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NPTEL ONLINE CERTIFICATION COURSE

Course
on
Analog Communication

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Lecture 05: Fourier Series (Contd.)

Okay so far what we have done is, we have seen a vector analogy and try to put that into signal space okay. so we are now targeting a particular signal must be represented by another signal and from the vector analogy we also know that in the signal space, if we can now, we also now know what is the characterization of two signals. So in a particular signal space if we can find out all the set of orthogonal signal, then we know that exactly a particular signal can be represented in linear combination of that entire orthogonal signal.

This is something that we have already understood and there coefficients also how to calculate that also characterized now okay. So we know that exactly how to evaluate those optimal c which minimizes the whole error and if I have the complete definition, that is called the completeness of basis set. So if I have complete definition of the entire basis set that are possible in that particular signal space, I will be able to represent a signal. So this Fourier, this was the background that Fourier started, means analyzing signal and this is how he started things.

So now, our target should be that how do we represent, or how do we find out the entire orthogonal signal in a particular signal space, so initial Fourier started with trigonometry signal.

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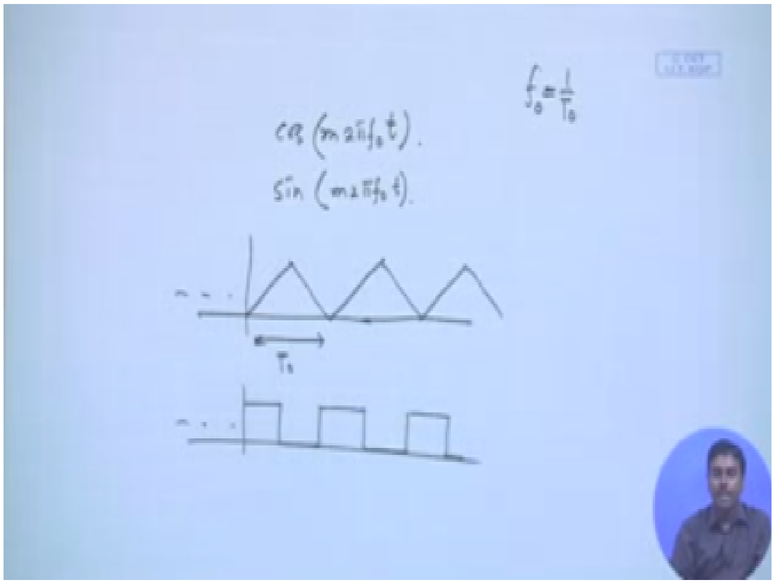

So initially he started with let say we take this Cos $m2\pi f_0t$ okay so he started with this and correspondingly I can also define Sin $m2\pi f_0t$ okay, so these two signal we are targeting so whenever m = 1, so it is just Cosine signal with frequency f_0, where f_0 is = $1/T_0$ okay, so T_0 is the time period of that signal. So whenever we are talking about Cos signal or Sin signal it is periodic signal we know that and the thing is that is this, what are the constitute signals at the periodic and what it will represent must also be a periodic signal.

So what initially Fourier devised, okay I can represent signals but all those signals must be periodic signals that I am targeting to represent, that can be any signal it might be a triangular something like this, let say this kind of thing but it must be periodic. Let say P_0, it must be a pulse and off course it stretch. So it can be any periodic signal and hidden as started saying can we represent this?

In a particular fashion, then he started enquiring the basic signal that we know. Even the electromagnetic waves that propagates, so can be represent all this signals with respect to this cosine but before doing that we need to know the orthogonal signals in that signals space, so what he started doing.

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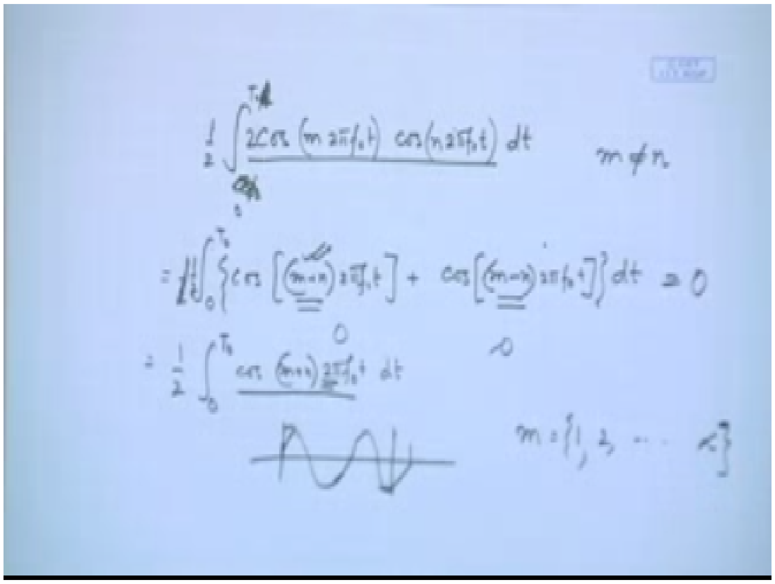

He started enquiring whether 2 Cosine sinusoidal, cos m2πf_0t for different values of m okay, so let us say 1 is cos m2πf_0t and another is cos n2πf_0t, whether these two signals are orthogonal or not. So if they need to be orthogonal what we have what prove, it is a real signal cos, is a real signal, so all we need to prove is and it is the periodic signal, so within one period if the properties is true, it will be true for all other period. It will be just the making the similar pattern.

So orthogonal means the integration must be 0, there multiplication and integration must be 0, that is what we have understood g and x will be orthogonal if gt x xt dt over a particular time period must be 0. So if within 1 period it is 0, the other period will also will be behaving similarly. So entire period or entire signal duration it should be 0. So this is what we wished to prove, is it true.

So we will probably do it from let us say +t_0/2 we can even do it from 0 to t_0 any periodical, we can do it from 0 to t_0, so over a single period we are trying to evaluate what would be happening if we do this integration because we know that, that is the prove of orthogonal, if this is 0 for two different signals off course, if it is same signal then we should not, we should get the energy back right. so if it is two different signals that means two different values of m and n, for this condition we are trying to see what will be the value okay.

So let see let us try to evaluate this, take out ½ it should be, now just put a trigonometry form to cosine into cosine, so that should be cos a + b+ cos a-b, so it should be cos m +n2πf_0t + cos m –n we are assuming that off course cos – that will be again become positive so, here either m is bigger than n or n is bigger than m it does not matter, it all will be same. Now let see the first integration 0 t_0 cos okay. So if m is ≠ n then both these are num 0.

So what is happening this is another cosine and whatever the f_0 is m + n or m – n it will be either 1 or bigger than 1, so over that period t0 if I integrate a cosine sinusoidal which is, see already 2π is there that means either it will just make the full cycle or it will make multiple full cycle and across the signal, if it completes a full cycle or multiples a full cycle always the integration what will happen, sorry okay this, always the integration will be 0?

As long as it is cosine sinusoidal signal and we know that it is either completing a full cycle, if the difference is, let say this addition will be more than full, it will be 2, 3, 4 or some integral number. So multiples of full cycle it is completing, so if I integrate that should be 0 definitely, so this part will be 0, even this part will also be 0 because, it is either it will be 1 full cycle because m0 = n, so m-n at least it will 1 and it might be even bigger than that.

So whatever it is, it is either a full cycle or multiples of full cycle, if I integrate again it will be 0, so the overall thing will be 0 if m0 = n. so therefore we know that all the cosine sinusoidal signal, with all the values of m going from 1 to infinite all orthogonal to each other. That is very important findings, so he could find in that signal space in the vector space if you remember when we had this page two dimensional, we had only 2 orthogonal vector.

Now we actually have infinite orthogonal vector right, any value of m you put, so m starting from integral value off course, starting from 1 and then going upto infinity, any value m or n you put, as long as m≠ n, all of them are actually orthogonal to each other. And these are actually called the harmonics. So you have a cosine sinusoidal, you have twice frequency cosine sinusoidal, 3 and so on. All those frequencies are all those harmonics 1^{st}, 2^{nd}, 3^{rd}, 4^{th}, are included and they are actually orthogonal to each other. The second part is, if I have a dc value which is just 1.

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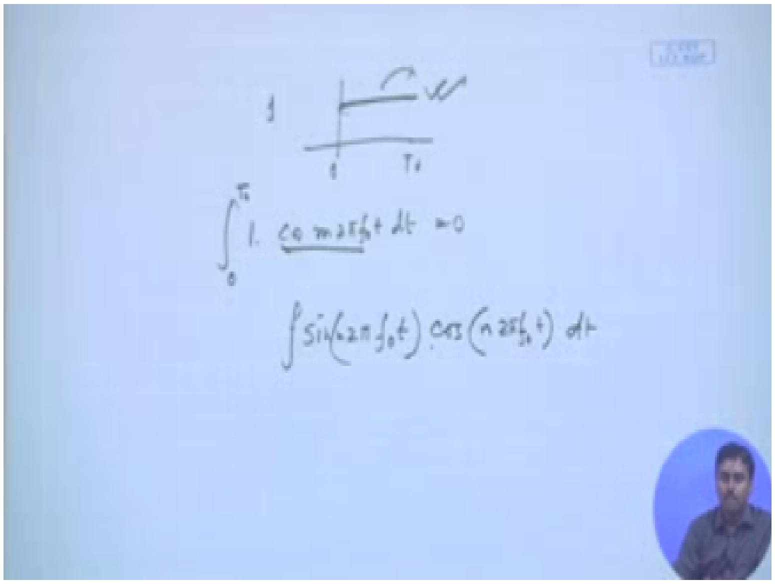

So between that 0 to t_0 whatever I have talked about, so it just takes the value 1 okay. If this is the case will it be orthogonal to all those cos, off course because what will happen, this as to be multiplied with all those cos and again it will be integrated, so if I just do the ∫ 1 x cos m2πf_ot dt and ∫ 0 to t_0 right, so again it is cosine sinusoidal with m taking value 1 to infinite. So therefore this is actually because the 2π is already there and f_o = 1/t0, so basically this is either a full period or some multiples of full period.

So it will others be again integrated to 0, so even this particular signals where it is just taking a constant dc value that is also a orthogonal to all those cosine sinusoidal, so the dc value which for a single period it takes same value so I consider all the periods it will just take a dc value, so the dc value even is orthogonal to all cosine sinusoidal right. So what we have already demonstrated.

And the next part is, this is also orthogonal to off course the dc as well as all those cos values, will now again show that if I multiply with some cos let say n2πf_ot for any value of m and n. now we do not restriction because even if m and n are equal these two are two different signals okay. So if I see this do integration with the same logic it will again become some cos or sin and then within again either full period or multiples of full period, the integration as to be done and it will always be 0.

So therefore what we have done is we could identify now few mutually orthogonal signal and I should not say few it is infinite okay. so all the cos m2πf_ot for all values of m positive integrals, all the sign to $n2\pi f_o t$ and dc value. These are all, infinite numbers of signals, are all mutually orthogonal to each other,. The other part which I am not covering over here which is little bit more involved, that if I take this whole set that is the complete set of this basis.

Like 2 dimensional vector we were saying, x and y that makes it complete because only with those two vectors I can represent any vectors. In this signal space for periodic signal we can actually prove, which will be little bit more involved, so we are just giving the outcome that if I just take consider these basis set, this actually make a complete basis set.

These are all the basis set that can be defined in this particular representation okay, so if I consider all of them I could actually finish representing all of them and any periodic signal therefore should be represent any periodic signal with respect to these functions therefore okay. So what should happen then, I know that now my.

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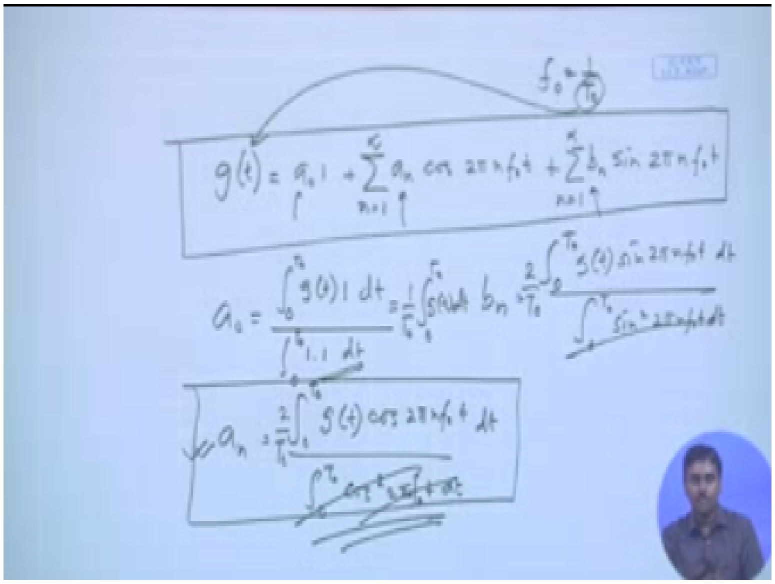

Any periodic signal gt I should be able to represent them linear combination with the linear combination of this entire basis functions right. so therefore I can write 1^{st} is dc, so a_0 x 1+ sum x cos $2\pi nf_ot$ where f_0 is actually 1/t0 and this t0 is the period of this signal okay and I have to take because all the basis has to taken, so I have to take a summation over all possible cause or in thing or all possible values of n + the sinusoidal is there, so dn sin $2\pi nf_ot$ that is the famous Fourier series representation.

So because we have proven, that these are basis set which are mutually orthogonal to each other and because we are also stated that I can get in that signal space, so this is actually a complete definition of the basis set. So therefore without error I should be able to represent gt as the linear combination of all these basis set, where the coefficient of linear combination are this a_n and b_n oaky and how do you evaluate these values? Let say a_0 how we should evaluate that.

This is something that we have already seen; the evaluation of a_0 should be with that dot product right. so all we have to do is gt with respect to odd whatever signal is, therefore 1 dt must be integrated over a period 0 to t0 and this 1 should be also multiplied with itself, so 1 x 1 dt from 0 to t0. This has to be done; as long as we are doing this we will be able to represent the whole thing right. So a_0 is like this similarly a_n must be whatever the gt is we multiply that with cos $2\pi nf_ot$, that is the Fourier coefficient if you go back and try to see whatever you have learn in Fourier series.

This is actually that is something we have already proven, this is the optimal a_n or representing that signal okay. So similarly b_n also will be we can get the formula 0 to t_0 it can be - $t_0/2$ whatever it is it should be that single period okay, so the result will be same. so again gt this time it should be $\sin 2\pi nf_ot$ okay, so in your Fourier transform you might have seen that this happens to be iff we do this integration it is just $1/t_0$, so a_0 becomes that is the whole formula right.

This if we do the integration it should be $t_0/2$ if you just do that integration there will be a ½ coming out and 2 going in and if you do that integration it will just find 1. So this should be 2/t, so therefore this thing goes away you get 2/ t_0 same things happens over here. So that is the famous flourier series and Fourier coefficient calculation which we could direct now proves. This is very clear to everybody, so once you have done this there is another representation, probably whenever we.

This was all fine, we know that any signal now we can actually represent with respect to corresponding sinusoidal. So what is actually happening all thee sinusoidal harmonics are coming into the picture, so any sinusoidal it is nothing but a infinite summation of different harmonics nothing else. Any signal completely is represented if you can evaluate through this, the appropriate coefficients will know exactly how to combine the coefficient appropriate will know exactly how linearly you have to combine those harmonics to get this particular.

So that is a very nice representation you actually now that, which sinusoidal with what strength you have to take and you have to add them to get this particular signal. In a way it is also giving us a strong to tool, you get a signal you immediately know what are the constitutional sinusoidal with what kind of strength should be present in that particular segment okay. So whenever start talking about the sinusoidal that it means; now we are talking what frequency components are actually there in this particular segment.

This is where the signal gets represented in the frequency domain okay, so if we guessed one example probably, we just represent this particular signal, so that the signal is something like this

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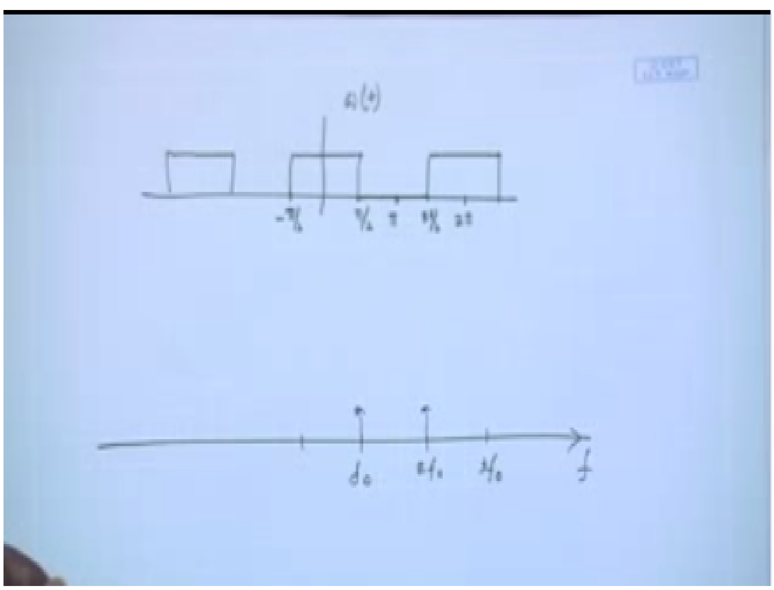

Let say it is ωt it defines from –π/2 +π/2 it is like that and then again it is π/2 starts again and so on. If I represent a signal like this which is the periodic signal and then if I have the way we have learnt the coefficient. If we just calculate that we can see all the Fourier co efficiency and then we can start representing this signal with respect to frequency component, so right now I will give the example for this because we still have not equipped our self fully with the frequency component.

But what will do we will take this example and come back after little bit of more analysis and come back and draw actually the frequency spectrum. We will see that what are the harmonics which are present and how they are present and accordingly we can actually draw the spectrum of this signal, an another representation not in time domain but in frequency domain. So basically we will be saying the representation will have like this, so f_0, 2 f_0, 3 f_0 what are the components have.

So it will just give another representation in frequency domain, so here it is just the frequency is increasing and whatever are the components of those frequencies which represent that we will be able to characterize. So this how Fourier series actually gives us the frequency domain representation of the segment. It is nothing it is just, it says for a periodic signal what are the whenever we say the frequency component we are actually saying what are the basis that it has.

So all those frequency component whenever we talk about, we plot the frequency domain, we are actually saying that there is a sinusoidal. So frequency domain representation is nothing but every frequency component we pick actually there is a equivalent sinusoidal that actually makes the signal. So you take of that strength sinusoidal you add all of them you actually get the signal back. So therefore there is equal representation, in time you see also in frequency domain we start seeing the similar representation of that domain.

It is just thanks to the Fourier, he could actually give the extra representation of the signal which you will see that it will help us many ways for the processing. So before going into that let us try to see another representation which is also Fourier representation, with respect to real signal, so right now whatever we have constructed these are all sinusoidal or dc value. So these all are real signal, now what we will do, we take a signal which is real signal gt.

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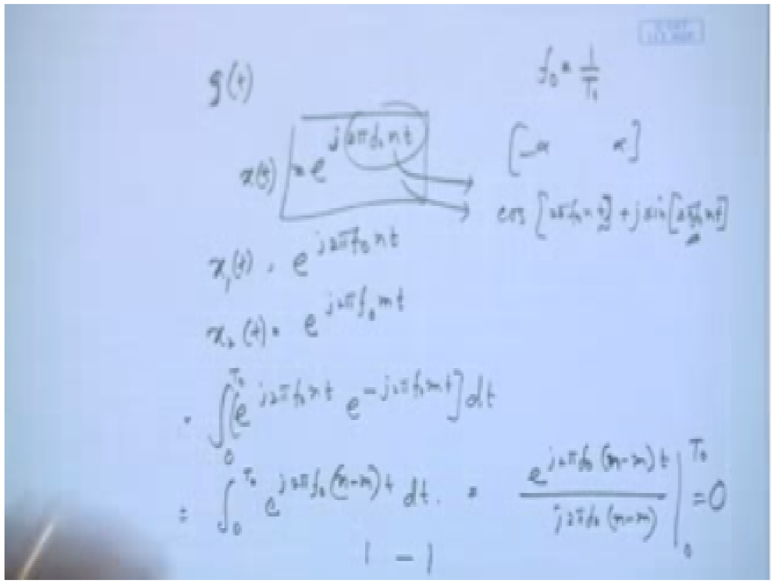

We will try now to representation with respect some complex okay, so first of all we will try to define the family of complex signal which are mutually optimal to each other, then we will say what is the again, then say what is the overall basis set. So that particular thing is represented has exponential complex signals. So it is like $e^{j2\pi f0nt}$ so this is test xt okay, so any xt is taking this value where n now can take integer but it can positive and negative.

So n now can take any value from – or + okay. Now let us first try to test whether these infinite numbers of signals we have got are they orthogonal to each other or not, that is something that you have proved. The signal is a complex signals because we have this complex part already, sop 1st we have proved that this is orthogonal or not, now we have implied the complex equality. So that means we take xt or x1t $e^{j2\pi f0nt}$ and we take another x2t $e^{j2\pi f0nt}$. Now all we have to prove is if m ≠n then should we get the = 0.

That is actually x1t x x2t start, therefore it should be $e^{j2\pi f0nt}$ x $e^{-j2\pi f0nt}$ and if you carefully see this is the complex signal that is also a periodic signal because this signal can be represented as cos this particular factor, so therefore again the fundamental frequency is related to this f_0 which is =

$1/t_0$, so the integration needs to be just done over the fundamental typing, we do not have to do that whole integration, so same technique will be applying 0 to t_0 the same signal.

So now this is let us try to evaluate this, so it should be 0 to t_0 $e^{j2\pi f0nt}$ right, so now this integration how much it should be $e^{j2\pi f0nt}$ / $j^{j2\pi f0n\text{-}10t}$. So now if we just t0 is 1/f0 right so this cancelled n ≠m right, so this cannot be 0 right, so this must be some positive integer okay. It is just $e^{j2\pi f0nt}$ into some integer that we can represent as cos and sin. Now cos 2π integer, so what do we get if I just represent this it should be2π that should be 1 right, so we get 1 for t_0 and for sin it is 0, so the j part is cancelled – I have to put this limit, so again I put 0, it should be 1 and sin it will be 0. So 1-1 gets cancelled so this becomes 0.

So whenever m ≠ n it is always 0, id n =m this becomes already 1, so we know that this particular things are all orthogonal as long as m ≠ n, that off course with himself it cannot be orthogonal, so with all other signals it is mutually orthogonal to each other. So this is something we now have proven, any value of m and n you take as long as m ≠ n we could from the basic principle of the orthogonality of the complex signal mutually orthogonal to each other.

And again we are saying we have proved we have sating that these are the all the signals that we required to represent all the entire resistance. So as long as we are taking that to be true we know that this xt can be represented with these things.

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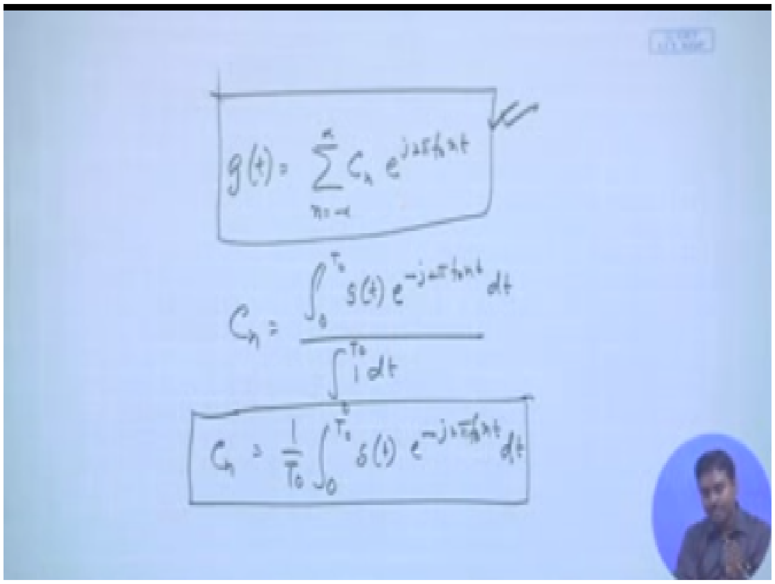

So now what we have to do is this gt ∑ some coefficient Cn e$^{j2\pi f0nt}$ and n goes from – to + infinity, that is famous Fourier series representation, where tht Cn must be evaluated with the same criteria of complex, so that must be integrated over that period gt x complex conjured of x, so that is why it becomes e-$^{j2\pi f0nt}$ / the this complex conjured, so they will cancel each other it will be just 1dt e-$^{j2\pi f0nt}$ this is the reason why the inverse 1 or the coefficient one whenever you calculate you have put e-$^{j2\pi f0nt}$, and whenever you are actually representing gt you get e+$^{j2\pi f0nt.}$

Later on from this series will go to Fourier transform and that is why Fourier transform always get e+$^{j2\pi f0nt.}$and inverse transform always get – or vice versa okay. so this happens due to signal, so no what we have seen that we can represent any signal again with respect to another basis set which are complex. In next what we will try to do is, we will try to get a relationship between these two representations. What the exceptional Fourier series it tells us and what the trigonometry tells us and what is the relationship between these two.