

NPTEL
NPTEL ONLINE CERTIFICATION COURSE

Course
On
Analog Communications

by
Prof. Goutam Das
G S Sanyal School of Telecommunication
Indian Institute of Technology Kanpur

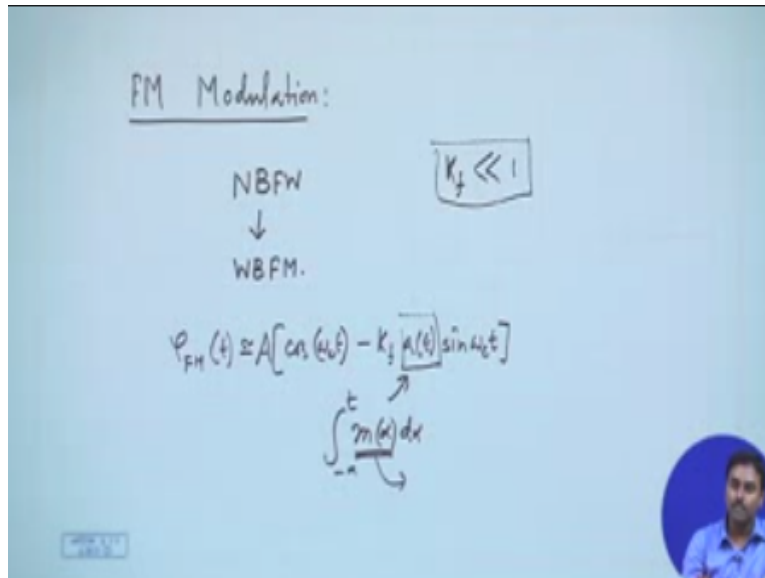
Lecture 50: Frequency Modulation (Cont.d)

Okay so in the last few classes we have I think already started discussing about the FM bandwidth and we have also discussed a FM modulation index, so that is something we have already discussed which is related to the frequency spread and in the bandwidth also we have defined what is the relationship between the overall in bandwidth and the frequency spread due to FM modulation as well as overall the baseband bandwidth of the signal that is being modulated okay.

So those things and Carson's formula for tone modulation what happens to him bandage we have we have seen it from different angle and has means started gathering information about what should be the FM band and there was some fallacy around the calculation of actual FM bandwidth that also we have discussed in detail, now what we will do is we will try to see how FM is being generated and means what are the different options of FM generation and then we will try to see how you can demodulate that.

So the means next 2 or 3 classes probably will be discussing about FM modulation and demodulation because we have seen that is the most important part, so that is where the signals and systems actually interact with each other and we need to define a system which does the proposed modulation or demodulation tasks, so we will start with FM modulation.

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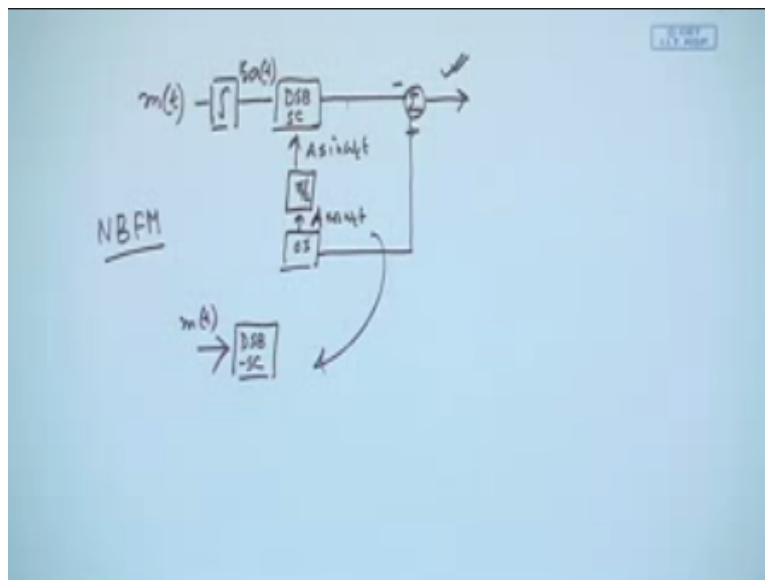
Okay so that is the first thing but before we start we should also characterize two things probably we have already started characterizing that we have told that there are something called narrowband FM which is called NBFM and the counterpart of that is wideband FM okay so what is the narrowband FM we have told that if the frequency deviation is very small that means especially that K_F value is very small much, much smaller than one okay, so whenever this happens for our FM modulation so corresponding FM is termed as narrowband FM and we have also seen that Taylor says with that tell us is expansion of that FM modulated signal.

We have seen that narrowband FM when this particular K_F into a T is much smaller we can actually represent it as this is something we have already explored approximately can be represented as $\cos \omega_c t - K_F \sin \omega_c t$ where ω_c is the carrier frequency of that means with which we are doing that that modulation and what is $\int m(x) dx$ that is actually the means we for doing FM we have to first differentiate it that or integrate it for FM and PM, so that is that part right so this is what we have we have already discussed about that okay.

So this K_F into $a(t)$ which is actually for FM it should be integration write integration $m \alpha \int_{-\infty}^t m(x) dx$ to T okay, so that is something which is already there taken care of so m also this is actually the message signal which we are trying to, so that is actually we have told that there are if you do Taylor series expansion there are other terms which are being neglected it because that will be because of Taylor series expansion there will be $K_F^2 a^2(t)$ and all other terms because this is already very smaller than much.

Smaller than so we are telling that $K_S a(t)$ it is much smaller than one so we can actually neglect that okay $\sin \omega_C t$ of course is bounded by 1 so modulus of that, so we are neglecting those terms as long as this is small so we get this particular things and then we have told how then the narrowband FM that modulation looks very simple it is almost like our this DSBSC or amplitude modulation kind of thing where you multiply this with this, so if you just now put that so what we have to do we have this mt.

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Whatever that signal is we put through a integrator we get $a(t)$ and then will you DSB SC modulation where we supply actually a $\sin \omega_C t$, so basically we will have a oscillator which will which will be generating a $\cos \omega_C t$ okay and will give a π by two phase shift to get \sin and then this has to be added Plus this is minus what you get is narrowband FM okay, so it is easier enough because what we have to do we have to just generate this a $\cos \omega_C t$ which is coming from this path.

So we are getting a $\cos \omega_C t$ now this is getting π by two phase shifted so we get a $\sin \omega_C t$ multiplied by $a(t)$ and then we are taking minus of that, so this plus and this minus so we get this particular thing okay so of course you say that somewhere cave has to be there, so we will put case either over here or we can include it over here, so wherever you wish to take that K_F it becomes like that okay so this is how FM narrowband FM as long as whatever that K_F you are choosing.

That is small enough and your means or I should say K F into a T that is small enough okay if that is happening you will only have too much of deviation and the corresponding one it do not really has to go and modulate the frequency with respect to the input signal because, we know that as long as that is small we have that expression okay so immediately you will get FM modulation for demodulation also it is very simple you just do the reverse process because it is almost like am modulation or FM modulation okay.

So right now will not talk about FM demodulation so we are just talking about FM modulation so this is the way we will be generating as long as the frequency deviation is small this is the way we will be generating narrowband FM okay, if you have to just do p.m. then you do not need this integration also we directly supplied right m(t) you directly supply, so m(t) you will be supplying and that directly goes into this DSBD, okay let us other things are same so this is that DSB yes other things are just duplication of this particular thing okay.

So narrowband FM and that will be corresponding narrowband pm of course you will have to again multiplied with K_p okay K_F for F_M K_p for fm kp for and K_p into Mt must be much smaller otherwise we will not have that approximation okay so this is the generation process now let us try to see what exactly is happening I have this particular term.

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$$\begin{aligned} \varphi_{\text{PFM}}(t) &= A \left[\cos \left[\omega_c t + K_f a(t) \sin \omega_c t \right] \right] \\ &= A E(t) \cos \left[\omega_c t + \theta(t) \right] \\ E(t) &= \sqrt{1 + K_f^2 a^2(t)} \\ \theta(t) &= \tan^{-1} \left[K_f a(t) \right] \end{aligned}$$

Bandpass Limiter

Let us say so I FMT which is a $\cos \omega CT$ sorry $-kf$ at $\sin \omega CT$ right so I have this what is happening this can be written as a sinusoidal I can write it as $t \cos \omega CT + \text{some } \theta T$ what is $E t$ that is actually or I can write it as a $E t$ so then $E t$ should be $1 + KF^2 A^2 t$ right because I am just trying to represent it as a sinusoidal okay, so accordingly we can put that and this is what we get because I can put that as \sin this as one as $\cos \theta T$ and then you manipulate it immediately we can also see that θT will be just $\tan^{-1} K$ into $a t$.

So this model means this particular representation we already know ok so from here to here if we wish to represent immediately a T and θT can be represented as this in FM what we expected was something like $A \cos$ some phase variation right, so the amplitude remains the same whereas in the narrowband FM of course there is a variation with respect to T of the amplitude this is what we are getting and the θT what we expected suppose if it is FM then generally θT should be just this portion because θT is $\omega C T$ plus this θT .

If it is original FM this should be K_F into integration $m \alpha d\alpha$ that is actually $a(t)$ integration of that from $-\infty$ to T so K_F into a T you should be expecting inside the phase right because we want a frequency which is proportional to the message signal whereas phase will be integration of that right, so this I expected but what I am getting is instead of getting it as K_f at I am getting \tan^{-1} of that as long as you can see already as long as K_f at small enough than universe of θ will be equal to θ .

So I have equivalent representation and as long as K_f at is much smaller than one I can actually make it as one okay but what we can also see from here means however small this is if this is not sufficiently small then there is a time variation of the amplitude however small it is there will be okay so first task when I am generating FM because I know whatever the frequency variation is keeping that intact can I make this amplitude constant so that the first thing because that that should be the way FM is modulated okay, so FM modulation whenever I do that amplitude must not vary with time here we have a variation.

So we should be somehow employ something some circuitry which actually keeps that amplitude fixed while the it will not touch the phase, phase will remain the same what is the process of doing that, so whatever amplitude variation we have so suppose FM modulated signal it might look like this something like that okay suppose it looks like this okay, so there is a slight

amplitude variation which should be generally constant that should have happened well that is not happening.

Let us say if this is the case what I can do I can pass it through a special circuit which is called band pass limiter what is this is first a hard limiter followed by a band pass filter okay, so let us try to understand what is hard limiter, so whenever that kind of signal is there so suppose I have this.

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$$v_i(t) = A(t) \cos w(t)$$

$$v_o(t) = \begin{cases} +1 & \cos \theta > 0 \\ -1 & \cos \theta \leq 0 \end{cases}$$

$$v_o(\theta) = \frac{4}{\pi} \left[\cos \theta - \frac{1}{3} \cos 3\theta + \frac{1}{5} \cos 5\theta - \dots \right]$$

$$v_o \left[\omega_c t + k_f \int_{-\infty}^t m(\tau) d\tau \right]$$

$$= \frac{4}{\pi} \left[\cos \left(\omega_c t + k_f \int_{-\infty}^t m(\tau) d\tau \right) - \frac{1}{3} \cos \left(3\omega_c t + 3k_f \int_{-\infty}^t m(\tau) d\tau \right) - \dots \right]$$

I have a which is some at cause let us say some θ T okay I am just trying to write that there is a amplitude variation there is a θ variation okay, so what it will do it will from this input it will generate an output V_{ot} which is something like this it will be plus one or minus one if this $\text{Cos } \theta$ whatever that is that might be a variation of time and all those things I do not care as long as this $\text{Cos } \theta$ is greater than 0 and so this is the hard limiter that means I had this signal, so whenever it is above 0 I must have plus one over there and whenever it is below 0 I must have -1 over there.

So this should correspond okay so it is just almost keeping the frequency variation intact but amplitude it is because it is just making it plus one or minus one so there is nothing in between so it is just making the amplitude constant okay so that is called the hard limiter, so this is this is the output of that so whenever $\cos \theta$ will be greater than 0 I have +1 whenever $\cos \theta$ is less than 0 I will have -1 so this is what will be the output okay, so after passing it through a hard limiter with respect to now of course there is a θ variation.

But we can ignore that if as long as we plot we are plotting it with respect to θ we can just try to see that what will be this output okay, so this V output if we say that is a function of θ how does that look like, so it is like this it depends on that $\cos \theta$ $\cos \theta$ greater than one sorry greater than 0 it is plus one and less than 0 it is -1 so with respect to θ if I plot so it will be just with respect to θ it is a square pulse with alternate plus and minus right with respect to θ if θ has some other variation.

I do not know because I am just plotting it with respect to θ how either varies with time I am not plotting it with time I am now just plotting it with θ so whatever the value of θ it will just look like a square pulse having values plus and minus, so that we have already represented if you just think about that square pulse we have already represented in Fourier series of course we are saying that it is with respect to T, now we are doing it with respect to θ so if I just do that if you remember that we have already derived that so that should be $\cos \theta - 1$ third all our terms will be there or all will be there $\cos 3\theta + 1/5 \cos 5\theta - \dots$ up to ∞ .

So this representation we have already done that in our Fourier series analysis okay, so as long as I have θ V output θ must be represented as this because that with respect to theta it looks like this something like this okay this is fine now I can write what is this θ this θT is actually the overall FM modulated thing plus the carrier.

Part okay so let us approximate that this is actually $\omega_{C_T} +$ let us say K_F integration minus ω to 2 T m α $d\alpha$ I can write into this way okay of course I do not have exactly this I have some Tan inverse of this one okay Tan inverse of K_F into at I I do have that but whether I have that I do not have that let us say that probably this is the case I can instead of this I can always write it is as Tan inverse of K_F into this which is at actually - ∞ to $T M \alpha D \alpha i$ even can replace this particular part as this rest of the part will be remaining same ω_{C_T} okay, so this whole thing I can now write as 4 by π .

Now cause of this so which is ω CT plus either I write it KF into this or Tan inverse whichever let us take this one it is just easier to write as long as they are similar and then $-1/3 \cos$ I will get three ω CT plus $3 K_F$ same thing and so on now all I have to do I have told that my hard limiter this is the hard limiter output I have to pass it through a band pass filter which matches with the FM frequency and the center frequency is ωC then what will happen all these terms will be neglected so whatever I will be getting.

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$$= \frac{4A}{\pi} \cos \left[\omega_c t + \tan^{-1} \left[K_f \int_{-\infty}^t m(\alpha) d\alpha \right] \right]$$

Indirect Method of FM Modulation

$$y(t) = a_2 x^2(t) \quad x(t) = \cos \left[\omega_c t + K_f \int_{-\infty}^t m(\alpha) d\alpha \right]$$

$$y(t) = a_2 \cos^2 \left[\omega_c t + K_f \int_{-\infty}^t m(\alpha) d\alpha \right]$$

$$= \frac{1}{2} a_2 + \frac{1}{2} a_2 \cos \left[2\omega_c t + \underbrace{(2K_f)}_{2 \tan^{-1} [K_f]} \int_{-\infty}^t m(\alpha) d\alpha \right]$$

That will be nothing but 4 by $\pi \cos \omega$ CT plus in actual I will get tan inverse K_F integration $-\infty$ to $TM \alpha d\alpha$ this is what I will be getting okay, so that will be after passing it through a hard limiter and band pass filter, so basically what I will do I will generate a narrow band of him then I wish to make the amplitude before launching it I wish to make the amplitude constant, so that is what I have done so here you can see that amplitude variation is gone okay so whatever the amplitude variation it was having that is all one I just have 4π and of course probably that a factor will be still there.

That a will be still there because that is that was already there in that okay, so a factor will be there and then I have cost this one as long as Tan inverse data is equal to theta I have FM modulated signal okay which will be happening if this is small enough so whenever we are generating narrowband FM generally it is augmented by this that band pass limiter okay the

reason is I can very easily do that I will always have narrowband FM whenever I generate I know there will be a frequency deviation which is a little bit non-ideal of course but as long as my K_F is adjusted.

That is correct but there will be also a amplitude deviation which should not be there and I have a very simple circuitry the way I have demonstrated just now just now you can just employ that and you can make the amplitude fixed atleast and there might be any variation of this frequency that's all right you will also see if the amplitude variation is there what is the consequence of that will we will demonstrate that also why we are doing that that will be more clearer but this is something you can do and that is what the modulator chain will do you will have first narrowband FM generator.

Followed by a band pass limiter and then you launch that signal okay so this is the way narrowband FM can be generated now let us try to see historically what Armstrong did he was actually taking that narrowband FM generated signal and then trying to make it wideband FM okay so let us try to see or appreciate how he has done that okay, so that will be our first exercise okay so what he has done so he has employed a method called indirect method this is of course we are targeting the FM modulation and doing it indirectly that means we will first generate narrowband FM from there.

We will convert it to wideband FM not directly generating it okay we will also see what is what we mean by direct generation this is the indirect method in the indirect method what he started doing is started exploring some of the nonlinear devices okay, let us see what happens so suppose I have a quadratic response nonlinear device, so that means if I have input x so the output will be this with this diodes transistor you can you can realize this with a particular biasing.

You can actually get a region where it is nonlinear okay so you can do that so therefore suppose I give you a narrowband FM signal, so let us say if I modulated signal the corresponding Y_T should be like this a to my x_t is $\cos \omega C_T$ plus ideally it should be \tan^{-1} that I am just writing as long as it is narrowband we can even write K_F okay, so that should be my x_t now I put it over here okay, so what will happen I will have square so it should be $\cos^2 \omega C_T + K_F$ so immediately what I can do because it is \cos^2 I can have two terms.

So one will be half a^2 plus there will be another term which is half $a^2 \cos^2 a^2$ of this so I will get $2 \omega CT + 2 K_F$, now he will be able to appreciate why he was doing that so what is happening there is a two factor which has come over here that something probably was not desired but we will discuss about that but what was desired was there is a two factor which has come over here okay why this is desired because, now we are actually going towards from narrowband FM to wideband of him that means the frequency deviation has to be increased.

So what has happened this will effectively increase the K_F and instead of $f K_f$ it will become $2 K$, so that means the deviation has become double whatever deviation I was having earlier in the narrowband FM that is becoming double okay, so it will keep on happening and the good part is if it is \tan^{-1} the two will be outside remember that, so this two remains outside and then you will have this \tan^{-1} this K_F into this part okay so inside \tan^{-1} it is still that narrowband of him, so you can still approximate that to be K_F into minus this sort of cave into integration $-\infty$ to the $m \alpha D \alpha$ right, so this particular part remains intact you are just multiplying actually were supplying terms multiplication multiplicative terms to K_F . So you are slowly increasing that K_F portion without hampering what was happening or what has been appreciated in nerve and effect okay, so this was his method what he did instead of doing this he went directly to in an eighth order non-linearity.

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$$\begin{aligned}
 y(t) &= a_0 + a_1 x(t) + a_2 x^2(t) + \dots + a_n x^n(t) \\
 &= C_0 + C_1 \cos[\omega_c t + \tan^{-1}(k_f a(t))] \\
 &\quad + C_2 \cos[2\omega_c t + 2 \tan^{-1}(k_f a(t))] \\
 &\quad + \dots + C_n \cos[n\omega_c t + n \tan^{-1}(k_f a(t))]
 \end{aligned}$$

ω_c
 A_f $n A_f$

So basically he told can I if I can get a circuitry like this suppose it is a 0 a1 xt so it has a net order non-linearity and a 2 X² T we have just taken this term as if we have assumed a 0 a1 and all other terms are 0 okay, so immediately if we just put it as whatever modulation we had what we will get and then rearranging them there will be two of that whole θT three of that θ T four of that θ T rearranging all those things I can just write it as C 0 + C 1 into Cos of course the C0 C1 will be some there will be some dependency with respect to a0 a1 a2 all those things okay so they can be means that can be a corresponding equations.

So that will be Cos ω CT plus of course it should be general ideally tan inverse so I can I can just write it that way it is just Tan inverse K_f let us write a(t) on the just for the brevity of it okay, so tan inverse this plus some c2 into Cos 2 ωCT+ 2 sorry the K_F should be there no that is fine okay so this is that then inverse K_f at right and then up to C_n Cos n ωC T plus n into Tan inverse K_F into a T right this is what will be happening now what I do I need a FM which has n times deviation .

So I will put a filter around this so at n ω C instead of putting it at Omega C I will be putting a filter around n ω C with a prop appropriate FM bandwidth because, now the FM bandwidth will be recalculate because it has n times deviation okay, so if RDR with narrowband FM it was δF now it will be n δF okay so earlier if this was the left, now it should be n into δ F so if I just put a filter over there I will be getting just this term so that is actually a FM modulated signal but what

has happened see this \tan inverse this one can still be approximated as K_F into at because the original K_F .

Into at that remains still small I can write that as K_F at only thing is that that gets n times multiplied so overall I get a wider band FN okay this is the best part of it okay so that is what Armstrong did in a very ingenious way and this particular technique is called as frequency multiplier it is an indirect method and you can see already it is doing nothing but multiplication of frequency it is generating that so it is generating your FM modulated signal at $n \omega C$ okay with a deviation which is n times the deviation of the corresponding narrowband FM okay.

So with that understanding we will try to see what are the shortcomings of this thing okay so that is something we will try to analyze in the next phase or next class and we will also try to see exactly in a practical scenario how do you design a FM transceiver all this multiplicative factor how we take into account if we have a target at frequency where the FM has to be put that means some ωC is targeted how do we actually manipulate that because here something is not in our hand we will explain that in the next class okay thank you.