so what will be that  $p_f$  how do I get that  $p_f$  this is just a delay.

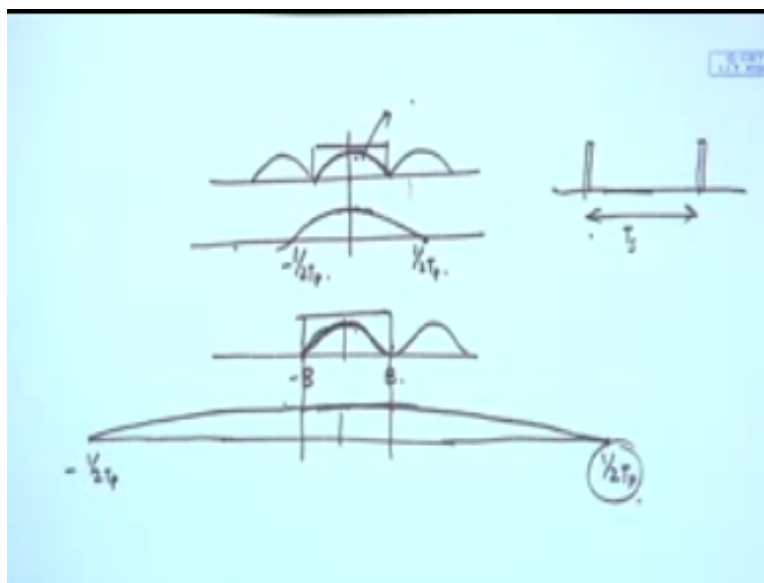
So you can even ignore that if I have my pulse described like this then I can be ignored that part, so I can as well write it as  $\pi t/T_p$  so that means it is a box function which gets repeated and that has a sorry it is not repeated, so the single falls we are describing it is a box function centered around 0 and it has the width of the  $T_p$ , so basically it looks like this okay so it is defined from  $-T_p/2 + T_p/2 + T_p/2$  and this condition satisfy, so immediately if this is the box function I will get a corresponding sinc function.

If so that should be  $T_p \text{ sinc } 2\pi f T_p$  right so this is my  $p_f$  so therefore my  $g \sim f$  must look like  $T_p \text{ sinc } 2\pi f T_p$  okay into  $1/T_s g_f - n f_s$  so this what I get so what is happening if you just try to see in this spectrum, so I have this structure which was the earlier structure as long as I satisfied the

Nyquist criteria so it gets  $1/T_s$  down if this is my  $-B$  to  $+B$  this is my  $G(f)$ , so it gets  $1/T_s$  down and this is till define from  $-B$  to  $+B$  and it gets repeated after  $n\pi 1/T_s$  or okay so that what is happening.

So this still remains so this is that part is gets multiply to this part okay so this is the disturbing part because what will happen, how does this looks like this is the sink function so that must be how does this looks like this is a sink function so that must be if I just draw it down so that is a sink function which is touching 0 and  $1/2T_p$  and  $-1 / T_p$  okay now what will happen sorry it should be done bitter on so it should be centered what we so let me draw it one.

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So this is and this is the center so this is  $1/2$  okay and  $-1/2$  and these two gets multiplied if you see what is happening earlier what we were doing we were just putting a band pass filter getting our signal band but now by signal is a composite signal multiplication of these two and unfortunately for me this is getting multiplied by this which does not have a flat characteristics

over the entire frequency band of interest, so what will happen there will be because of this multiplication there will be a distortion.

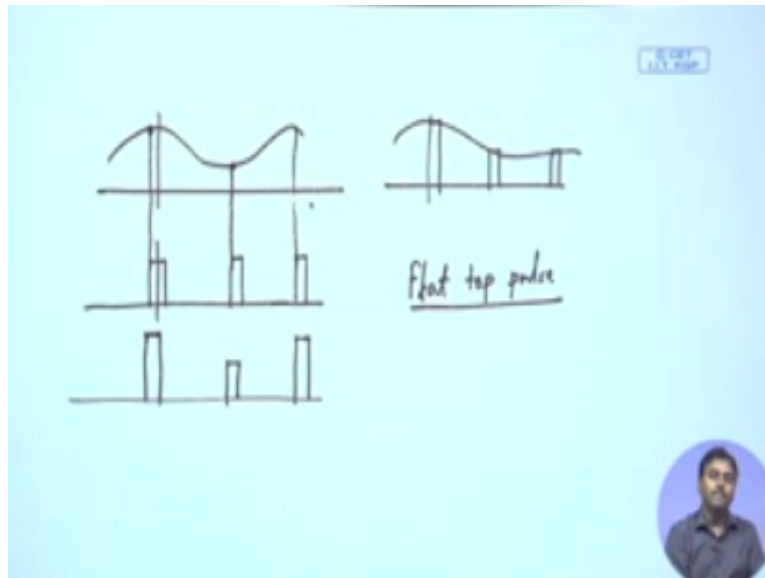
So this signal which will be created so whenever we put a pulse like that there is a possibility now we have explored that there is a possibility of distortion so what will happen once we multiply these two probably it will look like these things will be a little bit suppressed, so we will look like this should of course be symmetric so a little bit of distortion has been created it is not ideal for this one I can still put a band pass filter now  $-B$  to  $+B$  sorry low pass filter of  $-B$  to  $+B$  I get this signal back.

Which is actually a multiplication of these two so but our demand is this one okay so that is where we are having a problem now, now let us try to see how do I make this flatter so this will be flatter if this point where it is touching 0 stretches out, so it goes away then almost looks like a flatter so if I just draw it if this touch is over here that  $-1/2P$  to  $P$  and this is  $+1/2 P$  then it becomes so basically as it now you can see within the band of interest it is almost flatter so whenever I multiply with it the original shape will be in tact what this means so  $1/2 P$  I am increasing that means the  $T_p$  has to be reduced right.

So what happens to the pulse? This is my  $t_s$  and I am trying to reduce the pulse width so this will be exactly flatter when I have the pulse which has almost infinite similarly vanishing width okay which is I impulse function again so I know that if it is impulse then this will be completely flat and I do not have any distortion but I cannot generate impulse so what I need to do is I need to generate a pulse definitely finite with pulse because that is what I can do because then the energy will be restricted but I need to also ensure that the pulse width should be as small as possible because then they will be no distortion due to this modulation technique okay.

So this is something we have to keep in mind and this corresponding sampling is termed as flat of sampling so let us try to see in time domain what is happening and why it is being called flat of sampling.

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So I have this message signal I do a pulse which are like this some finite with pulse and at every instance suppose this is that instance I take the sample value and I represent a pulse with that sample value, next time I take another sample value and I represent the pulse with that amplitude of that sampling one next time again I get another pulse and I get a, so if you see the pulse with which we are modulating the top of the pulse remains flat because we have taken  $x^2$  pulse so a box of function.

Because the top of the pulse is flat and it just takes one instance of the sample value so the sampling looks like this I take a sample I maintain that okay, so this is being realized in practical circuit by sample and hold circuit so basically what we do we take a sample and we hold it for the pulse duration and that is how the top of the pulse becomes flat and corresponding pulse is called flat top pulse.

This kind of flat top pulse is done by sample and hold circuit okay so which probably we want to be discussing that because that is part of the circuit if you just do and look for sample and hold circuit you will get plenty of example of that different realization of sample and hold circuit, so with sample and hold circuit people do this flat of sense means sampling, or sampling with flat of pulses or pulse strain.

So the disadvantage of this one is what we can see that we need to really make sure that the pulses are very small that is the first thing second thing is probably there is a corresponding distortion which is coming and that is why probably we are making this pulse width very small

we have already demonstrate it back that  $1/T_p$  has to be very big that means the  $T_p$  has to be very small okay.

So the pulse width has to be small and with this whatever finite pulse width you take there will be certain amount of distortion because it is a overall spectrum is getting multiplied by pf so that is giving distortion to the base band signal as well and whenever we low pass filter it will get some amount of that distortion okay so that is the disadvantage of this kind of sampling, is there any other sampling so let us try to see if flat top sampling is one of them is there any other counter part of this that will be our next target. So the counter part of flat top sampling is call natural sampling.

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Natural Sampling

The diagram shows a signal  $g(t)$  (a sine wave) being sampled by a pulse train  $p(t)$  (a series of rectangular pulses). The resulting natural sampled signal  $g_s(t)$  is shown as a series of pulses whose heights are determined by the amplitude of  $g(t)$  at the sampling instants. The sampling period is  $T_s$ .

$$g_s(t) = \sum_{n=-\infty}^{\infty} Q_n e^{jn\omega_s t}$$

$$Q_n = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} g(t) e^{-jn\omega_s t} dt$$

$$= \frac{1}{T_s} \left. \frac{e^{-jn\omega_s t}}{-jn\omega_s} \right|_{-T_s/2}^{T_s/2}$$

What is natural sampling? So natural sampling is something like this I have the signal I have pulse train okay so whether probably this pulse amplitude is unity assume to be unity so it has it is a pulse train so it has pulse from  $-\infty$  to  $+\infty$  so what I do, in the previous one I was taking a sample and then multiplying by that pulse okay or I was with that sample amplitude I was representing a pulse okay so just that pulse was getting multiplied by that sample amplitude.

Here what I will do, this  $g(t)$  I will actually multiplied with this means pulse train so let us say that is  $g(t)p(t)$  okay or let us say  $g(t)p(t)$  because this is the  $T_s$  okay, so I will be multiplying it, what will happen? Wherever it is one it will be fall in because I am multiplying, so because it will be multiplied by 1 so this will be sampled all right but during this duration it will just follow



because it is multiplication one with this one so it will just follow the way signal is whenever the pulse is on, so it will just follow that signal so it happens to be this way so basically as you can see you again connect these things your signal will be degenerated okay.

So this is called the natural sampling the natural sampling means within the pulse duration you are actually allowing the signal or the pulse shape or the top of the pulse to follow the signal that is what we are trying to do okay so can I now characterize it in frequency domain and then tried to see the way we have done for this that si something will we try to do.

So that first of all we have to characterize this thing so  $Q_t(t)$  what is that that is a periodic signal right so if it is the periodic signal I can always write infudious series so  $n$  from minus infinity to plus infinity sum  $Q_n$  are you can write it as  $d_n$  the way we are writing  $e$  to the power  $j n \omega t$  where  $\omega$  is equal to  $2\pi f_s$  where  $f_s$  is  $1/t_s$  right.

Because that is the fundamental frequency so we will get this where the  $Q_n$  the way  $Q_n$  is calculated so that should be  $1/t_s$ - the pulse is defined so if this is the pulse from  $-t_p/2$  to  $+t_p/2$  is defined  $q_t e^{-jn \omega t} dt$  so we can do this so if we just evaluated this what you get  $1/t_s$  okay and you have because this  $q_t$  remains 1 within this so I will have 1 from  $-t_p/2$  to  $+t_p$  because this is pulse is having unit amplitude that is what we have assumed so it is just you integrate it so  $e^{-jn \omega t}$  okay so that is what we are getting.

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$$\begin{aligned}
 &= \frac{2}{n\omega T_s} \left[ \frac{e^{jn\omega T_p/2} - e^{-jn\omega T_p/2}}{2j} \right] \\
 &= \frac{2}{n\omega T_s} \sin \left[ \frac{n\omega T_p}{2} \right] \\
 &= \frac{T_p}{T_s} \text{sinc} \left[ n\pi f_s T_p \right] \\
 \hat{G}(f) &= G(f) * \sum_{n=-\infty}^{\infty} \frac{T_p}{T_s} \text{sinc} (n\pi f_s T_p) \delta(f - n f_s)
 \end{aligned}$$

And then if we evaluate this, this happens to be okay so if I just do simplification after putting  $t_p/2$  and  $-t_p/2$  so I get  $\frac{2}{n} \omega_s t_s e^{jn \omega_s t_p/2} - e^{-jn \omega_s t_p/2} / 2j$  I have taken this form because it will be our sin function so I can write  $\frac{2}{n} \omega_s t_s$  this is  $\sin n \omega_s t_p/2$  and this I can manipulate so basically to bring out these things so that I can get a sinc function so finally if I just do that manipulation I will get  $t_p/t_s \text{ sinc } n\pi f_s$  so I have put  $\omega_s$  is equal to  $\pi f_s T_p$ .

This what we get okay so  $q_m$  are all being represented now, now what is we have got by  $q_t$ s okay so what will be the corresponding frequency response because  $Q_t$ s is nothing but this  $q_n$  e to the power this  $\sum$  - infinity and + infinity so this is just become impulse function at frequency domain at those frequency and  $\omega_s$  or in  $f_s$  okay.

So if I now means what we are tried to do is if you just see this sample  $q_t$ s is getting multiplied with  $g_t$  okay so if it is multiplication in because we have already characterize the Fourier transform of this one because we have got Fourier  $c_t$  and then you do the transform you will get all kinds of function with the strength as  $q_m$  corresponding strength okay.

So now if these two signal are getting multiplied in time domain exactly to create this so therefore Fourier transform of this one will be convolution of  $g_f$  and this one fine so that if I represent that has must be  $g_f$  which is the original missing signal convolution  $T_p/t_s \text{ sinc } n\pi f_s T_p$  so this  $n$  goes - infinity to + infinity.

And I have the delta that every  $f_s$  corresponding  $f_s$  right so this is the Fourier transform of that  $q_t$  because it is just exponential so corresponding they will be creating delta at every  $f_s$  so I get and the strength will be corresponding sinc so this is what we get now  $g_f$  convoluted with this  $\sum$  and this convolution I can inert exchanged so  $g_f$  can go inside then  $g(f)$  convoluted with  $\Delta$  again it becomes just  $g(f) \cdot n f_s$  so I can write this as  $t_p/t_s \sum_n \text{ sinc}(n\pi f_c t_p)$  this becomes of  $g(f) \cdot n f_s$  right so that happens to be my overall Fourier transform okay so now what you can see is this.

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$$\begin{aligned}
 &= \frac{2}{nT_s} \left[ \frac{e^{jn\frac{T_p}{2}} - e^{-jn\frac{T_p}{2}}}{2j} \right] \\
 &= \frac{2}{nT_s} \sin \left[ \frac{nT_p}{2} \right] \\
 &= \frac{T_p}{T_s} \operatorname{sinc} [n\pi f_s T_p] \\
 \hat{G}(f) &= G(f) * \frac{T_p}{T_s} \sum_{n=-\infty}^{\infty} \operatorname{sinc}(n\pi f_s T_p) \delta(f - n f_s) \\
 &= \sum_{n=-\infty}^{\infty} \operatorname{sinc}(n\pi f_s T_p) G(f - n f_s)
 \end{aligned}$$

That the Fourier transform is nothing but at every  $n$  so basically whenever  $n=0$  I get  $g(f)$  so  $n=0$  means it is at the center frequency  $n=1$  means right side-1 into left side so like this so there will be frequency tom which has so it will be just exactly  $g(f)$  so the frequency spectrum remains the same all it is that it as the strength which is sink function and  $t_p/t_s$  so this sink function what happens this will be bigger and the next one will smaller and so on they just keep on going like this.

So this  $t_p$  means the strength of them follows a sink function at every location of phase so this is 0 this is  $f_s$  this is  $-f_s$  and at every location of  $t_s$  the tip of them actually follows sink function so what is happening I am getting two advantages over here one is that if I now employ a low pass filter this has no distortion because this is just exactly equivalent to  $G(f)$  with that sinc multiply okay and the other advantage I get if I do original sampling what was happening my power was getting distributed everywhere right where as here the central lobe which is the lobe of my interest is getting more power and the side lobes which I should reject anywhere are getting lesser power because it has a sink function so the overall strength of that follows a sink function.

So that is the good thing because most of the power are concentrated on the low pass band and I will be employing filtering to take this so most of the power I will be getting while demodulating it is first thing second thing is unlike the flat of sampling it does not go through distortion these are two most important advantage of this one and it has another advantage which we can easily

see that because the strength of is proportional to  $t_p$  so what is happening as I increase my  $t_p$  this strength in the central lobe will increase okay that is a very good thing because what will happen as you increase  $t_p$  there is also another counter effect this  $t_p$  will increase so this will factor will increase because it is a sink this as you increase  $t_s$  this will go to 0 very soon in the side lobe.

So basically the side lobes are getting decreased even more so what is happening as I increase  $t_p$  I will be getting more and more power over here and let us try to see what happens if I increase  $t$  suppose if I have a signal like this I want to sample it so I sample it with these falls let us say 50% you could so whatever so this is so my  $t_p$  is just  $\frac{1}{2}$  of this  $t_s$  so say this is  $t_s$  so what will happen it will just follow the signal over this  $\frac{1}{2}$  and then it will be 0 and then again over this  $\frac{1}{2}$  it will follow this so I have seen that as I increased to off course I will be getting more power consider over here which is very obvious us.

Because if I just make it full that means  $t_p=t_s$  then the entire power because it will just look like this same signal entire power will come to my central load is that something I want probably not be cause why against sampling we were doing sampling to get those central get those sample values and in between you are freeing the time to do time division multiplication so more  $t_p$  we take probably we will have less opportunity multiply anything else so there is a trade of between these which we can immediately see so probably with this we have clear bit of understanding about natural.

And flat of sampling these are the two most important sampling that are available practically so next what we will do we will try to see what kind of circuitry we can put for doing this natural sampling okay so in the next class we will discuss little bit of circuitry of these modulation techniques and then we will probably go to means digital version of this sampling out pulse sampling modulation this is called PCM.