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Course
On
Analog Communications

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Lecture 54: FM Noise Analysis (Contd.)

Okay so let us come back to the same equation that we have derived in the previous class that is the x (t) which is in the fm demodulator circuit.

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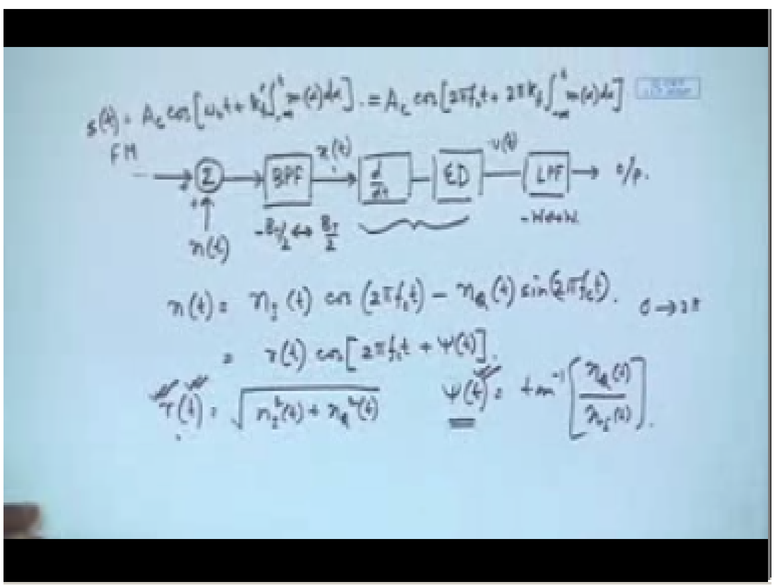

That is where we are now so now we want to see that effect of this thing okay so after the band pass filter we have already characterize the signal plus noise so this is the mass boundary to characterize this let us try to draw a phase diagram which is famous in FM so what we will do will try to draw the parser diagram in phase means you know already will be actually putting the phase in angle and the amplitude will be put over along the radius of that so we have two dimension diagram now in any phases the reference is very important so here what we will do will take the reference as this one signal difference so that must be the reference where we should start okay.

So this is something we call as let us see that is called as ϕ(t) okay so we are calling this particular part as ϕ(t) okay so we can even write this as $A_c$cos2πfct+ ϕ(t) we just writing that as a phase rtcos2πfct+yt okay this is the reference so therefore I can put AC over here because in phases it whatever the angle now this angle is as taken as reference so that must be the 0 angle with respect to that we will have to put this okay what is the angle of that one okay and these are in phases this will to two vectors right which has this angle and this angle so this is the reference angle therefore what is the angle of this one of this phases that must be this minus this right that is the angle so that must be 2πfct gets cancelled.

So ψt-ϕ(t) that becomes the angle of this one okay with respect to this reference so remember that we have taken the reference which has to put this so that becomes the 0 so therefore this angle this entire angle because the phase is just something cos something so therefore that amplitude gets into the radius of that particular path and the angle will be corresponding angle will be response so it is just the polar plot where the amplitude is the arc of the polar quadrant and θ is the θ it has okay so with this reference.

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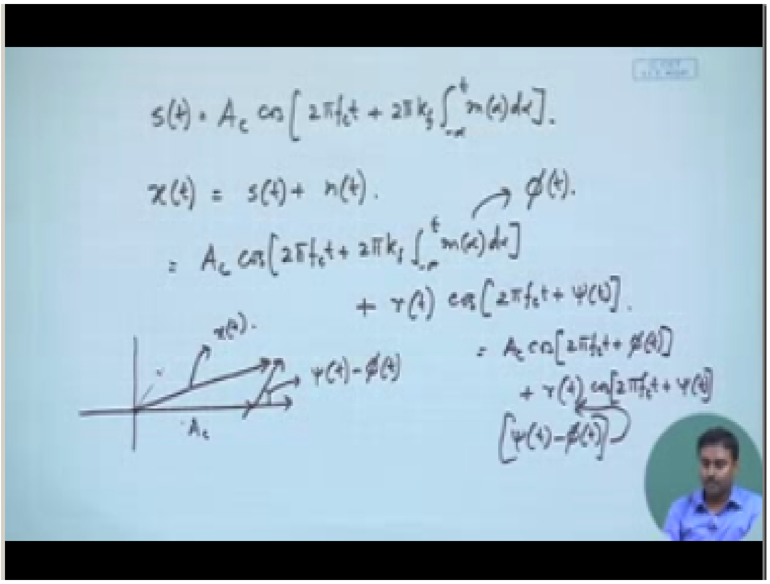

We have this amplitude as ac and we get this one drawn so this will be having amplitude rt whatever that is and this will have a phase of $\psi t-\phi t$ now that should look like something like this okay because we are adding these two so in factorial we can write from here same phase so this angle should be this angle so therefore this must be $\psi t-\phi t$ right so overall this x(t) must be the resultant one resultant phase so that must be by x(t) okay so this is just of that pole okay so immediately that must have some angle okay.

Let us call that as $\theta(t)$ off course it is reference to this one so this must be $\psi t-\phi(t)$ right because the reverences is already $\phi(t)$ so therefore if this as some angle $\theta(t)$ therefore the actual angle between these two should be $\theta(t)-\phi$ right so now I can write this $\theta(t)-\phi(t)$ how do I represent that so basically I can see from the phase diagram that 10 of this must be this divided by this if I put a particular from this point to this point this entire thing divided by this entire thing so just this angle should be $\tan^{-1}$ or maybe you can write it fresh.

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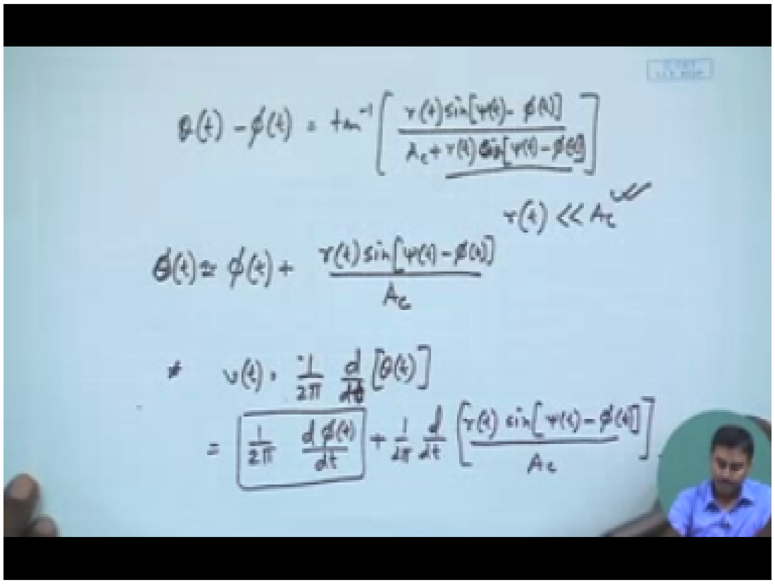

So this angle should be $\theta(t)$-ϕt=$\tan^{-1}$ so let us see the diagram this should be divided by this, this should be r(t) sign of that sign of this angle and what is this, this should be ac+rt cos of this angle this internal angle so I can write it as r(t)sin(ψt)-ϕ(t)/ac+r(t)sin ψ(t)-ϕ(t) okay so that's all fine now whenever we are doing analysis of noise we expect that noise has to be that is lesser power from here to my actual signal or that ac so basically my r(t) should be always much lesser than ac that is approbations we can always take so immediately I can neglect this part so I can write this θ(t)=ϕ(t) this $\tan^{-1}$ of this divided by ac again it is r(t)/ac.

So I can again neglect because that angle will be smaller because sin only can get up to +1 so the maximum value will be r(t)/ac so which is again r(t)>ac so $\tan^{-1}$ of θ must be equal to θ so I can write this as r(t) approximately r(t)sinψ(t)-ϕ(t)/A_c okay so this that is the approximation which is a valued as long as this condition is okay so I get my overall θ(t) which is actually the phase of the resultant signal that means that x(t) if you see over here that is the phase of x(t) what discriminator will do or what the FM demodulator will do it will actually take that phase differentiate whatever that is that must be my signal okay.

So discriminator output should give the differentiation of this phase right so basically my output signal let us called that as let us say we take this diagram so that is v(t) so the output should be v (t) should be according to my understanding of this discriminator circuit and should be differentiation of this phase but remember the differentiation of the phase gives you ω variances so if I need to have frequency variances which we are doing for this calculation so there must be

$1/2\pi$ d/dt $\theta(t)$ that must be what we expect after the FMB mode so let us try to see that so it should be $1/2\pi d\phi(t)/dt+d/dt$ let us first try to evaluate this part what was my $\phi(t)$ so $\phi(t)=2\pi kf$ for what is $\phi(t)/dt$ integration.

And differential get cancelled out so it should be $2\pi kf$ and if I differentiate that should be so I must expect this, so v(t) if I write so $1/2\pi.d\phi/dt$ that is this one so that should be $2\pi kfm(t)$ then 2π gets cancelled I have $1/2\pi d/dt.r(t)/ac$ what we have derived this is exactly my message sequent that is very good so this must be accounted as noise overall but the problem is if I just try to put that as suppose nd(t) so what do I get as noise nd(t) that must be $1/2\pi$ $d/dt.r(t)\sin\psi(t)-\phi(t)/ac$ what I can see is there is $\psi(t)$ and $\phi(t)$ $\psi(t)$ is related to the noise it is completely related to the noise no problem no issue with that but $\phi(t)$ is still related to a message signal so in that noise there is a part of signal already.

So there is where we have to be little bit worried this noise some message signal part now again we will talk about something which will not prove because of time constraint so that was is that came the derivation came from a famous paper written rights at 1963 if you are interested go to that paper it is very famous paper for FM noise analysis so what he could prove at this particular noise is independent of some message signal okay so whatever the message signal is it is independent of that but not only that he could also prove that see this $\psi(t)$ we have already we have shown that it is actually without prove we have that it is actually means it is random process which is having uniform distribution between 0 to 2π right.

So what he could prove that this also which is independent of message signal even though $\phi(t)$ is there it will be independent it will become independent and this is also uniformly distributed between o to 2π so basically what I can do instead of taking that part we can just take this as $\psi\pi(t)$ so this was the approximation we could take and he could prove also that this approximation is valued so therefore I can write this without loss of any generative ac is a constant that goes out so d/dt of $r(t)\sin\psi(t)$ can you identify this what is this, this is actually quadrant set of go back to me that filter representation of band pass signal you will see that $r(t)\sin\psi(t)$ is actually the quadrant tom of the noise okay so that is NQT so therefore immediately.

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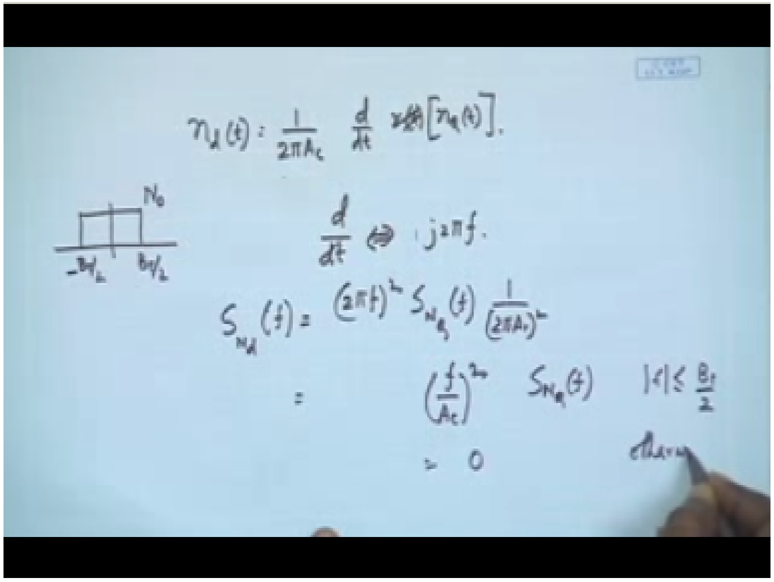

We can say the FM noise which is NDT is nothing but 1/2πac and then it is d/dtr(t) okay so this is something we can already say so this 1/2πac differentiation of nq(t) so now nq(t) we already know right so nq(t) is something like this if θ0/2 is the noise strength so it should be n0 because it has to be shifted left at right every we have already done that so it will be n0 over the desire battery so that band becomes dt/2 up to bt/2 okay fine so this is the overall NQT now let us try to see what this means NDT so NDT is nothing but this NQT pass through a ideal differentiate because that is the operator which is being operated over a NQT.

So it is nothing but our circuit which is an ideal differentiator and pass your NQT whatever you get at output that is NDT with this factor, the factor is alright you need not have to bother of that factor so that is what is my NDT right so let us now try to see ideal differentiator what is the Fourier transform of that is actually j2πf so this is the Fourier transform of that we have already proven that ideal differentiator also are linear time in variance circuit that we already okay so if we have a random process input to the this is something that we have proven input to a linear time invariance circuit then the output random process will be mode hf^2 where hf is the transfer function of that particular linear time invariance circuit or whatever it could into the input sine or post that density.

So basically we can write density of this ndf must be equal to the hf which is $2\pi f^2$ snq(f0 and also you have this part so therefore it should be $1/2\pi ac^2$ so this must be that characterize my noise after the discriminator circuit so what do I get from there so I get this snq(f) is already there we have 2π gets cancelled so I get f/ac right so this is what I will be getting and this is defined remember the filtering was already employment so this must be defined from -b(t)to +b(t) so this is true if my $f<b/2$ and this must be 0 because otherwise there is no noise because it has already been cancelled due to the ideal nature of band pass filter.

So that must be 0 otherwise so can I now plot this okay so if I plot this snq(f) is something like this which is constant between –b(t) to +b(t) but snd(f) that has f^2 so at 0 frequency 0 it must be 0 and then it should have a parabolic thing so it must be something like this so this is the FM after demodulation FM noise characteristics so it never remains similar as AM any other AM counter part because in AM counterpart always whenever we were analyzing the noise it was like this but this is what is happening if you think little but carefully we have already proven that interference effect if you take all this noise as it small, small at a particular small band, small amplitude interference what FM does it captures it right.

And closer to the FM it captures it more that means it suppresses it same performance is happening over here as you can see closer to the central frequency it is actually surprising those voice and as you go away from the central frequency it gets its some significant model okay so FM due to the capture effect only here the physical meaning is again that same capture effect we have talked about but we can see now mathematical that is what it is happening so FM due to capture effect actually shows this particular this kind of representation let us now try to see what will be the overall noise okay.

So now this is my noise and at the output we are also getting the signal part was kfm(t) so this is the signal and the noise whatever NDT we have talked about that is density now these two things will be putting now already my signal has come into the base band up the modulation it is just MT as the signal is defined from –w to +w I have noise which is defined to the band pass filtering and after that whatever demodulation has happened my noise is still defined from –bt to +bt now this is FM bandwidth, FM bandwidth we now already proven to formulas that it is weight narrow band or white band whatever it is BT is always bigger than W okay or bt/2 which is one sides FM bandwidth that is always bigger than that W which is the message band.

So I for sure I know this bt/q is greater than equal w so I can now employee because my message will be somewhere here only within w beyond that whatever is there that is not required for my signal and I can also say due to FM capture effect that sure the noise is having higher power so I should employee a low pass filter around that message signal okay going from –w to +w so that is what I will be doing next I will be passing it through a low pass filter of bandwidth w so it will actually take from -w to +w so this will definitely have less sine so if I now integrate this positive density whatever I get that must be the noise power let us try to calculate the noise power.

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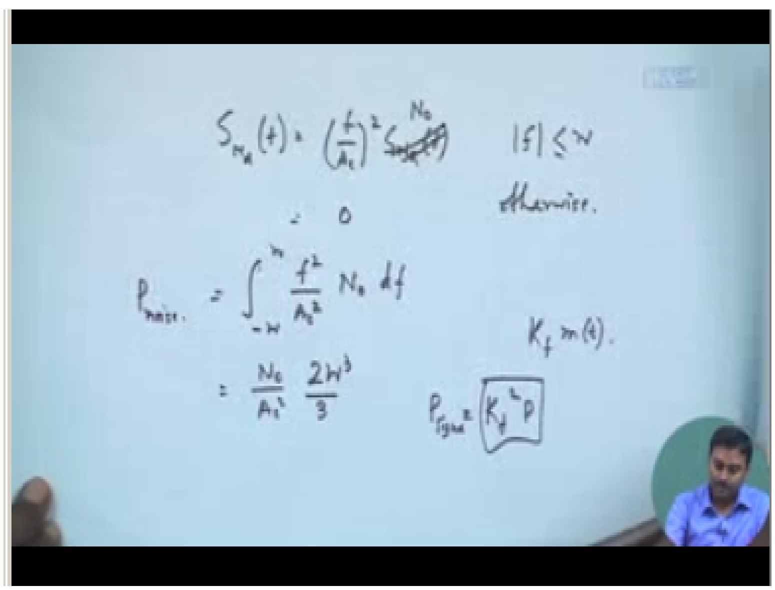

So I can now define after doing this low pass filtering this Snd(t) must be something f/ac^{2} snq(f) where f<w because it is scripted to that because I have already done the low pass filtering and it must be 0 otherwise so what is sn qf in that is just n0 so I can instead of this I can write n0 right so what is the overall power we know from density you have to just integrate it so here it is defined only from-w to +w.

So the overall power of noise should be integrated from –w to +w is f^2/ac^2n_0df that must be my overall noise problem so no/ac2 then if you put w so that become actually $2w^3/3$ is the noise problem if say corresponding signal power so I had kf.m(t) so the overall signal power is t so it should be $kf^2.p$ that must be my signal power therefore what is the signal to noise ratio for FM.

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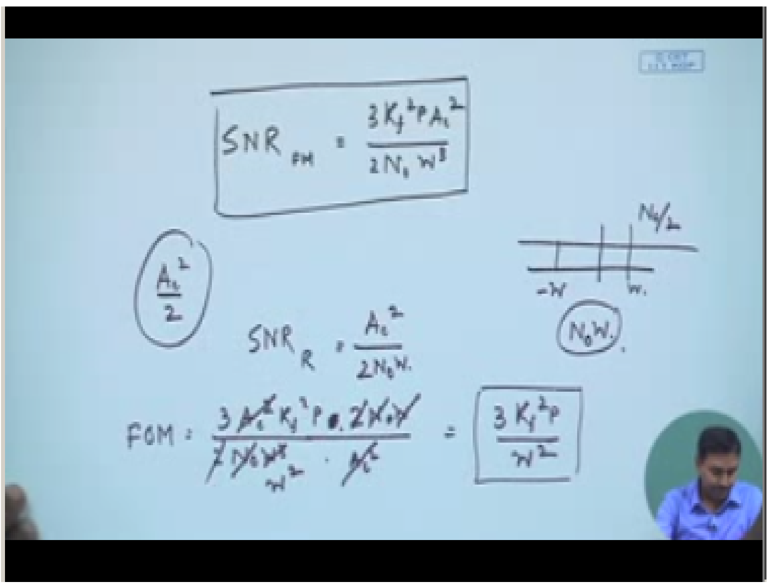

So let us say $SNR_{fm}=kf^2pAc^2/2n_0w^3$so this is what we get as SNR now you have to do the figure of merit calculation for figure of merit calculation what we have to do in the base band we have to transmit whatever we have transmitted for whatever power we have put for FM transmission FM was just AC something is there okay so what will be the power of that that will be the $c^2/2$ because whatever inside that is just face that will not continue to the power of this signal so $Ac^2/2$ will be the power that will be transmitted in the base band.

And what will be the noise, the noise goes from my it will have a strength of $N_0/2$ and it goes from –w to +w so what all noise will be N_0w if you integrate okay so that is the noise power so therefore overall SNR for the reference 1 should be $Ac^2/2N_0w$ so therefore the figure of merit that must be this divided by this right so figure of merit must by $3A_c^2kf^2p/2N_0w^3$ so we are left with $3k_f^2p/w^2$ right that is the figure of merit probably from here we are not able to appreciate what is happening in effect that is still something which we still do not know let us try to evaluate it in terms of tone modulation then probably we will able to match it with respect to aim what happens okay.

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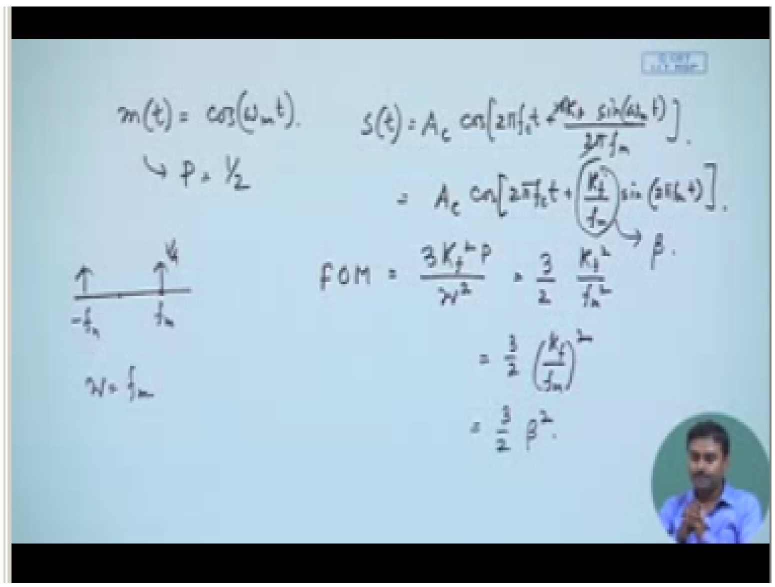

Let us try to do on a tone modulation so in tone modulation what will happen by m(t) that is actually nothing but some cos(ωmt) okay this is just a tone modulation this is sinusoidal and I am trying to transmit so immediately I can calculate what is the power of that so that means p should be1/2 because is just a cos so p should be ½ so this is something I know okay what will be my $s(t)=A_c$ cos($2\pi f_c$t+kf and then if this kf is represented in terms of let us say ω so then that is kf otherwise there should be 2π all those things okay.

So if that is the case so kf and then I need to have a integration of this one so I just to a integration what will happen if I do integration that should be sin(ωmt)/ωm now if kf is represented in frequency that must have 2π here also can write 2π fm so this is my representation so I can write this as $A_c$cos[2πfct+kf/fm sin(2πfmt)] okay so that is my representation now the figure of merit which we wanted to calculate what is that it is actually $3k_f^2p/w$ okay what is w for me that is actually a FM because the message signal is having just a impulse of strength ¼ at +fm and-fm so what is the overall bandwidth that should be this one.

So w becomes FM right P becomes ½ so I can write this as 3/2 so I have kf^2/fm^2 can you now identify what this is, this is actually the β for tone modulation at kf/fm is the β or the we should say modulation index or frequency deviation index okay so this must be the β so I can write this as $3/2\beta^2$ now what I see particular relationship between the FOM figure of merit with respect to β the good part is here if I increase β FOM increases that is one thing what does that means that means if I increase β I am actually going towards a wider band FM because the frequency deviation that means the kf value is getting increased with respect to FM that is the frequency deviation is getting increased which is actually termed as wider band FM.

So if I make the FM more wider that means I increase the bandwidth of the FM I get a benefit in terms of noise cancellation because it is figure merit gets improved so this is what we wanted to discuss that FM is the only modulation scheme none of other modulation scheme will probably do that that with I can exchange my frequency with respect to noise performances so if I exchange means I can make this watts and get a better benefit so basically I can increase the FM bandwidth where I guar tee to have a better performance in terms of noise conservation this is what is happening in FM let us try to compare this what was the best FOM figure of merit that we could get for AM.

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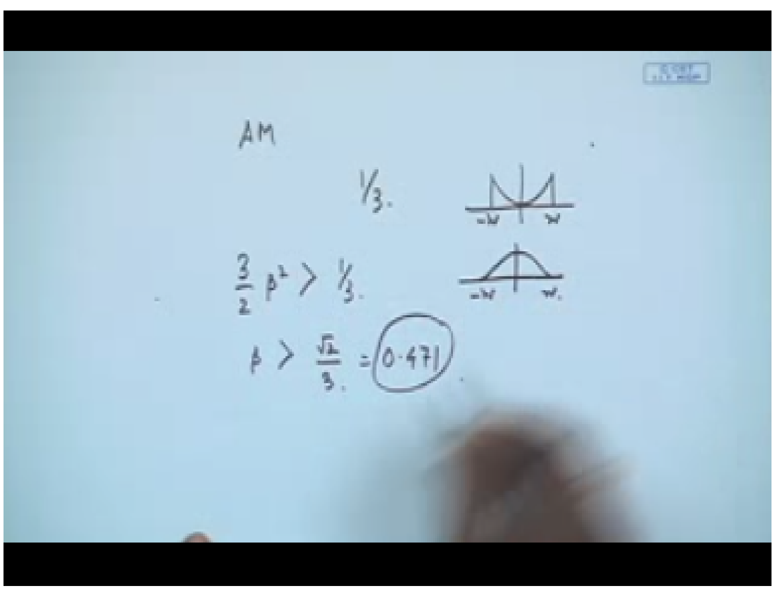

That was 1/3rd we have proven with a tone modulation so what we can write that this if this has to be anything better than this 1/3rd my $3/2\beta^2 > 1/3$ okay and then immediately I can get a $\beta > \sqrt{2}/3$

which comes out to be some 0.471 so any β which is greater than this will always give the benefit in terms of noise cancellation so will be happening I can take to any β value where this will be much bigger okay so basically I have now controllability parameter or a controllable parameter which I can just keep controlling to get better and better noise performance.

So in FM we can actually do we can make the bandwidth wider off course spectral efficiency will be reduced but to get better signal quality that is always possible where as in FM you can never do that if you increase bandwidth more noise will be coming inside and you will get a better watt performance and you cannot decrease the bandwidth because the bandwidth you take for AM that is the optimal one anything you decrease your signal will be destroyed so there is no option over there no variability over there where as in FM you can do that.

And this also the reason why people have preferred wider band FM or to narrow bandwidth counter part because in wideband FM you can a better noise cancellation and that is why we have seen that Armstrong tried so hard to generate wider band FM with series of circuit from a narrow band counter because generation of narrow band was that time easily when receiver was not there and then they could actually reproduce wider band FM by doing this it is also give us another advantage if we just try to see what was the FM noise performance this was the noise spectrum where as the message will be so this is from –w to +w where the message will look like this.

So from –w to +w or it might have nothing have 0 whatever it is we can see that more noise are added at the higher frequency to call back that what we can do we can do some pre processing at the signal level we can give something where the signal higher frequency components are boosted because those frequency component are much better off in terms of noise because they are getting adding with lesser amount of noise whereas the higher frequency components are getting more amount so if we just boost them little bit what will happen will get a better signal to noise characteristics at the higher frequency.

And then after doing FM demodulation and everything we can actually whatever boosting we have done we can just reverse that so that means after noise cancellation on everything that has been employed we can just redo the whole thing and we will get probably a better analysis because noise power will be already same whatever will be coming in and after that if we do a reverse whatever filter we apply at the beginning if we just reverse it we probably get a better performance in FM.

So that is what people have employed which is famous they termed as pre phases and de phases so you do a preprocessing of a frame signal to give it a better noise cancellation so next class probably we will be discussing this and how much it gives benefit with the practical circuit okay thank you.