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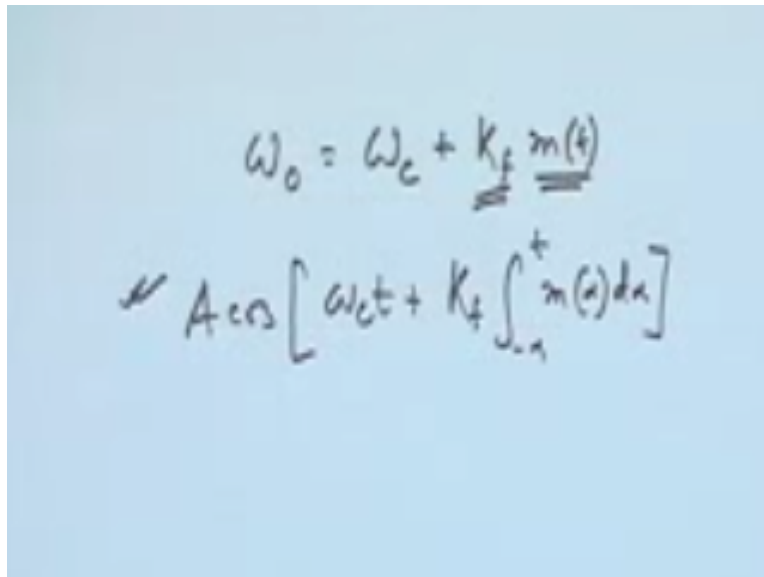
NPTEL ONLINE CERTIFICATION COURSE

**Course
On
Analog Communications**

**by
Prof. Goutam Das
G S Sanyal School of Telecommunication
Indian Institute of Technology Kanpur**

Lecture 52: Frequency Modulation (Contd.)

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The image shows two handwritten equations on a light blue background. The first equation is $\omega_0 = \omega_c + K_f m(t)$, where ω_0 is the instantaneous angular frequency, ω_c is the carrier angular frequency, K_f is the frequency sensitivity, and $m(t)$ is the modulating signal. The second equation is $A \cos \left[\omega_c t + K_f \int_{-\infty}^t m(\alpha) d\alpha \right]$, representing the FM signal waveform.

Okay so what we have started exploring is the frequency output of the oscillator we are targeting which is we have proven that that should be $\omega_c + K_f m(t)$ we have defined what is ω_c and what is K_f okay so this is something we have already discussed right now FM generation once we have this circuit that means our particular oscillator with an inductor which is fixed and a very cap where the it varies with the input that bias voltage okay.

So once we have that we will be getting the output oscillation frequency is accordingly vary okay so what I will do if you now start giving our input voltage as your $m(t)$ into that very cap immediately the output oscillation that we will be getting that must be FM modulated signal

because that will be the m_f immediately the oscillation frequency suppose it generates a \cos some frequency so that must be this okay so and immediately you will get if this is the frequency the phase will be $\omega_c t$.

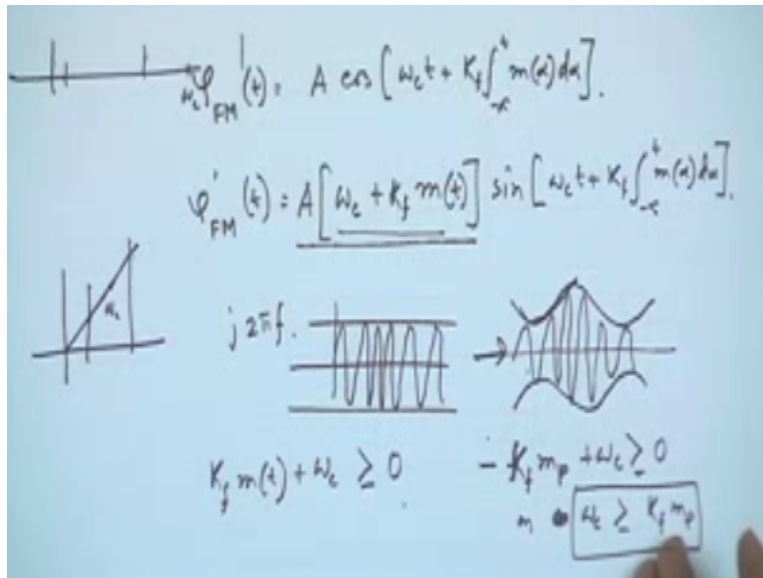
This is something we have already explored that should be K_f integration - \int to t $m_f \alpha$ $d\alpha$ that is actually the FM modulated signal so that gives me a direct modulation where this K_f is something which is chosen by that particular very cap right so whatever I choose as my VCO I will be getting accordingly the FM deviation and everything so I can I can choose my parameter accordingly and I will get my FM so I have to choose accordingly the free running frequency so that FM as well as the K value and that will give me the current correct output right.

That is called the direct method so you can see already lot of complicated that were arising from narrowband FM and with an indirect method to generate wide band FM that goes away and of course there also there was a problem that narrowband FM it is not if it is not sufficiently narrower then there will be a \tan inverse of $K_f \times$ at term right so instead of just K_f into a t terms so that that problem was there whereas here there is no problem like that so this is called the direct method of FM generation okay.

So let us try to see now how do you do FM demodulation okay so our next target should be FM modulation is almost done so we have learned narrowband FM how that can be generated then for wider band FM we have seen two methods one is direct one is indirect so in the indirect method we have to just put frequency multiplier and we have seen also how to adjust the center frequency as well as frequency deviation whereas indirect method it is just choosing a proper VCO parameter okay.

Sorry the direct method it is just choosing a proper VCO parameter and it is very easy just across the very cap you give your input voltage and you will be getting your corresponding FM modulated output at the output of that VCO okay so let us now try to discuss about FM demodulation again you will see just the mathematics tells us what should be the FM demodulation suppose I have a ψ FM t .

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And what it is it is a $\cos \omega_c t + K_f$ right this is my ψ FMt I want to demodulate it the first thing that I will do is if you carefully see this if I just differentiate this signal so let us try to do this what do we get so simple differentiation you do so whenever we differentiate it should have that chain rule so differentiation of course must be giving me sign and then inside part also has to be differentiated right so inside part if I differentiate this must be $\omega_c t$ must give me ω_c because I am different shading with respect to T right.

And then K_f is a constant differentiation and integration actually cancel each other and this will give me $m t$ right so that is the whole thing that I get after differentiation and \cos must be also differentiated so that must give me \sin of $\omega_c t + K_f$ right so what we can see that FM modulated signal if we just differentiate what happens it will generate another sinusoidal but the whole signal varying part comes into the amplitude okay now what we do whatever the sinusoidal so it will actually look like this suppose ideal differentiator how does that look that is generally $j2\pi f$ okay.

Differentiate a DDT if you take the corresponding Fourier transform so that looks like this so ideal differentiator should be something like this constant means if I put the amplitude part of it so that should look like this it is a linear function of f okay so if My ω_c is somewhere over here and it remains linear over there means if it is a ideal differentiator so I should be expecting something like this from that is the output okay and after this how the signal will look like so I had initially FM moderated signal.

So which was having something some modulation so it was varying with respect to the amplitude of it is just the frequency deviation if I just differentiate what happens this frequency variation also comes into amplitude whenever there is a higher variance variation of frequency so basically I will have accordingly a higher amplitude and correspondingly if there is a lower variation I will get a lower amplitude right.

So this will be means once I pass it through our differentiator it will look like this and inside also same pattern will becoming something like this okay this is something which will be happening so whatever is happening what we can see that message signal is almost in the envelope so all I have to now do is envelope detection nothing else as long as I can ensure that envelope detection gives me a signal back as long as this is not doing a zero crossing we have already learned that in a modulation.

So what is the condition that this will not be crossing zero if this remains always positive so that means I have to write $K_f m(t) + \omega_c$ must be greater than 0 right this is something I will have to write now what is the minimum value of this where it might cross 0 that means the minimum of this must also this is true for all T that means the minimum of this must be also greater than 0 so minima let us say that is $-m_p$ or that is $-m_p$ let us say okay.

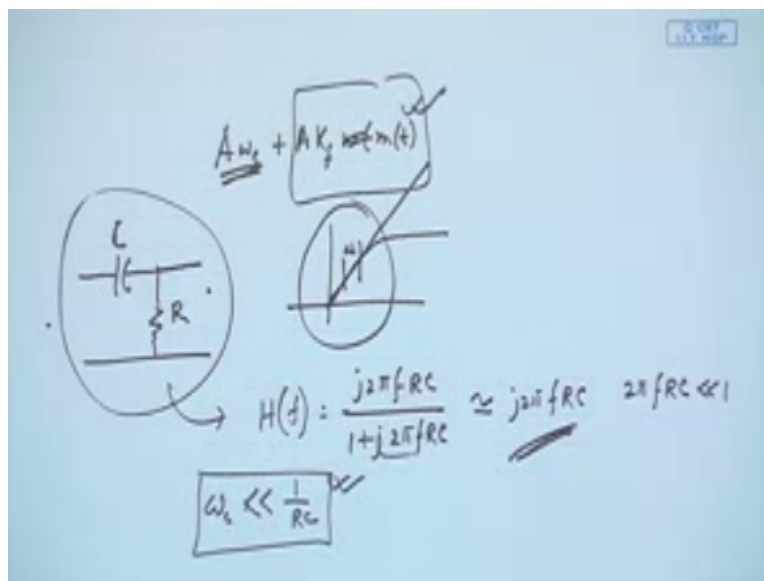
So then I can write $-K_f \times m_p$ where m_p is a positive number - of that is the minimum $+ \omega_c$ must be > 0 or ω_c must be \geq to K_f into m_p so this is the condition I get what is K_f into m_p that is the frequency deviation we have talked about okay so basically my carrier frequency must be bigger than frequency deviation this is something which always I will be doing because if the carrier frequency because I know from Carson's formula that the bandwidth is more than 2 times of this deviation if my carrier is not even bigger than this particular thing there will be a FM right.

We are actually putting at the carrier ω_c now the bandwidth is definitely bigger than my Δf twice of Δf right so or $\Delta \omega_c$ if this is already bigger than my ω_c then what will happen this will come even beyond zero and there will be FM so definitely I whenever I do FM I will make sure my ω_c is bigger than that Δf at least at it has to be bigger than $\Delta f + B$ according to Carson's formula right so it has to be done if that is being always done so this condition will always be there that is prevalent we know that this will be happening.

If that is the case I know that the envelope will always remain above 0 it will not have any 0 crossing so I will actually this is guaranteed that I will never have to cross 0 so if I just detect the envelope I will get my signal back so that makes the FM demodulation pretty simple all you have to do you have to pass it through ideal differentiator followed by a simple envelope detector the one we have designed for a modulation demodulation.

So this is what happens and you know that it is guaranteed as long as you make sure that the FM modulated signal that you generate that is not creating aliasing that means ω_c is bigger than at least that $\nabla\omega$ you are pretty sure that your envelope will be above always and if you just track the envelope you will get your message signal back. So tracking the envelope will give you $\omega_c + K_f \int m(t) dt$ this term will be gone because you are just tracking the envelope that sinusoidal variation will be gone.

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So you will just get $A\omega_c + AK_f \int m(t) dt$ sorry ω_c now this is just a DC term you block the DC with a capacitor will get this part which is the message signal so indirect sorry this method of means

this differentiator induced method of FM demodulation that is very simple all you have to do is you have to find out ideal differentiator circuit now we will discuss about that that is little bit difficult to find out the ideal differentiator okay.

So generally what people have done means you cannot actually get an ideal differentiator that is not possible in circuitry every capacitor will put there will be some spurious resistor in that capacitor there will be some other effect and always you will see that it cannot work as an ideal differentiator so that is not possible there is nothing called ideal differentiator what we will do we will employ something like this R C circuit this is actually a high-pass filter right.

So how the high-pass filter will look like it will it will actually look like this but there is a region generally high pass filter we are not bothered about this roll-off generally we want to neglect that roll-off in high pass filter we are most more concerned about the where the filter response is flat but this is further for the FM demodulation we are actually bothered more about this roll-off so we want to see where exactly it remains linear that is where it almost behaves like an ideal differentiator.

So basically we have to target a particular frequency zone or we have to design our high pass filter in such a way that in the frequency of interest which is this ω_c and around that ∇f^+ and ∇f^- right so that region it remains linear okay so that is something we will have to find out so let us for our ideal this particular filter let us try to see that what is the transfer function so that should be $j2\pi fRC$ if you just put it accordingly so this is what we get okay hf will be just this okay now this can be approximated as $j2\pi fRC$ if this $j2\pi fRC$ is much less than 1 because then this term will be neglected.

So there will be only 1 so I get this is almost like a ideal differentiator so this will happen if this condition is valid so now I can get a condition on my ω_c okay so what I can do this $j2\pi fRC$ let us put it as ω_c so ω_c must be much less than $1 / RC$ okay so this is the condition I get if I choose it accordingly then the ω_c will be falling in this region where it looks like ideal differentiator so all I will have to do is I have to choose a RC corresponding to my ω_c that has been put over there accordingly I put my RC so maybe I can fix C and then try to find out what should be my R and then try to find out the RC value which is much bigger than this ω_c .

Then I know that around that ω_c it remains linear because this particular transfer function characteristics will be valid that approximation also will be valid and it will almost behave like an ideal low-pass filter sorry high-pass filter sorry ideal differentiator right and then if I just pass my signal through this at the output I will get a differentiator or differentiated output at that particular frequency so this is all that I will have to do it is simple enough all we will have to do is we have to design our RC accordingly and then try to get a particular differentiation circuit.

Once this is being done I know that the means envelope tracking will be very simple whatever we have done at a.m. that has to be mimicked over here okay this is one way of means demodulating FM.

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FM Demod using PLL

$$\sin(\omega_c t + \theta_i(t))$$

$$\theta_i(t) = K_f \int_{-\infty}^t m(\alpha) d\alpha + \pi/2$$

$$A \sin\left[\omega_c t + K_f \int_{-\infty}^t m(\alpha) d\alpha + \pi/2\right]$$

$$\theta_o(t) = K_f \int_{-\infty}^t m(\alpha) d\alpha + \pi/2 - \theta_e(t)$$

$$e_o(t) = \frac{1}{c} \dot{\theta}_o(t) = \frac{1}{c} K_f m(t) - \dot{\theta}_e(t)$$

There is another way that is FM demodulation using PLL okay so if you remember that for PLL we have also used a VCO right that was how we have designed PLL so for FM modulation we have already used VCO now for demodulation also we will be just using PLL and PLL also as VCO in it so basically FM modulation demodulation both will have key component as the VCO so once you have VCO you can do both modulation demodulation if we can appreciate this particular circuitry.

So let us try to see how FM the modulation can be done with a PLL so this is another application of PLL which is coming out earlier we have seen that for carrier tracking probably PLL is very

good and with that target only because we are talking about phase lock loop which was with the target of carrier locking okay our phase and frequency locking we have discussed about that effectively.

So now we will try to see the other application of PLL which is FM demodulation so let us say the PLL generate a it is input this one to which it gets locked that is a sinusoidal so that is something like $\sin(\omega_c t + \sum \theta_i t)$ is the input so what will do will actually give this FM signal if I modulated signal to the PLL input so what will happen to this $\theta_i t$ that must be whatever phase it is getting so FM is a $\cos(\omega_c t + \text{the FM modulated part that } K_f \times A_m \times T)$ okay so that means that $\theta_i t$ must have this $K_f \times A_m \times T$ because its cause and PLL input take sin.

So there should be a $\Pi/2$ shift so there therefore $\theta_i t$ should be $K_f \int_{-\infty}^t m(\alpha) d\alpha + \sum \Pi/2$ if this becomes $\theta_i t$ so immediately I can put my FM input should be a sin this $\omega_c t$ so I can put $A \sin(\omega_c t + \text{this } \theta_i t)$ which is $K_f \int_{-\infty}^t m(\alpha) d\alpha + \Pi/2$ right that immediately becomes cos so it is $A \cos(\omega_c t + \text{this } \theta_i t)$ that is actually a FM signal so therefore if I wish to put FM signal to PLL input and if we correspond it we get my $\theta_i t$ as this one right this is fine.

Let us say the output phase error that is generated due to the locking that is θ_e of course this should be small enough we will see that so basically therefore what is θ output t which is being generated after the VCO means the PLL output that should be this input phase by - this $\theta_i t$ right so input phase is $K_f \int_{-\infty}^t m(\alpha) d\alpha + \Pi/2$ and - this $\theta_e t$ right so that is what it will lock to and it will get this output phase but what we also know that this $\theta_e t$ if we have if we now try to see the error signal that is being generated by PLL.

So this that is $e(t)$ that is nothing but the differentiation of this output phase so and with a factor $1/C$ which is the factor of PLL right so $1/C$ differentiation of $\theta_e t$ now let us try to differentiate it so it should be $1/C$ and if $\theta_e t$ we have to differentiate so this term will be gone I will have a differentiation of this one and I will have differentiation of this one so differentiation of that that one is K_f and if I just differentiate integration difference cancels each other so I get $m(t)$ right.

And I will get differentiation of this now that is exactly the error frequency because phase differentiation is the error frequency PLL if it is properly locked then frequency error must be zero so this I can almost say it should be $1/c K_f m(t)$ which is very good because that is actually

the message signal with some constant factor which I do not bother so basically at the error of PLL I get FM demodulated signal if I give at the input of PLL FM modulated signal.

That is whatever analysis we have done for PLL that actually directly comes from means it directly comes from there if we means we just take a PLL circuit earlier we are not bothered about the error signal of PLL we are not bothered about that.

Now in this particular demodulation what we will try to do we will try to suppose at that time we were bothered about the VCO generated output okay now we are actually looking into the error signal that is being generated after the PLL after the loop filter so what we have to do is will give as input the FM modulated signal we do not actually tracking we are not tracking anything so we are not worried about the VCO generated output we are not worried about now.

Now we are not worried about that we take that from the low filter whatever is coming out we will take that and we could prove mathematically again you can see why we are able to use this because mathematically this is getting proved so it is the circuit almost operating the way it is defined its transfer function and everything operating on the signal this is what it is giving so once you prove that mathematically you know that with this circuit I can do this things.

So we can see that the error signal is actually becoming the differentiation of that particular thing okay so differentiation sorry differentiation of this output and that becomes happens to be proportional to my message signal so that means at the error I am getting my demodulated signal right so what we have so far done is by we employed two methods of FM one was direct method one was indirect method indirect method we could be used VCO indirect method we had to do a lot of things a lot of multiplier part lot of nonlinear circuit and all those things.

And then for demodulation also we could see there are two methods one is through differentiation followed by envelope detector which is simple enough so for differentiation you will probably have to employ a linear part of a high-pass filter okay and the other part is just use PLL for demodulating FM signal so these are two things that can be done to demodulate FM signal okay.

So far we have analyzed about FM bandwidth and how we can generate or demodulate FM modulated signal and FM demodulated signal okay so this is something we have analyzed but first of all we need to understand that why we should study FM or why one should employ FM so this is something we will now be exploring so we will try to see or try to appreciate that FM

has some good characteristics initially people who would have thought probably it is the bandwidth efficient protocol sorry bandwidth efficient modulation technique. But that is not the case we have already proven that that this is not probably as bandwidth efficient as any of the a modulated signal but there are some advantage and this is something which we will be exploring next so the first thing is we have already discussed that there are any modulated signal that will be transferring that has some effect when it is passed through a channel right.

So what are those effects the first effect we have discussed is if the channel is nonlinear okay so that is the first thing which happens so we have also seen that if we have A modulated signal and if there is a non-linearity there will be detrimental effect this is something we have already appreciated and we have seen that okay so what we now wish to see that for FM is there any effect or is it very means very much superior in terms of modulation that any non-linearity in the channel can be just rejected by FM okay. So that is something we wish to see.

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$$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + \dots + a_n x^n(t)$$

$$x(t) = A \cos \left[\omega_c t + K_f \int_{-\infty}^t m(a) da \right]$$

$$y(t) = C_0 + C_1 \left[\cos \left[\omega_c t + K_f \int_{-\infty}^t m(a) da \right] \right] + C_2 \cos \left[2\omega_c t + 2K_f \int_{-\infty}^t m(a) da \right] + \dots$$

So let us try to appreciate that so let us say we have an arbitrary non-linearity so $y(t)$ is some $a_0 + a_1 x(t) + a_2 x^2(t) + \dots$ sorry $x(t)$ is the input $a_2 x^2(t)$ some order non-linearity in the channel now what will happen I will be launching FM signal so FM signal that means my $x(t)$ should be $A \cos \omega_c t + K_f \int_{-\infty}^t m(a) da$ so whatever discussion we are doing that is equally valid for FM as well as p.m. right.

So both are equivalent that that is something we have proven instead of integration you will be putting just empty over here okay so if I just put this $x(t)$ what will be my $y(t)$ there is something we have just done for our direct modulation right so what will happen if I just put it over here I will be just seeing something like $C_0 + C_1 x$ sorry \cos of course this A can be taken inside C so do not worry about that so it should be $\cos \omega_c t + K_f \int m \alpha d \alpha + \cos C_2 \cos^2 \omega_c t + K_f^2 K_f$ and so on up to a n^{th} term but what has happened.

Now this FM modulated thing after it passes through this particular channel will probably get all these extra higher terms and I my receiver what generally that will have at the means at the front end of receiver we have already talked about that it wants to neglect the effect of noise so it will have a band pass filter where that band pass filter will be centered at ω_c and the band will be just FM band so if I just pass it through that band pass filter these things all will be neglected so all those higher frequency term where this is getting contaminated because the frequency deviation is getting twice thrice and all those things.

They will be all canceled out what will happen I will only have this particular thing so even if I have channel non-linearity I do not bother about it because FM automatically due to that band pass filter will cancel out that effect of non-linearity and it will get pure FM modulated signal even after passing through a nonlinear channel so that is a very big advantage which FM has or FM has that edge over amplitude modulated signal because in amplitude modulated signal if you multiply it that $m^2 t$ term will be coming out because it is in the amplitude.

So that multiplication will create a multiplication in the signal also and then that will create problem for you whereas that is not happening over here okay so that is a big advantage which we will have whenever we have a channel which is slightly nonlinear so in that channel if you just put FM that is more protected compared to your any form of A modulated signal okay, so this is the first thing where we could get some advantage of FM will see if there are interference see whatever I have told that is actually creating a source of interference also.

So if the channel is nonlinear and I have here probably at ω_c my FM signal but at $2 \omega_c$ there might be some others FM signal where due to these things in the channel they will be all created so this will create interference to them however small that non-linearity is this a_2 a_3 coefficients are small they might be smaller but they will still create some effect at those frequencies so those are treated as interference.

So what we will see in the next class that in presence of interference how FM survives okay so that is something we will try to appreciate in the next class and then at the end we will also try to do noise analysis and we will be able to prove that in any time in any channel FM is much better in terms of noise cancellation compared to any of its AM counterpart okay so that is something we will be proving and with that we will probably end our discussion of FM okay thank you.