

NPTEL
NPTEL ONLINE CERTIFICATION COURSE

Course
On
Analog Communication

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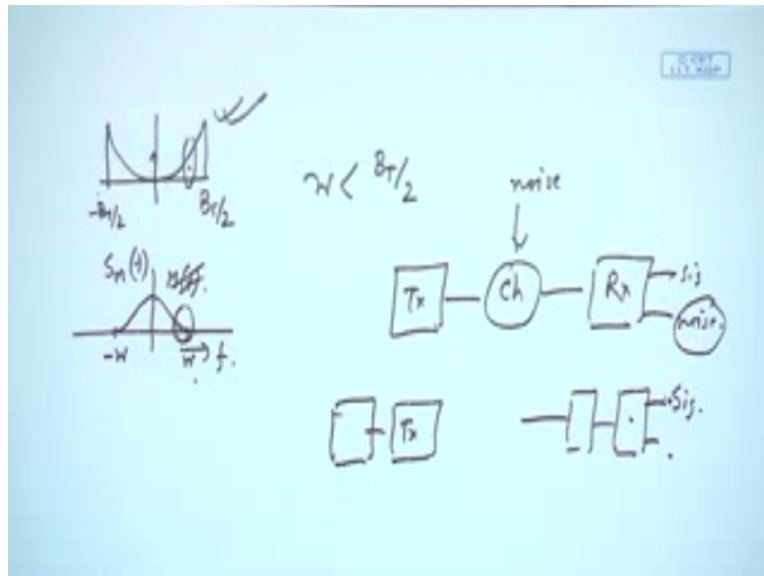
Lecture 55: FM Noise Analysis (Contd.)

Okay so what we have done so far is means discussion of FM and it is noise analysis so we have also proven that in presence of noise FM performs better in a way that we have also shown that FM is probably the only modulation scheme where we can actually exchange the bandwidth efficiency with the noise performance what does that means that means if we increase the noise band width or sorry if you increase the FM bandwidth that means make a wider band FM that is what we can do if we do that immediately we can see there will be a positive effect in terms of noise cancellation okay.

So more wider the FM bandwidth better noise characteristics will be able to see so this is something which has been already proven we have also done that for a tone modulation we have shown how better it is in terms of noise cancellation with respect to a.m. and what parameter set like the modulation index which is directly proportional to the FM bandwidth or frequency deviation.

So how we can set that to make it better than a so this is something we have already demonstrated and at the end of last lecture we could also show that and even before also we could show that FM has a noise cancellation or even interference cancellation when the interfere assignment is closer to FM signal so this is something which has happened and especially that was proven that FM noise due to the demodulation process looks like this.

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So basically the noise spectrum or power spectral density looks like this if this is minus $b(t)$ by 2 to plus $b(t)$ by 2 so that is something we have already shown so what effectively we mean by this that if this is the FM carrier and around which the FM modulated signal is there so something closer either noise or interference very closer to that central frequency of that FM modulated signal that will be canceled out whereas as we go away from the central frequency the noise power spectral density also gets boosted up as well as interference.

Also we have seen that that is something which is to be noted for FM so this is typical of FM now generally what will happen whenever we within this there will be the means this is the low-pass equivalent noise characteristics right so if I just plot the signal spectra that should look like this okay so this is my M F power spectral density our power spectral density let us say s_{MF} okay with respect to frequency okay so we know that it is band limited probably that is the band so minus W to plus W and of course W must be less than $B(T)$ by 2.

So that is a condition which generally happens we have already shown that and this BT is actually the actual FM band width whereas W is the message signal bandwidth so FM bandwidth is always greater than that W that is something we know so what happens in this side at the higher frequency generally the message signal shows that it's because it is been limited so there the spectrum component will be having lesser amount of power okay.

And whenever it is transmitted after the receiver the discriminator circuit followed by a low-pass filter what we will see and this position noise is more enhanced due to FM demodulation and this

is the point where signals are means having less power so if the signal is having at this frequency of course overall we have already analyzed but at this frequency we just concentrate on the higher side higher frequency side of the message signal or message band.

We can see the noise power is higher whereas signal power is so maybe something can be done over here okay so what is that something the something is like this see noise is getting added at the channel right that has nothing to do with the transmitter so I have a transmitter followed by a channel through which it propagates this is where noise gets added and followed by a receiver after that I get my signal as well as noise okay the composite part of it so that this particular noise after going through means added in the channel and going through the receiver that looks like this okay now I can do something to the signal what I can do is effectively because I know that noise will be already higher at the higher frequency.

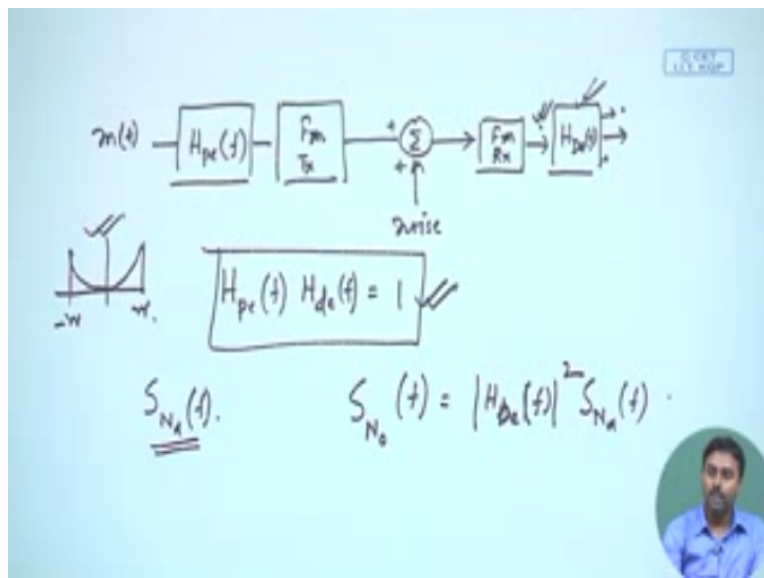
So I can deliberately boost the signal at the higher frequency okay that boosting will not boost the noise because that boosting will be done at this transmitter so which is independent of noise before even noise gets added I boost the signal so that that signal is already of high power then the noise gets added due to the demodulation process noise will be as it is but the signal has been already boosted at the transmitter side so at the receiver side signal will have that boosted things.

Now what we have to do of course the signal if I boost it the relative amplitude a different frequency level has been changed so basically that will distort the signal so if I have boosted it I need to put a D booster or some filter which is just cancelling out this boosting at the receiver so the fun begins when I start putting that so if I start putting that so suppose I have a receiver after the receiver I put that reverse thing at that time what will happen the signal it will come back to original.

But because this is actually reducing the overall amplitude or overall power of the high frequency term so what it will do to the noise, noise is as it is it has not gone through some boosting so it will be as it is so here if we are D boosting it or if we are reducing the amplitude of the high-frequency part so noise will go through that reduction in overall power so at the output I will see the same signal because it was boosted at the transmitter so before transmission I have this boosting and here it is the reverse operation.

So I will get the same signal but this noise power will be much more reduced so this is a technique which has been employed in FM which helps means combating noise even better okay will probably show with the practical almost fast order this boosting and D boosting filters which are called as pre-emphasis and D emphasis because we emphasize the higher frequency before transmitting and the emphasis is just doing the reverse so pre-emphasis then D emphasis so we D emphasize it and bring it back to the original after the transmitter.

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So this pre-emphasis D emphasis actually gives you with the practical implementation of pre-emphasis D emphasis will be showing that almost 30 DB gain in terms of signal to noise ratio okay so means 30 DB okay so will that is what we want to achieve let us see how that can be done okay so the overall thing is there should be a H pre-emphasis or pre-emphasis just before the transmission so our message signal $m(t)$ must go through this and then followed by whatever FM transmitter we have any transmitter is okay.

And then it goes to the channel where the noise is being added with the modulated signal modulated pre emphasized signal okay so then I go to the receiver which is FM receiver okay followed by of course I have to cancel that pre-emphasis so I have to put a h d emphasis filter and that should be my transmitter receiver chain and what we should also realize that this h PE f into h de f this must be one because they should just cancel out each other okay.

So whatever the transfer function of the pre-emphasis the emphasis must have inverse of that so that they just cancel out each other and the message signal passed through this chain of pre-emphasis and the emphasis will actually have the same quality spectral quality as well as time to mean quality okay so this is something which has to be happening okay so if this is the part then let us try to see after the receiver okay after going through the emphasis what should be my signal and noise signal should remain as it is as we have discussed earlier but the noise part will go through this.

So basically we have already told that this and F is the noise power spectral density at this point okay so that goes through the de-emphasis filter so it must be passing through the DM filter and that is what the noise will be so if I just add the output if I wish to get SNR that must be passing through a filter so which is also pre-emphasizes the emphasis will be means proving that they are generally chosen to be linear time-invariant so I can again put that whatever random process theorem we have proven that any random process with a power spectral density if it passes through a particular filter which is linear time-invariant then the output power spectral density will be the input power spectral density into mod hf square.

So same thing will be happening so if this is that $H df/de$ so that must be H capital $d e$ sorry F square s and F which was the noise power spectral density over here this is that part which we have already derived that looks like this right so and that is defined from minus W to plus W right because this is after employing the low-pass filter also this is the entire it should be FM so entire FM demodulation chain okay.

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$$I = \frac{\text{Average noise power without P/D filter}}{\text{Average " " with P/D "}}$$

$$\text{noise power}_{\text{with}} = \int_{-W}^W \frac{k N_0}{A_c^2} f^2 |H_{de}(f)|^2 df$$

$$\text{noise power}_{\text{without}} = \int_{-W}^W \frac{N_0}{A_c^2} f^2 df.$$

$$I = \frac{2W^3}{3 \int_{-W}^W f^2 |H_{de}(f)|^2 df} \quad \checkmark$$

So that is after the demodulation and after following means after putting the low-pass filter of the message bandwidth so that should be the noise spectral density okay so this is what happens now let us go back and try to see what is the overall noise power okay and what we will try to evaluate that what is the relative benefit by doing this the emphasis okay so let us say I have this particular thing which is defined this is actually average noise power without pre-emphasis de-emphasis so this is without pre-emphasis or de-emphasis filter and the ratio of average noise power with pre-emphasis de-emphasis filter.

So I am just trying to see if I means put this pre-emphasis de-emphasis what will be the noise power at the output provided that same kind of message signal and same kind of noise is being added in the channel if I do not put it what will be the output noise power so I am just trying to take the ratio of this, this will be the improvement because the signal power remains the same only the noise power will be reduced by this factor okay.

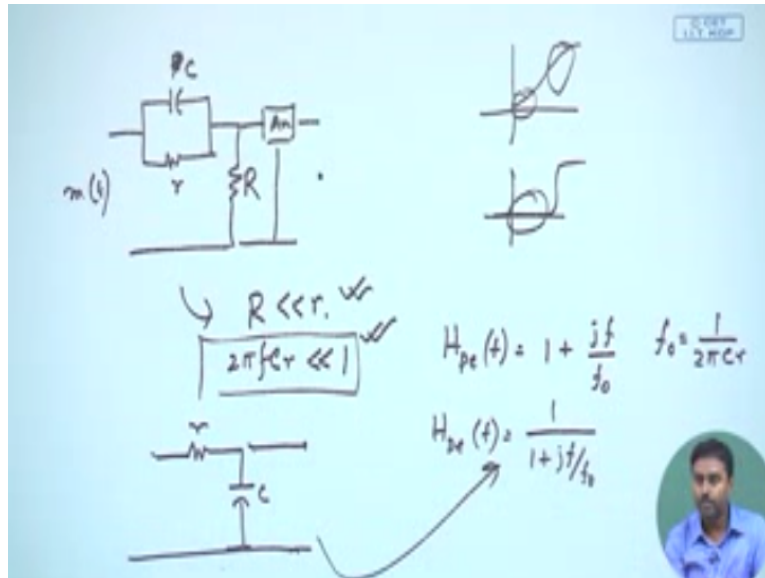
So this is something I am trying to calculate so what will be the noise power with pre-emphasis de-emphasis so this is with P D so that must be we know already this relationships and F^2 I have to integrate it from minus W to plus W right it is valid from minus W to plus W so all I have to do is that only so if I just do we have already derived the noise power spectral density S_{DF} so let us put that so that was equal to n^2 by F^2 now along with that there will be a multiplication term of H_{DE}^2 right and this must be integrated.

So that is the overall power spectral density this was the power spectral density for $N(f)$ in that $N(f)$ and multiplied by this square then I have to integrate it from minus W to plus W right so this should be my overall this one and what has happened for noise power without PD we have already calculated so that should be noise power without PD this is something we have already demonstrated so that is just minus W to plus W and $0s$ is square FS square $D F$ right so this is something we have already evaluated.

So to get I we have to take the ratio of these two so therefore that I should be after doing all this calculation what we will have we will be having so without one on the top and with bottom so that should be this so this should be $2W$ cube so after doing this simplification three and I will be only left with this integration $F H de F$ that's my I right so and $n_0 AC$ square gets cancelled because this is a ratio of noise power so I will be just I have to evaluate this okay.

So let us try to see if we have a practical filter for this pre-emphasis ND emphasis how do we means what should be the performance okay so let us say we employ a pre-emphasis filter which is almost which looks like a high-pass filter because I want to really pre-emphasis means the higher frequency I should boost and lower frequency should have means there should be suppressed so if I put a practical high-pass filter which has a nice means smooth slope something like that.

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So it will actually boost this higher part and it will mean reduce the lower part sufficiently right so a high-pass filter if we just give one example so this is C small R and followed by of course we have to amplify it also so R and that is the amplifier that is the input impedance of the amplifier okay so this is that high-pass filter part okay so if I just put my message signal over here whatever at the output I will be getting that is actually the pre emphasized message signal okay.

What is the corresponding transfer function so we have to also consider that this R is much, much lesser than this small R okay so that is something we will have to ensure and also the cut off frequency this $2\pi f_c r$ that must be much, much less than one so it must be much lesser means this particular overall thing okay let us see an R so this is these two things we have to ensure just to make the high-pass filter so that at the low lower band also it is passing something else otherwise the message signal will be completely surprised.

So I do not want that so I just want to emphasize the higher frequency part so it will definitely what I will try to do the high-pass cutoff frequency I will bring it very close to zero means it should be closer to that so that the higher frequency is getting a little bit amplified or means and the lower frequency is getting a little bit surprised so this is something which is happening not completely getting surprised okay so I do not want a high-pass filter which is like this okay.

So then the lower frequency which is the actual message signal will completely be gone so I do not want that so this that is why this restriction I will be putting and accordingly they should look


like $1 + jF$ divided by F_0 where F_0 is $1 / 2\pi CR$ so this is just that filter characteristics we are putting so if $H_{PE}(f)$ is this immediately $H_{DE}(f)$ becomes $1 / (1 + jF/F_0)$ so of course that should have a reverse characteristics and fortunately that can be realized so if we just put our over here and C over here.

So it is the reverse thing low-pass equivalent part okay and if I just put that this will have a transfer function if you try and look at your filter designing skills you will be knowing that that should be the transfer function of this particular filter so we have already designed HP and HD and that condition that this multiplication must be one that is being satisfied ok the way we have taken the transfer function and it is as you can see from the filter these are all first order filters okay so be at high pass low pass these are all first order filters if we have this then can we now calculate the I so that I which was we have derived the formula is this in that formula I think we have missed something so earlier.

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$$\begin{aligned}
 I &= \frac{2W^3}{3 \int_{-W}^W f^2 |H_{DE}(f)|^2 df} \\
 &= \frac{2W^3}{3 \int_{-W}^W \frac{f^2 df}{1 + \left(\frac{f}{f_0}\right)^2}} = \frac{\left(\frac{W}{f_0}\right)^3}{3 \left[\left(\frac{W}{f_0}\right) - \tan^{-1}\left(\frac{W}{f_0}\right) \right]}
 \end{aligned}$$

$f_0 = 2.1 \text{ kHz}$ if $W = 15 \text{ kHz}$.
 $I = 22$ (13 dB)



When we have shown that square was missing so it should be F square HDE right so that must be the overall thing and now just put that filter HD EF you already know so if I just put that filter transfer function so I get minus W to plus W F square DF divided by that filter characteristics mod HD F square so that is f by f 0 whole square okay now it is just doing this integration which is a simple integration so this can be means you can separate them out and you will get an inverse integration so if you just do that it will become finally I am just giving the final form so that should be W by F 0 minus tan inverse W by L 0.

So this F 0 has to be manipulated it should be written as 1 plus so here F is there take F 0 out so this will become F 0 minus F and then this plus 1 minus 1 so this plus 1 that gets cancelled so this will be just DF integration so that will be F which gives you this minus 1 by 1 plus F by F 0 say F by F 0 you substitute with some, some variable and then do that it will be just an inverse this okay so that is the that we know already that is a simple method of doing it okay so this is what is happening this is becoming and if we just take F 0 to be 2 point 1 kilo and if we take W to be 15 kilo Hertz for a typical FM transmission.

So you must be seeing that this is not just wise this is actually our whole band of even musical instruments also comes under this so 15kilo means voice was just 3.4 kilo Hertz so this is actually a wider band message signal okay which includes as well as means wise as well as some musical instruments which has higher frequency component so if that is the case and we choose F 0 to be 2 point 1 kilo Hertz so it depends on what F 0 you choose because we can see already

this I is function of F_0 so if we choose this thing then immediately I becomes 22 and in DB terms that is 13 DB that is what we have told that in a practical FM case we can actually reduce the noise.

So this becomes 22 times higher that means with without this thing pre-emphasis de-emphasis whatever noise we get that is 22 times iron and things with pre-emphasis emphasis so basically 13 DB higher noise you will be getting at the receiver if you do not employ pre-emphasis GM faces so that is a big benefit so a sign R will be accordingly you can go almost 13DB means you will have a cart in DB margin.

So this is the advantage that you get using FM because FM can also give you this facility so it already had we have already seen that it has a nice noise cancellation not only that you can employ more things to cancel further noise okay so and that is the reason why means days people used to transmit just voice at that time probably amplitude modulation was still okay but when people started transmitting all kinds of musical instruments music and everything.

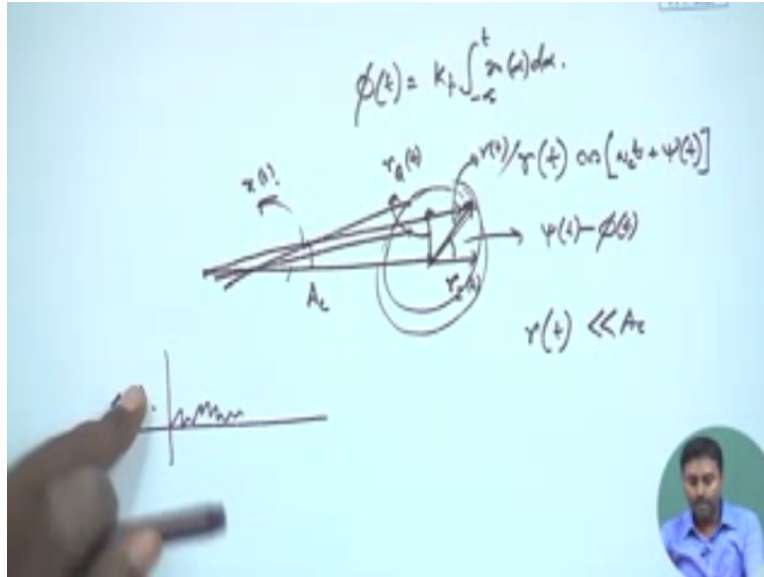
Then the means overall message bandwidth was not restricted to just three point four kilos or 4kilohertzso it means went beyond that and it went up to 15 kilo 20 kilo hertz at that time if you employ a modulation huge amount of noise was coming in and that was really killing the quality of the signal so FM came up very handy because in FM we could see more the bandwidth better than cancellation so more clarity will get that is what happens okay.

So I think we have now almost proven the strength of FM in a way so what next we'll try to see that means from this perspective it looks like FM is all means implementation of FM is all it is just one of the best modulation technique that can happen and all those things ok but means none of the story in this world are like that it is not always means everybody o a particular part is always winner so there are some hind side of FM we will try to also means just if not quantitatively at least qualitatively clarify them in this particular part of the course.

So we will now try to see another interesting part of FM which is called FM means if FM gets captured by the noise it is very bad so that is something we'll try to capture over here which is called the threshold effect of FM or it is also termed as click noise in FM ok so when that happens if you remember when we were doing FM analysis in one place we were drawing that phase or diagram and we have also means mentioned that probably the FM noise signal the

amplitude of the noise or overall power of the noise is much lesser than compared to the signal part.

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So this is when FM does very nicely or performs do perform very nicely but when noise becomes comparable to the signal that is when all the problem starts coming ok in FM so let us try to appreciate that so what happens if you remember that phase or diagram we have drawn so this was A_c which is the signal and that is in the phase or what we have said that is means the phase or phase is the reference so but because FM has A_c then $\cos \omega_c t$ plus that K_f into integration $M \propto D \propto$ so that that entire thing is the difference on top of that we were adding noise so has we have said that it can be represented as $R_T \cos \text{some } \omega_c t + G_T$ right.

That is something we have said so this is that means after doing that quadrature and phase representation of band pass noise then we could represent it in this fashion okay so and we could see that that will be somewhere over here where this angle is this site e^{-} that fight whatever FM phase is where that FM modulation term comes so that πT is actually K_f integration minus infinity to T $M \propto D$ right we have discussed that so this becomes that t minus πT right and on top of that this is actually the R_T and that is our I_T and that is actually our Q_T right this is the overall noise plus signal okay treasure and this is something we have characterized that is the X_T right now let us say what will happen to our FM okay.

So suppose around this I start means what will happen this noise is uncorrelated and it is independent it is probably one of the most random things that is happening so this noise amplitude as well as the angle will keep on varying this will almost remain the same whereas on top of that this will be varying so this tip wherever it is this strip will be wandering around that tip it can be anything depending on how the noise amplitude is so this may wander within this circle problem okay.

And the amplitude also might slightly become bigger or smaller but as long as this amplitude R_T is much, much lesser than AC whatever wherever they wander they will be remaining very close to this particular point only that X_T on the okay so they might wander around but they will remain over there so basically the phase as you can see overall phase will not be deviating too much okay because they are they are small in amplitude they are varying on that location wandering around that location.

So overall phase we will still be joining some point over here random point over here so the Fed does not get change hugely so basically if I just track the face it will be wandering around that value so this is probably that 3:30 but what will happen if this amplitude becomes bigger then the circle that it actually wanders around is becoming much bigger and then what might happen we'll see probably in the next class that what might happen that might really create a huge amount of effect in the phase variation.

And the phase variation is actually this that is the θ_T which you are trying to track where the FM is modulated differentiation of that as is actually the demodulated signal so if this varies a lot there will be some impulses which will be generated so this impulse noises are actually termed as click noise will we will see that so if noise amplitude is very high then probably whenever you are d modulating FM you will be seeing those clicking sound in-between randomly happening so that is actually termed as FM noise okay if I am click noise so we will discuss about that in the next class more on these things and then we will also talk about threshold effect okay thank you.