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**NPTEL ONLINE certification course**

**Course**
**On**
**Analog Communications**

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**Lecture 56: Sampling Theorem**

Okay so we were in the last class we were actually discussing about the click noise okay, so that something one topic we have started. So will not go into the details of it but let us give you some basic idea of what that click noise is so basically we have drawn that pharos diagram right.

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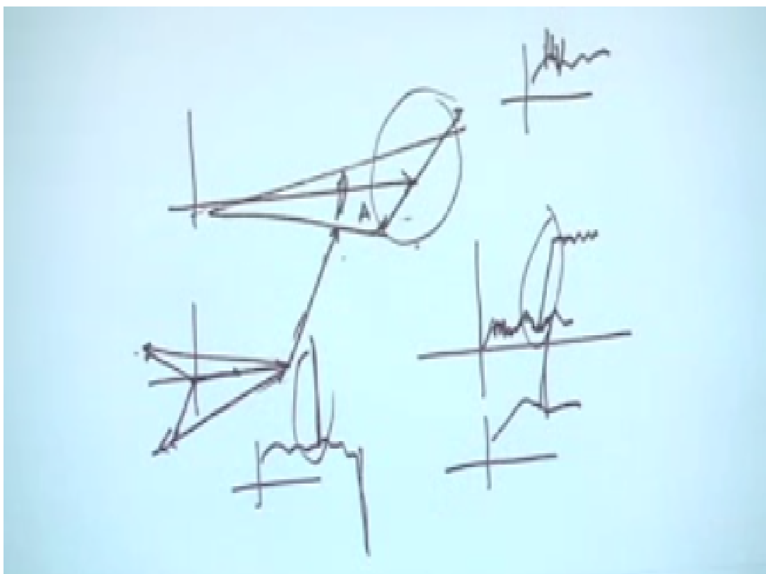


So this is that AC that is that point on top of that the noise is being added. Now the problem is as long as this noise amplitude is smaller than A there is no problem but if the noise starts getting suppose this is my A and this is the noise amplitude okay. Now what might happen if this is big enough this might even go in this direction so what is happening to my phase? it was on this direction okay so the phase this is the reference phase which is over here this immediately goes into reverse direction.

It might even go into this direction okay so the phase becomes so basically there might become a random variation of phase which was not happening over here if it wanders I know that it will still be remaining on that same around theta okay, so it is not getting a huge variation due to noise okay. So if you just plot that theta so probably I wish to have a theta variation like this on top of that there will be small noise induced theta variation okay and I will differentiate it they will put them out even more and I will get my actual message signal variation.

Whereas what will happen over here once there is a variation and if means if this goes from one particular suppose it is in this direction now noise is random so it can immediately in the next this one it can go into this direction, so there will be almost a PI phase shift okay so there is a possibility now because the noise amplitude is higher, so there is a possibility that randomly after sometimes suddenly a pi phase shift happens. So what will that means this particular thing will have a certain PI jump okay.

And then again it will probably track again another PI jump might be there or it might have again come back, so all these slopes if you differentiate it what will happen they will start creating suppose after differentiating I was getting this but this particular point because there is a sudden thing, there will be a impulse noise which will be created as many pi shifts are happening okay either in the positive or negative whatever happens there will be positive impulse or negative impulse.

It depends on what kind of things are there but whatever it is there will be a impulse noise which will be coming and they will be coming randomly because noise characteristics you do not know when it will do that pi shift okay. Here even if it does PI shift my noise due to noise this variation will not be huge it will just be a small amount of modulation but because in the face there might be a PI phase shift, so there might be a click noise which will be coming and this clicks are random in nature when they will be coming that depends on noise characteristics okay.

So what is the threshold effect so this is called click noise so basically what you will see that after FM demodulation it will have that original message which because it has a nice noise cancellation, so overall noise cancellation will be good. So the message signal will be smooth but suddenly there will be impulse and then again smooth suddenly there will be impulse, so this will be creating a cracking noise in the receiver. So whenever you are hearing there will be cracking noise one after another this might happen whoever means have listened to FM radio you might have heard those clicking sounds okay very rarely happening.

But what will happen if the noise amplitude is too high and it remains too high and the noise nature actually varies this means this arrival of Pi phase shift it happens too often, so within a second if there are too many clicking sounds so this particular part whenever this within a second how many clicking sounds are being hard that increases then that is called the threshold effect ignition. So at that time probably the message is still being demodulated very nicely but there are too many impulses on that.

So basically our means here whenever we hear it probably that clicking sounds they are too often so that will be only reverberating inside our here, so we will not be able to hear whatever is actually being transmitted the message signal will have no consequence it will be just a collection of clicking sounds one after another coming. So that is when we say that FM goes into threshold so there are analysis that with what SNR value FM goes to means threshold zone what is the number of clicks in one second that will be coming.

So all those things are there probably will not bother our self due to time constraint in characterizing those things but it is good to just know that FM has that kind of effect okay, so it is not all as we have thought whenever the signal-to-noise ratio becomes smaller. so it either goes very close to unity or even less than unity then we have a big means challenge in receiving FM signal okay. So I think with that we have almost finished our discussion of FM so whatever we wanted to discuss about FM we have finished that.

Now we are in a position to actually compare FM and AM with their full scale so what is the bandwidth efficiency what is the corresponding noise characteristics or noise cancellation characteristics how it behaves in terms of nonlinear channel characteristics what kind of interference cancellation technique they, they can employ so all those things already has been evaluated okay. So now we will probably start with another form of analog modulation it is still being called analog modulation because that was probably the transition phase between analog and digital transmission.

So we will start we will try to cover also that that particular analog form of means modulation technique which is called pulse modulation technique we have already mentioned it in the beginning of our course.

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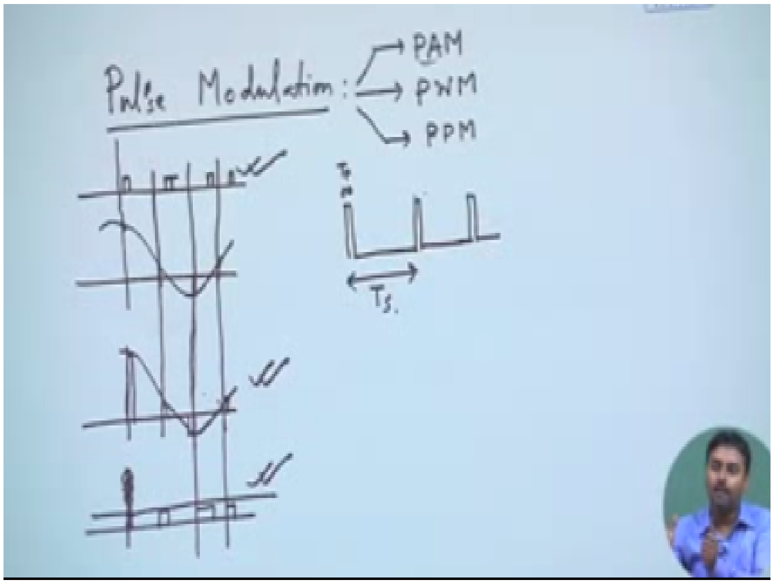


So today we will try to means it is called pulse modulation okay so this is something we will try to cover but to understand pulse modulation will have to probably start appreciating some of the fundamentals of this pulse modulation or saw the signal aspect our signal theoretic aspect of this pulse modulation. So that is something which we will be doing first means dealing with them and then we will come to the fundamentals of pulse modulation okay.

So pulse modulation has three types like MFM they had or AM and angle modulation amplitude and angle modulation they had their own types like forever angle modulation we had pm and FM like amplitude modulation we had means amplitude modulation without means suppressing the carrier then DS CSS BSC then vestigial sideband, so here also there are three techniques so one is called pulse amplitude modulation PAM the other one is called pulse width modulation and the other one is called pulse position modulation okay.

So it is almost analogous to basically frequency phase and amplitude okay so if you see PAM that is actually like this whenever we say pulse amplitude modulation it is in terms of some discrete pulses, so the entire modulation happens with pulses pulse means it is on for some duration and rest of the duration it is off probably okay. So that is generally termed as pulse modulation so we will be modulating generally with in terms of this kind of pulse train which is just this these are periodic signal after every let us say $T_S$ amount of time it repeats.

And the pulse is on for some amount of time let us say T P and T s - T P time it will be off so the modulation happens with these pulses like we had carrier we have pulse train over here to do the modulation, so it almost analogous to the carrier okay and when we say pulse amplitude modulation that means if I have a signal the signal will be covered in the envelope of this pulse so that means the pulse amplitude will be modulated according to the signal. So if I have a pulse over here that will a higher amplitude next if I have a pulse over.

Here that will have a smaller amplitude so it will be just like this if you take the pulse tip and connect them they will create the envelope of the signal, so again there will be a pulse that will be negative and so on so if there is a pulse that will be positive so if you just connect the tip that will just look like the original signal okay. So that is pulse amplitude modulation whereas pulse width modulation is you, now start varying the width of the pulse okay.

According to the amplitude of the signal message signal so basically what will happen if the amplitude is higher it depends on how you wish to modulate it so you can you can take a decision that if the amplitude is higher I will have lower width pulse and if the amplitude is lower I will have higher width pulse, so what will do probably here if you do pulse width modulation here the pulse width will be smaller which means now you are actually modulating that Tp okay.

So that is something we are doing over here the amplitude is lower, so probably the pulse width will be little higher and over here amplitude is even lower so pulse width must be even higher then again it is becoming smaller as it goes up probably the pulse width become means of course we should have same amplitude. Now it is almost like now frequency modulation okay, so the amplitude of the pulse remains similar it is just the width you are modulating the width of the pulse okay and the other thing is the pulse position modulation which is almost like a phase modulation.

So basically you position your pulse depending on the amplitude of this particular thing so basically whoever will be having higher amplitude probably they will be closer to the this pulse starting time okay so what will happen if we just draw that at this point because the amplitude is higher. So pulse will be closer to that pulse starting time at this point because this is lower, so this will be further away the pulse now the width will be same amplitude will be same it is just the position of the pulse which is being varied here, it is more negative so it will go much more away from the this one again it will come closer or something like this okay.

So this is called pulse position modulation this is called pulse width modulation and this is called pulse amplitude modulation okay, so this is something which we will be trying to employ okay, so let us now try to see as we have said that before we do this we need to have a mathematical foundation of this pulse modulation. So that we can analyze their characteristics and everything else so that signal processing part of that or probably the signal analysis part of that has to be dealt first before we even jump into these modulation techniques.

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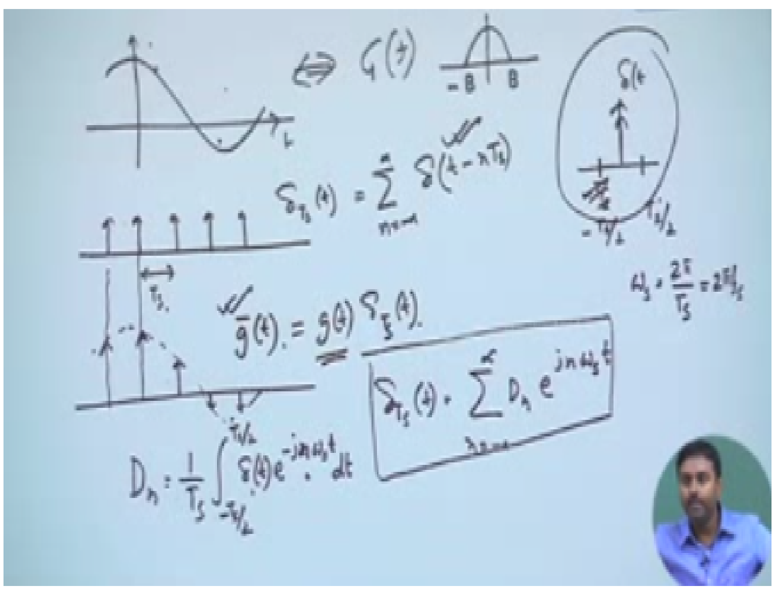

So if you just see we have probably discussed this what is actually a pulse modulation, so impulse modulation what you are trying to do suppose I have a message signal. So whenever we could have actually transmitted this message signal okay that is without modulation we are transmitting it. Now instead of doing that probably I will have some theoretical foundation that not all the time because this is a continuous time continuous amplitude signal okay, so this is time and this is amplitude it might be voltage it might be whatever it is whichever parameter we put for this.

So that is taking any value and also the time every time the signal exists so if some theory can tell us that probably for this signal description I do not need all the time, so I just need some portion of that time to define that signal or maybe some discrete values of time where that has to be justified rest of the time I do not have to have a definition of this signal and I can still reconstruct the signal as it is.

If we can do that then probably will have some modulation format which is similar to pulse modulation, so in the pulse modulation basically what we are saying is in discrete time, so time actually we are making a discrete in discrete time we are trying to get the signal and send the signal, so in some part of the signal, rest of the time we are keeping it open where others can modulate. So it is also a mod multiplexing scheme like infrequency MDM we have told that infrequency domain we can multiplex things this is actually where in time domain we can multiplex things.

So that is called TDM so we will discuss more about it later on but that is what we try to do that time domain and we actually discretize it and only take discrete-time events or time samples to represent the whole signal so that is what we are now targeting. Later one when from here will be transiting towards more of a digital transmission what we will do we will not only discretize the time we will also try to discretize the vertical axis, so then our amplitude also will be having discrete values and we will take those values and represent them in a digital format.

So that is actually what is termed as digital transmission so that is why I told that this is one step ahead towards digital communication, so you are just discretizing or trying to discretize the time or trying to free up the time that you do not use it for whole time to represent the signal, so that is something we will try to and now theoretically analyze. So this was first done by nyquest most of you must be knowing about Nyquist sampling theorem, so that is something we will try to demonstrate.

Now okay so what is Nyquist sampling theorem that actually tells that if I have some trains of impulse function or δ function, so which looks like this so this is a periodic signal where at a particular interval let us say that is $T_S$ I put a δ function okay so this i represent as δ $T_S$ T okay which is nothing but a train of δ function. So we can write this as δ t - n $T_S$ where n goes from - ∞ to - ∞ okay, so this is just at different location in time there are δ function okay.

And it is a periodic one after every $T_S$ it gets repeated so the basic signal is something like this from - $T_S$ by 2 sorry - $T_S$ by 2 - D s by 2 this is just a δ function and it is defined from here this gets repeated ok so basically that is the Train of δ function and sampling theorem says see we have already told that we want discrete values of it, so sampling theorem says that we multiply these two so whenever we multiply with δ if you multiply what happens it just picks that value at that point wherever that δ is defined.

So it will be actually picking those samples at those δ locations so the output will be something like this if I just draw the envelope, so this is where1 δ so at that point δ of the strength which is following this gt will be captured and so on okay, so this I can write as δ t okay, so what is this this is nothing but our gt multiplied by δ tst right so this original function multiplied by this train of δ function if you just multiply these two we get a sampling okay.

So sampling is nothing but we are discretizing in time so basically we are picking that sample value at that location and then ignoring all others between this $T_S$ and again at the boundary of $T_S$ we are picking another sample value which is having amplitude exactly equivalent to that at that point whatever GT has sampled well or amplitude value, so it is just picking those samples. So we have got are presentation so this is actually a sampled version of our signal okay now let us try to see what happens in frequency domain, so that is what we want to understand.

So this GT what is the corresponding frequency domain representation what we are saying is this gt has a frequency domain representation of gf which looks like this which is defined from - let us say B 2 - B so eventually we are saying the signal we are sampling that is a band limited signal and a low-pass equivalent signal, so it is a low pass band limited signal having maximum bandwidth or maximum frequency component up to B. So it has a band width of P so that and that is the gf that is the Fourier transform of that particular signal okay.

Now if we wish to do Fourier transform of this one what we know it is the Fourier transform of this Fourier transform of this and because they are multiplied in time domain it should be convoluted in the frequency domain this is something, we know so first we already know the Fourier transform of this gt so we need to understand what is the Fourier transform of this del $T_{ST}$

that is something we will have to first do the n only we will be able to characterize the Fourier transform of that g hat t okay or g bar t okay.

So this del $T_{ST}$ what is this is actually this looks like this it is a periodic signal with this as being repeated every time with a period of T s so therefore I can represent this as a Fourier series, so if I can do that so what I can say is it is actually summation n equal to - ∞ to - ∞okay Dne to the $\omega^{ST}$ where ωs is just 2 pi by $T_S$ or I can write it as 2pi fs okay where fs is 1 by t s okay so I can from my Fourier series understanding I know that any periodic signal can be written like this where what is $D_N$ must be calculated as Fourier has said for Fourier series analysis.

So that must be integration over the period which is- $T_S$ by- 2 + $T_S$ by 2 and then we have to multiply with the function itself which is δ T so δ T into e $^{-j}$ n ω s t $D_T$ right that should be my $T_N$ what is this it is δ T multiplied by this that must be at T = 0 whatever value of this is there so that is Z that is 1, so 1 now you integrate this okay. So basically what will happen if I we have also seen the property of δ T if it is integrated from - ∞ to- ∞ with a function it will just give me as output the value at 0 whatever it is.

So δ T if is if it is multiplied by a function and then integrated over the entire duration I will be getting this particular function, so if I integrate it from - $T_S$ by 2 or from - ∞ - it will because δ T is 0 anywhere else okay so I can always write that as if it is integrated from - ∞ to -∞ and then I will be getting at T equal to 0 whatever value I do get okay so that should be what I was telling that at T equal to 0 whatever this value that should be the value, so 1 so it must be 1 by Ts right so DN becomes 1 by $T_S$ so it is not a function of n anymore. So it is independent of n so I can actually take this outside the summation.

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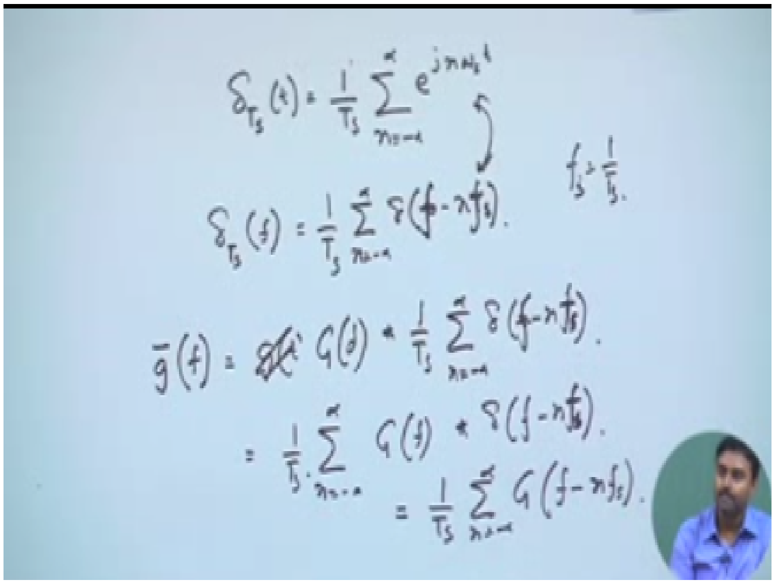

So what I can write this δ $T_S$T I can write as 1 by $T_S$ because I can take it outside the summation n - ∞ to- ∞ e $^{j\omega st}$ right I can write it this way what is the Fourier transform of this let us say that is del Ts this will remain the same it is a summation of exponential each exponential if I take Fourier transform what it creates it creates a δ function in the frequency domain at that location wherever it is okay, so at that frequency location. So basically this will be just a sum of δ function so this happens to be right this is something we already know that if I have this means I can I can get a time period accordingly which is 1 by means 2 pi by $T_{S.}$

This can be represented at TS it will be creating δ this is something we have already known this is the relationship Fourier conjugate relationship to the e $^{j\omega st}$ will create a δ function at FS okay so this I already know so it will be just a summation of δ function that is very good, now I need to find out this g bar f which is gf or maybe capital gf convolution of this which is 1 by Ts summation n - ∞ to -∞ δ t - n TS right I can write it this way now the convolution is our integration and this is a summation these two does not depend so I can interchange them.

So gf I can take inside there is no problem in that because this summation has nothing to do with F so I can write this as 1 by $T_S$ summation n - ∞ - ∞this is nothing but gf convolution δ sorry this should be it is a frequency domain representation, so that must be fright so that is f - n $T_S$I have done a wrong thing this is actually if I am putting it at f it should be n fs okay where fs is one by $T_S$ so that is the Fourier representation right if I put it in Omega it will be n Omega s okay.

So that is that is the wrong thing I have done so that must be fs so this is what we are getting now we know anything convoluted with δ must be giving me the same thing at where the δ is okay so the convolution theorem with respect to δ this will give me $T_S$ summation and - ∞ -∞ this will be just gf – n fs that is what happens so if just that g goes into every fs . So in frequency domain what will be the corresponding representation?

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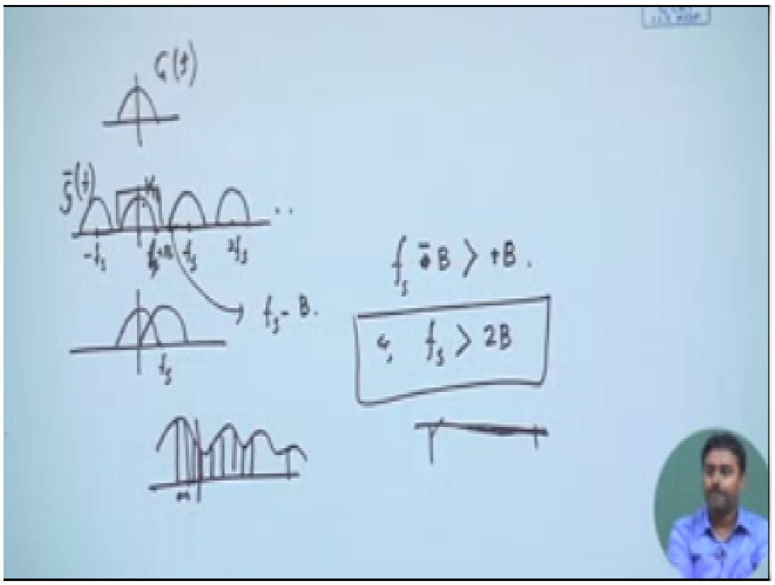


If I just try to see so that was my gf now I am trying to get this g bar f right, so how that will be represented this is this entire thing gets repeated at every fs so this is 0 fs also this will get repeated at 2 fs because n goes from - ∞ -∞ so at two fs also this gets repeated, so this is a phase this is two fs and so on what is the strength of them so the strength should be this one by $T_S$ so that must be the strength so this should be 1 by T s and this gf gets repeated everywhere even in the negative side also.

This will be at – fs I will have that same repetition right so I will have this reputation from -∞ to - ∞. Now the thing is that if I wish to keep the signal intact that means after doing this I can still reconstruct this signal how in the frequency domain if you see how I can reconstruct this signal if whenever I am shifting this if the $f_s$ is chosen carefully, so that this and are separated they are not overlapping. So you can immediately see if my $f_s$ is too small suppose I put $f_s$ over here, so what will happen this is getting repeated and this is getting repeated now these things are getting superimposed.

So if $f_s$ is small then I will have problem, so what should be my optimal $f_s$ so that I can still separate them out with a low-pass filter that is all I will have to do right, so for that what is this point this is $f_s$ - B sorry this is - B just and what is this point this point is $f_s$ - B so I need to ensure that $f_s$ - B is greater than- B so I need to just ensure this $f_s$ sorry $f_s$ - B is greater than – B or $f_s$ must be greater than to B that is the famous Nyquist theorem, so as long as whatever the bandwidth of the signal if twice that if I means twice of that.

Which is to be I do my sampling frequency that means I choose my $T_S$ in such a way that my sampling frequency which is 1 by $T_S$ that becomes bigger than this to be I am fine I know that I can put those samples and then I can transmit it those samples if I just pass through a low-pass filter which has a bandwidth of B it will be able to in stress that signal back, so I can reconstruct that signal I am not losing any information so I can still recover that signal back it is only when this condition is violated I know there will be a super imposition and my sampling is not giving me very nice reconstruction of the signal this is quite obvious.

Suppose I have a signal something like this okay and then if I just take the samples like this what will happen if I start try to reconstruct without knowing what has happened inside so I will just reconstruct it this way right so it will take out all the variation whereas if I start getting samples like this it will capture the entire variation.

So basically I and criteria tells me that at least what should be the means what should be that $T_S$ what should be that value so from here I can calculate one by $T_S$ must be greater than to be or I can say this $T_S$ must be less than 1 by 2 B, so that is the maximum value of $T_S$ 1 by 2B I cannot really go beyond this if my $T_S$ becomes more than 1 by 2B I will be losing some of the information. So my sampling will not be good representative of my signal, so as long as I am while sampling I am considering this I know that the sample will be a true representative.

So this was the sampling theorem which has given a huge impetus for modulation because with this only we know that probably the entire time domain I do not need I can just pick some discrete values of time, where I represent the signal and that is good enough ok so what we will try to do we will come back and try to see yes they are good enough those samples are good enough we will try to show that theoretically.