NPTEL

NPTEL ONLINE certification course

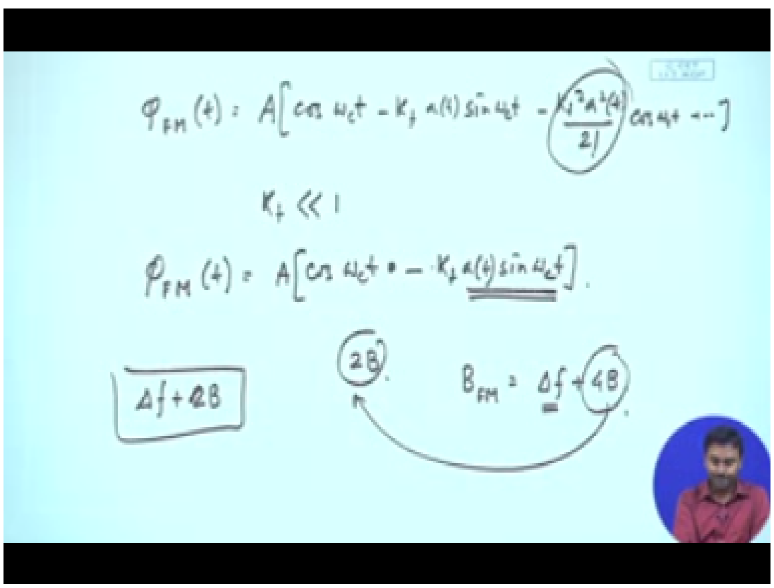
Course
On
Analog Communications

by
Prof. Goutam Das
G S Sanyal School of Telecommunication
Indian Institute of Technology Kanpur

Lecture 49: Frequency Modulation (Contd.)

Okay so in the last class probably we have already started discussing about FM bandwidth so let us just explore a little bit more on FM bandwidth but before that for our own advantage we would like to give some intuitive understanding of narrowband FM you remember.

(Refer Slide Time: 00:45)



We have derived this formula that ϕ_{fm}(t)that was actually A[cos ω_{ct}-kfa(t)sinw_ct-kf2a2(t)/2coswt..∞] what we have said that initially people started saying that we want to have a very small deviation and that will create a narrowband FM at that time people who are thinking that narrow band FM means it is almost zero bandwidth okay so that Δ F will be zero. so with that understanding we say that K F<<1 very small value of K immediately what happens all these higher terms actually goes away square to cube - up to ∞ goes away and the ϕ_{fm} becomes this which is almost like AM okay though we are calling it FM it almost means the functionalities almost looks like FM okay.

So we have a carrier term over here which is this and then we have just 80 x sine ωct it was 4 a.m. it was $\cos\omega_{ct}$ and there was a plus sign just that difference otherwise it looks like a AM okay so earlier that was our amplitude modulation right how on earth this is becoming a frequency modulation this is something comes to our mind every time we ask this but remember here also the amplitude variation will be very restricted because the K value is very small okay so a multiply it by this KF into 80.

So whatever that 80 variation is it will be very small it will not be very significant so the amplitude variation as we can see it will be restricted okay and because we have got it from the FM formula so this will also have corresponding frequency variation will later on when we will be drawing FM phase diagram probably you will understand this right now we are not means we are not trying to prove that part but this faithfully represents that above 1 and that was according to us FM signal so we know that at least we could prove that this does not have too much of amplitude variation that is for sure okay.

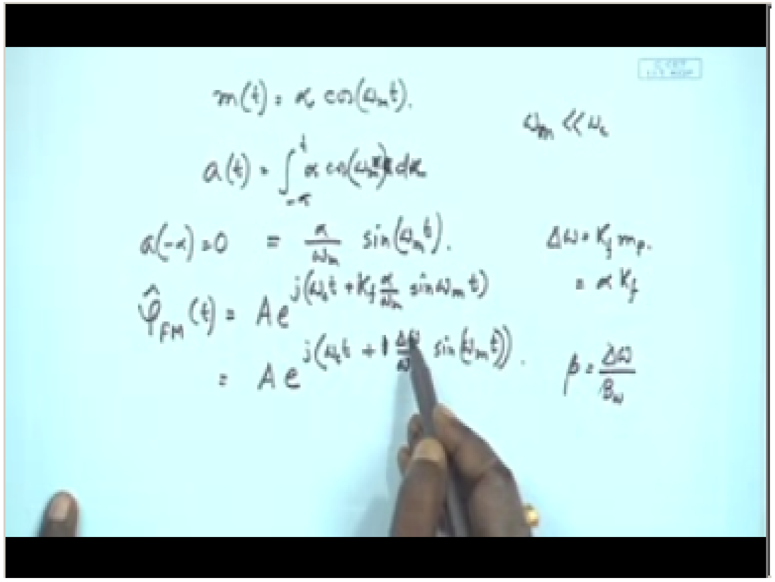
So that is something but there will be some amplitude variation once we start truncating it that all those higher signal actually they were balancing those amplitude variation so once we target truncate that however small there will be some amplitude variation we will also see when narrowband FM will be generating how do we really control that part so that something also we will see but this is what happens at this instance if I now talk about because we were talking about this narrowband FM we have just demonstrated because we wanted to say talk about bandwidth so we were talking about bandwidth.

So at this intense if this is the FM earlier understanding was narrowband FM must have frequency deviation almost zero so the overall bandwidth should be zero but this is coming out to be narrowband FM so what's the bandwidth of this, this has this modulation term so it must be having a bandwidth of two B if B is the bandwidth of that so immediately I can see if I just make narrowband FM that means ΔF tending towards zero or very low value of K F then this particular function tells me the bandwidth of that FM must be 2b but what was our derived formula.

So β FM in frequency in F domain not ω domain angular frequency domain so that was ΔF+ 4b right now make ΔF tends to 0 so this ad hoc formula tells me this is 4B but from here direct from FM I can get that this must be to be right so there is some fallacy in this still but what I can understand because this was a DA formula so I should not believe in this formula directly I know that this formula should be almost correct but it should not completely believe on this formula will till I derive some very accurate kind of formula but with this ad hoc formula I can say that probably the overall bandwidth because what we were doing we were wherever the means that sink function goes to zero we are taking up to that.

So maybe that is over estimation because I am almost going towards zero and then after that all the other ripples will be smaller and the ripples that are coming from internal sink this is just the boundary sink internal sink there will be even smaller okay so probably we are overestimating it so it should be intuitively we can say because this must be to be whenever I put ΔF tends to zero so in Duty B I can say it must be this is the upper bound of that so it must be something like this we will come back to this okay so this must be the FM bandwidth okay let us try to see how we can deal with that okay.

(Refer Slide Time: 06:50)



So to deal with that let us try to define one thing which is called frequency deviation rate ratio it directly comes from that particular formula where we are saying that probably this FM bandwidth should be this ΔF+B wait a second what is my derivation correct Δ F+ 4b yes so we are saying ΔF+4b right okay so this particular deviation I am taking this sometimes you will see this derivation means this ΔF I have taken it from the entire deviation so it sometimes taken from zero how much it varies in the positive and negative form so that way they write also to Δ F so according to the formula gets modified okay.

So if I write that way then it becomes 2Δ F + 4 B okay if the delay definition is not the entire deviation it's just from the center that ωc how much in the positive half and how much in the negative half it goes so if I just take it that way so it becomes this or we can write - Δ f means 2 into Δ F + 2 Band we have said that it maybe not for me it may be too big so with that approximation b FM must be - Δ F + B okay so one of this formula will be correct we do not know right now we have derived this one but we are saying that probably from narrowband FM this should be the correct one okay.

Now whatever it is the derivation ratio is defined as this I take this B out then I will have this thing ΔF divided by B plus one this particular thing is called termed as deviation Radio ratio okay this is actually tells us how much deviation I get in frequency divided by what is the bandwidth of the original message signal so that ratio is defined as a term called β okay in our next derivation probably this beta will be useful so this is actually the deviation ratio that we are talking about so now let us try to do this okay suppose I have a message signal which is actually a tone modulated signal or which is a tone or which is a single frequency.

So cos ωt it so I am not now this was by the derivation of Carson’s since how he could derive FM bandwidth so we are just going through his derivation so here what we are saying and earlier we are taking any FM so any message signal you could take and then we are trying to derive the whole form so now we restrict yourself we are saying probably we are just test it for a tone modulated signal and we are do the proper derivation I think the proper bandwidth derivation so we are saying that a message signal is just a tone or a co sinusoidal at frequency ωm so this is ωm and of course omega M must be much, much smaller than ωc.

So this condition is valid okay so if this is the message signal first I have to for constructing if I might have to first construct a tokay so what will be a t that will be the integration from minus ∞ to that particular T value now if I wish to do that it will be minus in ∞ to t α cos ω MT sorry α D α or I should not say α that should be some other things so let us say it is X DX okay because α is already there so I have taken a dummy variable I just have to take a dummy variable which is X so M X DX okay I mix inside this now this integration if I wish to evaluate I already know how to evaluate that but at minus ∞ it will create problem for me.

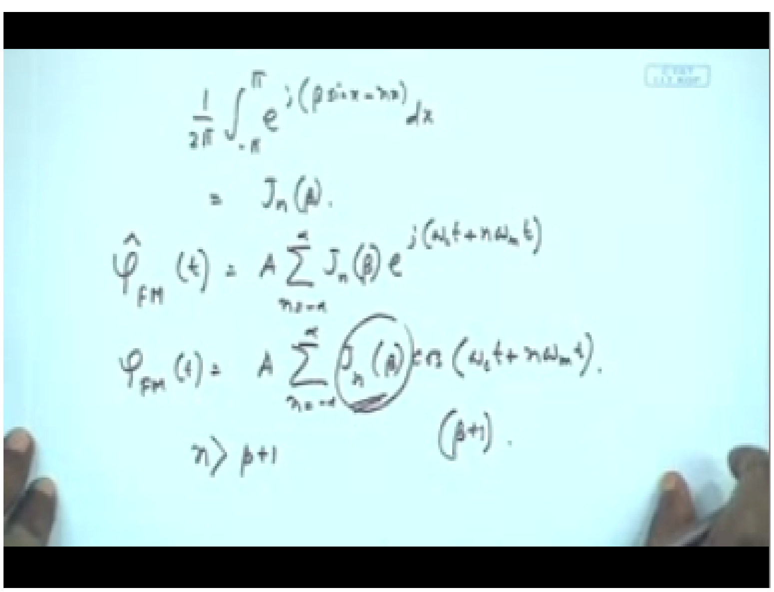
So if I just say that at minus infinity I already know that this particular thing will be means this a T will be already zero so this is something if I know means we are just assuming that at minus c ∞ we know this okay so if I just assume that at a minus ∞ I know that is zero so then immediately I can write this as integration of that and putting as T so this should be α divided by ω M sine M now in place of X I will put T because at minus ∞ I know it is zero so this is my aT right so now I wish to evaluate the bandwidth of it so what I will do I will first go to the actual FM signal so let us do that the way I have done that ψ_{fm} (t) let us try to evaluate this part.

So this is actually $Ae^{(i\omega ct+kf\alpha/\omega msin\omega mt)}$ so KF into alpha divided by ω M sine ωM T right I can write it this way now I can also write we have already derived this Δω is actually KF into MP this is something we have already derived so this I can write as α into KF because MP comes α this is empty okay what is the maximum value of this that is actually α and minimum value minus MP is just minus α so MP becomes alpha so I can write Δ ω in terms of α and KF right so this is something I can write.

So KF into α immediately I can write as Δω so I can write this as e^{-J}ω CT+ KF into α I can write as Δω/ωm sineωmt okay now what is β that deviation ratio that was actually Δ For if I represent it in terms of ω that should be Δω divided by the bandwidth in ω domain okay now for this particular message signal what is the band or bandwidth it is just a to neat ω M so the bandwidth must be that on the right so B must be ω M so I can write this P as ω m and Lω is this one.

So Δω by ωM must be my β so β Δω and B must be ω M so this becomes my ω so I can write this as a Ae^{J}ω C T+ β sine ω MT if I separate it okay so that is my five hat FM now try to see this signal because I have sinusoidal it is a periodic signal right with frequency ω or this angular frequency ωn so because it is a periodic signal what can I do my Fourier series cell it tells me that this can bere presented as a Fourier series expansion okay so that understanding tells me that.

(Refer Slide Time: 15:44)



I can write this $e^{J\beta sine\ \omega mt}$ a Fourier series expansion with coefficient DN and the frequency I know already ωm so this must be e^{Jmt} and this n varies from -∞ to +∞ okay wait a second I have not written it correctly yeah that is correct so this must be the case so basically at every ωm I will be getting this term probably that M I have not written over here okay so it must be that every frequency term I will be getting okay with respect to this and yeah it is not n it should be n.

So for every value of n I will be getting a corresponding N and I will be getting that frequency term right now this DN how do I calculate so DN becomes the Fourier integral so that should be 1/2π so that is or 1/T so the period is basically the overall period is ωm that is in angular frequency so it should be ωm/2π or the time should be 2π/ωm so it should be -π/ωm to + π/ ω M and 1/T means it should be ωm/2π and the integration goes this is the signal itself it will power $e^{J\beta sin(\omega mt)}$ right multiplied by $e^{Jn\omega mt}$ right that is the Fourier series analysis DT right.

Now just put mince pie by this T by ω M as X so immediately what you will get is this becomes 1/2π goes from -π e-$^{j\beta sin}$x e^{-jnx} dx r I can write π2π$e^{-J\beta sine}$ x dx okay this particular integral is not something that can be calculated easily okay that has to be numerically evaluated but this has a name this was my vessel and this is called Bessel function of means first means this is in eighth order actually so because this is there so it is called an eighth order Bessel function so I can write this as this integration 1 /2 integration minus π to π $e^{J\beta sine}$ X minus nxdx this can be written as J in β.

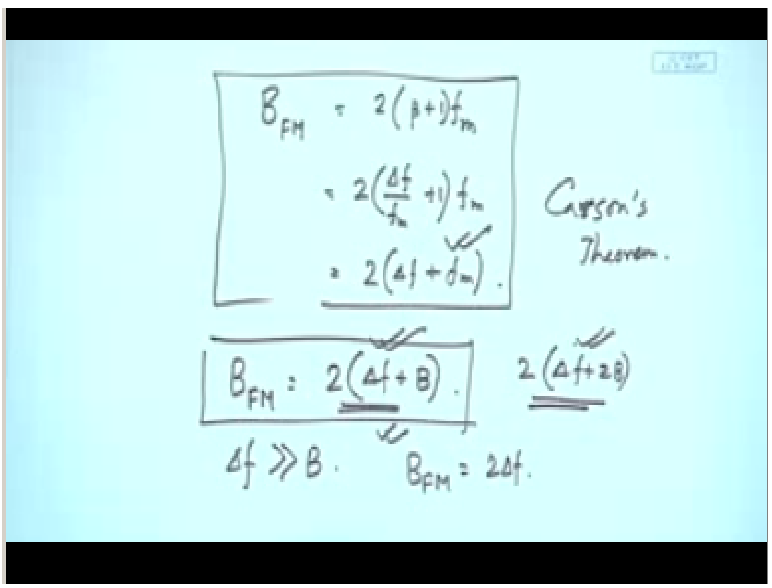
So this particular integral I can write it as J and be beta right so immediately what happens by representation that phi f m hat t what happens to that this DN can be now represented as this so immediately I can represent this whole thing as summation n goes from minus infinity to plus infinity j n β which is that coefficient and I will have e $^{j\omega ct}$ was already there multiplied and here it should be plus ωmt which is coming due to the Fourier representation the Fourier series representation so this is something what we see immediately I should take the real part of it so I will be getting this just the cost component of all of these come things so ϕ FM T I can write as a summation n equals to minus infinity plus infinity J n beta cosine of ωct + n ω M T.

So you can see that it is actually creating C at ω C I have something and then plus n minus n all infinite terms it is creating okay so this is something which is happening so again we can say probably the overall frequency is infinite okay so that is something we can immediately from here we can say because it goes from minus a filter same field all of them will be populated but if you carefully analyze this J and β it is revealed that as n goes bigger than β+1 the significance of this or the value of J and β becomes insignificant so this is something from the basis function plot for different values of β and n you can you can actually see that okay.

So for every n there is a β+1 means beyond which that n value is not really significant okay so what we have to do we have to so for every value of n the overall significant term we can actually evaluate okay so what we can say the significant sideband should be up to β+1 because n this value of n for a particular β value that β value is defined already right so β value is coming from the FM modulation Δϕ have given and the land with of this one.

So once I provide a β value I know in becoming beyond β+1 that is becoming insignificant so basically what happens to my bandwidth the overall bandwidth becomes from ω C I take β+1on this side and β+1 on this side that must be the overall band okay so I can immediately write that my FM bandwidth for this tone modulation must be 2 β+1 okay so this is something I can immediately write so this happens to be my overall FM bandwidth but this is just a number.

(Refer Slide Time: 23:17)



I still have not really accounted for band width each one of those numbers I take so this many numbers of frequency I will be taking what's the separation between them that is actually either ωm in ω domain or FM infrequency domain or F domain so if I really wish to call this a bandwidth I need to multiply this with FM because that many terms I am taking so how many overall this thing I will be taking that should be this multiplied by FM okay this FM okay or I can write that as ω M divided by 2 π right so if you start writing that way you will see that if I just write it so suppose let's keep it as FM so I multiplied this so this becomes 2 now this beta RDR we have represented with respect to the ω okay.

So β is just a ratio either I do it with respect to ω or B it will be just 2 π gets cancelled so I can even write β with respect to ΔF and corresponding FM so I can write that as they left by FM + 1 into F M right I can write this way immediately what do I get ΔF From this is the famous Carson's theorem so immediately you can see for tone modulation the overall bandwidth of FM is nothing but 2 into that Δ F which we have defined as frequency deviation due to the FM modulation itself plus the bandwidth of the message signal here it was tone modulated so FM becomes the bandwidth of it okay what he could do is to prove this only that for a tone modulated signal this will be happening.

And we have seen that for a generalized modulated system probably this will be ΔF Plus here it should be means overall for B so it must be taken as to be inside okay but what we have seen also that narrowband FM tells me that this should be plus B so that is why this formula was more accepted there was no formal proof of this because it is very difficult for any other things so there was no formal proof of FM bandwidth overall but people could understand that this must be this okay so that is why finally they could take the overall FM bandwidth was 2 into Δf +B and there was the other formula where it was 2.Δ F+2B okay.

So one of them is acceptable this was more acceptable because the it came from the tone modulation that's first thing where we did proper bandwidth calculation and second thing is this actually for the narrowband FM this defines the correct bandwidth okay where this formula does not define it correctly it gives me 4b which is not true for narrow band FM when Δ F tends to 0 okay so that is why historically this bandwidth definition was more widely accepted.

And then we could see all kinds of relationship with respect to this if my Δ F becomes much, much bigger than B the thing that people were actually saying that the frequency deviation should be the FM bandwidth that actually happens where Δ F is already that frequency it is not the narrowband part it is the wider band part where you have a huge deviation that means the K value is already big huge deviation on top of that the bandwidth that is coming from this particular thing the actual message signal bandwidth that will be insignificant so there I can say this B_{fm} is nothing but 2 into the deviation.

Okay as long as the deviation is taken from Ωc2+1/2 of whichever not the whole deviation then it will be just enough okay so that is something we have already told so all the understandings were correct but in some perspective okay people thought that it is in finite bandwidth yes it is in finite bandwidth but some of the part of that bandwidth becomes insignificant which we have seen through the Bessel function proved also for tone modulation that Bessel function itself whenever you take your end greater than that β+1 that becomes insignificant okay that is numerically seen okay.

So that has been observed or you could also see that from the perspective of the our derivation that we have taken sampled values and then try to derive it then also we could see that means no not that part actually expanding that FM and then getting all those a square a cube a to the power n term and then n tends to ∞ that becomes convolution of all those 80 that also takes it to infinity so infinite bandwidth conjecture was correct there's no doubt about it but the thing is that some of the things are insignificant that is also something we have proven that this a to the power n KF divided by factorial n actually goes to 0.

And then we could also see that it is not Δ F because it is Δ F if the bandwidth is already that Δ F variation is already sufficiently large initially people thought that it should be the left which is true if it is already wider band FM that means it already covers a huge amount of bandwidth then we can say it is just enough okay but initially what people thought with that logic that it is Δ F they could actually make the delay very small and they were thinking that it should be just 11 but that was wrong because Δ F is always supplemented with a β and if Δ F goes to 0 that to be at least will be there.

So whatever happens that to be factored will be there so it will never be that is what we wanted to prove that it will never be anything better than double sideband it will at least be what sir than that whatever it is so bandwidth wise what we can prove now FM is probably not as inefficient as people who are thinking that it covers the entire band up to infinity so it is not as inefficient a that but it is much inefficient in terms of spectral usage compared to any other FM counter sorry a.m. counterpart so this is something we have clear understanding now.

So that is where after this bandwidth calculation and all those things pins the reputation of FM got dented initially people started jumping into FM because they thought this is bandwidth efficient but that got dented because of this derivation so now we have seen that probably FM is not as good what we will try to do in the next few classes will try to prove why FM is still good in terms of noise cancellation interference cancellation and channel non-linearity so all the impairments that comes from channel okay, so thank you.