NPTEL

NPTEL ONLINE CERTIFICATION COURSE

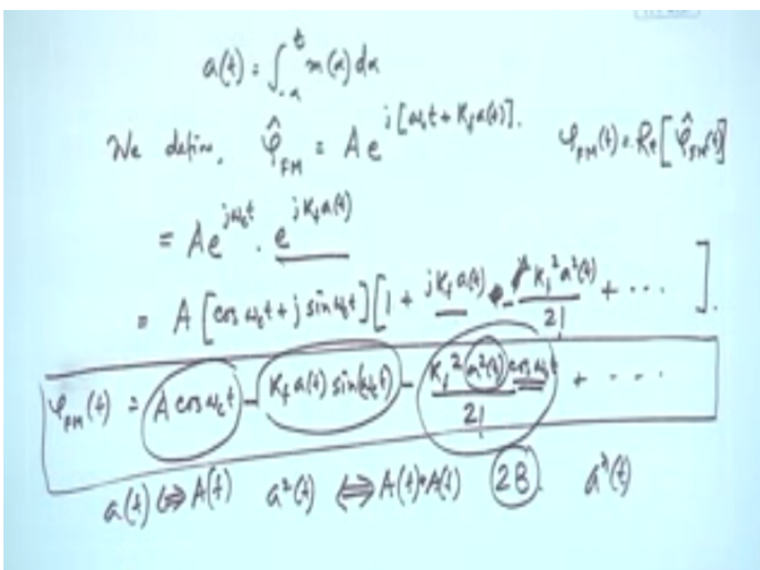
Course
On
Analog Communication

By
Prof. Goutam Das
G S Sanyal School of Telecommunication
Indian Institute of Technology Kharagpur

Lecture 48: Frequency Modulation
(Contd.)

Okay so coming back to the calculation of FM let us try to do calculate that one suppose because of its FM so what we have will have to do we have already have seen that we generally any co-sinusoidal whenever we represent we represent with respect to the over al phase so I should represent it with the integration okay.

(Refer Slide Time: 00:55)



Because whatever empty al we have already seen that form and it should be a cos some ωct+ integration of mα dα right into k so let say that integration part I represent with another thing a(t) so m(α) dα this is just for representation we do not want to always write that integration form so we have just taking that as another function of t because I am integrating up to t right.

So then we define something we will see why we are defining this so which we are not calling as ψ_{Fm} but ψ_{Fm} ^ why because I am defining the complex signal so I will define it like this A $e^{j\omega ct+K*at}$ it is very easy to see if I just take the real part of this I must be getting Fm the real part of this will be it is cos+j sin so I will get the cos part.

So that, that is the a cos ωct+ k*at where it is this one that is Fm already discuss that so I just I am trying to see what will happen to this okay so I know that my ψ_{Fm}(t) Fm modulated signal should be the real part of ψ_{Fm} ^ (t) right is something I know already in the background so let us try to see I can write this as jωct $e^{jkea(t)}$ right now what I do I expand this in an exponential series okay nothing else and this I write as cos ωct+j sin ωct will see why I am writing this.

So I just representing both of them in two different forms nothing else one is I am just taking another one I am putting the exponential series so this should be 1st term should be 1 next term should be this jk_fa_t right divided by factorial 1 which is 1 then + this square it should be j^2 k^2 a^2t/ factorial 2 and so on all other had terms up to infinity right.

So what you can see every odd term okay wherever I am doing means this is probably 0^{th} term this is the 1^{st} term the 2^{nd} 3^{rd} so every in that way if I just represent every odd term will have means that will be an complex one now I should say that be imaginary term right that j will be multiplied here it will be just j^2 so I can put that as minus okay.

So it be like that and I have this so therefore whenever I write this I can just take the real part of it of this multiplication so there is something I can start doing what I see wherever this j term is there that should be multiplied with sin and wherever there is no basically j term that should be multiplied with cos that must be this so therefore immediately I can write ψ_{Fm}(t) from this understanding I will just take all those multiplication where I have real term.

So the first one will be A cos ωct*1 the second term should be this multiply by this okay so that must be minus because j^2 will give you minus k_f a_t okay so that multiplied by sin ωct right and so on right there will be other terms like minus kf^2 a^2t cos ωct/ factorial this multiply by this and it will just continue okay as many term of their which are free of j all the real terms so that must ψFm.

So what I have done is I have got an expansion of ψ_{Fm} utilizing this okay carefully see it has a co-sinusoidal which is a carrier term the next one is just like okay by A(t) that integrated signal whatever it is that is as if you are doing with co0s but it is by sin okay but the spectrum will look like the same okay so there will be a translation around ωc which will have a band equivalent to this band of A(t) right.

Then immediately you can see that at least Fm modulation will have a band equivalent to integration of the signal now what we can also say very carefully see suppose this m(t) that is the band limited signal right what do I mean by A(t) that means m (t) is lineally pass through the integrator okay.

So if m(t) is band limited the integrated version also should be just band limited because it just it is getting pass through linear circuit so if I just means this signal pass through the linear circuit so what will happen wherever it is 0 there it will be 0 say if it is band limited wherever it does not have any component it will also not have any component so if that is band limited this should be also band limited with almost equivalent band okay.

So if that is the case then I know that this is A(t) also will be band limited with equivalent band because integrator is just nothing but 1/j2πf right so that something we can just represent if I just put that immediately I calculate band, band will remain the same so a(t) in the frequency domain if I try to see that has equivalent band as m(t) or m(f).

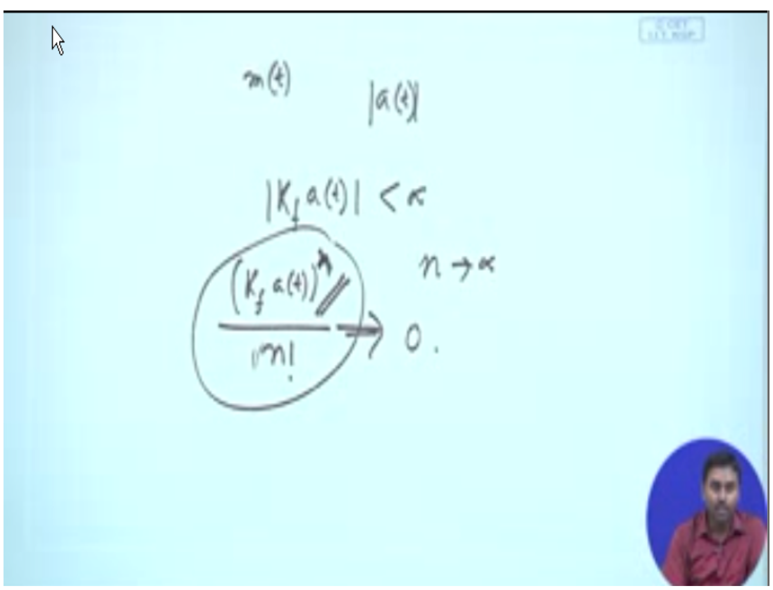
If I do this modulation so first two term if I just intake I have a carrier and I have already almost similar like dsp modulation which takes the entire band so initially whatever we have said that fm probably might have 0 band that is not true from this analysis we have already seen but not only that rather stories to you just start in inspecting this one this is also shifted at ωc so it around ωc but what is the band it is having the signal which is square of a so it is square t what happens in the frequency domain whatever is multiplied in time domain in frequency domain if suppose a(t).

Other frequency domain are transform is A(f) so therefore in frequency domain that should be converted right now we have already proven this that two things convert it the band will be both the things are same so you converted into pixels the band becomes twice okay so this will have now 2B band and it will have all the higher order terms up to infinity.

So there will be the term were I will get A to the power nt which will have n times the band and then n goes to infinity so that I the effecting band of the Fm that goes to infinity this is whne people started getting discourage them about it they could see that from the analysis it is very clear that Fm band is infinite.

So whenever I do this modulation because all this square cube terms and order terms are being already embedded okay so I am getting all every frequency component getting populated by this okay so that was one apprehension people had for Fm okay so this is I hope this is particularly clear right so I get infinite that is alright.

(Refer Slide Time: 09:53)



But here there is the catch let us try to see that if suppose this message signal the regular signal which is bounded let say that is the case so what is a(t) it empty remains bounded and it is a regular signal when the integration of it also must be bounded so this is something we should be because otherwise the signal means the integration goes to infinity.

Then doing transformer is not possible okay so that is not our regular signal or any signal that can be handling in communication systems because that will not be possible okay if we sih to calculate the power and all those things it will go infinity right so our energy will go to infinity so that is the kind of signal we are not expecting that kind of signal okay so basically hat will happen even the differentiation due to that our integration to that will remain bounded okay.

So that I can always say this is bounded this is does not approach to infinity I can then say kf which is constant into a(t) so that is also remain bounded okay so this must not this must eb there is the infinity I can always write that okay now a bounded value what we are getting the term if we try to inspect which as k(f) a(t) to the power m right.

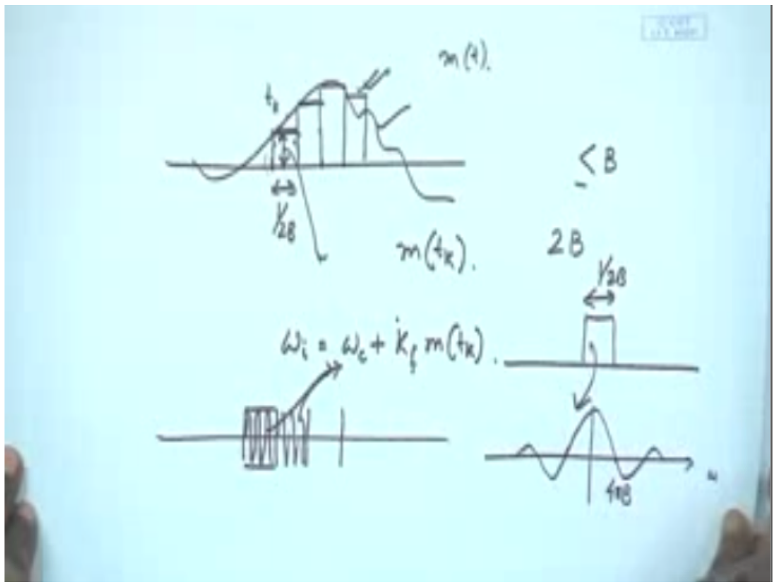
So I have k(f) a(t) to the power n divided by factorial n we know that this polynomial term does not go as fast as this factorial term that is true because we are polynomial term whatever we are multiplying it is the same thing getting multiplied n times where the factorial terms goes much faster okay.

If that is the case this is already bounded so this will go faster than this one okay once I already know this then I can say that as n approaches to infinity this will approach to 0 okay so all his not lost over here what I can see though I have infinite band at the strength of that and whichever is created at higher order term it was going to infinity because the nx terms was creating and into b right.

So as goes to infinity this band goes to infinity but what I could also see that the nx terms as n goes to infinity that gets vanished so basically the importance of that will be vanishing the small and then I can conclude that most probably all this terms are not important some of those terms will be important with that there will be some band which might be bigger than of course b.

That is something which is true but it will not be infinite most probably okay so this is just intuition that is coming directly from the fm signal analysis we still have not prove anything about it expanded we just doing it just to exert something about the band okay so once we have this understanding can we now go about analyzing the fm band okay.

(Refer Slide Time: 13:35)



So let say I modulate the signal which looks like this okay so this is my m(t) or this is my signal I modulate this signal okay first of all probably we still have not proven that will resolve the sample so we have not proven this particular thing but it just say that you most of you later on will also prove that most of you know that this particular signal as long as it is band limited.

That means the band is less than B then I have effective sampling which is two times of this one okay so 2B if I just sample this signal those samples are already represented it of this entire signal I do not have to really take as long as it is band limited I do not have to really take all the samples infinite number of sample that every time instance.

I can just sample it at an instance of ½ B or even higher okay the sampling rate at least has to be this lower than this sampling rate will not represent the signal faithful this is something will prove but this is the sampling theorem okay so if I just sample it to this I will still have I will just take this samples and those samples faithfully represent the whole signal.

What is that means I can actually start constructing box of that of course you can see because I have taken the box little bit bigger actually it will come from the band with actually gives all this slope so accordingly the band will defined and accordingly by sample size or this simple interval would be defined.

So this is not properly drawn the if I just plot this into frequency domain I will see probably much higher frequency terms of because of this variation and due to that ½ will be much smaller but whatever it is there will be ½ B which faithfully represent it I can just take those samples and I can make this sample top flatter so I will say that particular flat sample signal whatever I am this box if I just put one of the another whatever I get that actually faithfully represent this signal.

I am not saying any wrong thing over there because I know those samples are to representative of the whole signal will be proving that also sampling interferences say that so basically I can say at this particular signal can even be represented as those boxes okay now what happens at this instant what is the height of this box.

That is actually at that teem instant whatever sample value iterate so that must be at suppose this is t(k) value at t(k) whatever message signal I get whatever sample value I get and in Fm what will be doing will be actually picking those samples and with those samples you will be generating a frequency right.

That is exactly what Fm is so general Fm says that you take every samples and start creating the corresponding frequencies but we are saying because the message can be only represented it by these samples faithfully so I will take that sample and for that amount of period will keep the frequency same which is determine by this m(t).

So at that m(tk) what will be the instant in your frequency ωi that should be because the frequency so it should be ωc+K(f)*m(tk) this is what we have said already for a fm that at any time whatever the message signal it should be proportional to instant frequency should eb proportional to that, that is what we are doing and that k(f) is the proportionality constant.

According to our fm modulation so at this instant I will be just creating a frequency which is exactly this and because I have represented this as a box signal so that means this duration the signal will stand remains m(tk) so therefore the frequency will remain m(tk) so I can now say that the entire signal if I do a modulation this will just the a pulse over which duration this frequency should prevailed.

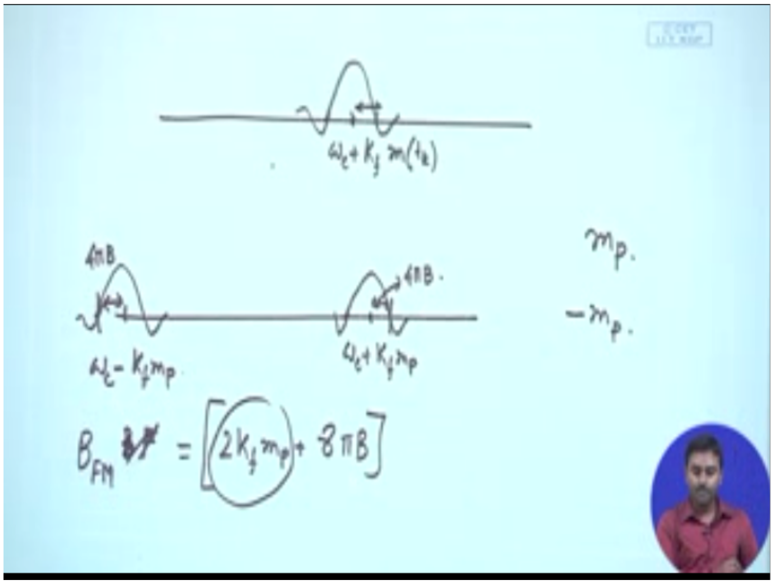
The frequency of this so next time whenever I put I have the change frequency which will be at that instant whatever m(tk) value will be that particular thing so accordingly I can represent the fm modulation so I can just sub divided into those pulse and modulate that with the frequency that comes out of it.

At that instant whatever value it can take so this is the faithful representation of fm modulation I can immediately say from the way we are constructing that advantage you get is now the actually have pulse which is getting modulated at a frequency which is known okay so how does the pulsed will be represented in suppose I want to see the frequency domain representation so what will happen this particular pulse what is that it is the unit pulse okay if I go to the frequency domain how that will look like that will be our sink function if 1/2B is the width of that pulse because of that sampling theorem.

So what will happen this one by that should be the point where it will be cutting means the frequency axes in the positive half as well as minus half one by that should be the frequency axes where it will be cutting the frequency axes okay if I take it to ω that should be 2π into that so 1/1/2 be should be 2B*2π that should be our that 4πB so I can represent this as sink function at this point is 4πB if I put it in ω right.

But what is happening that particular pulse is getting amplitude modulated is a pulse and then I have modulating it to the sinusoidal right of frequency to it so immediately this will be shifted modulation means that pulse getting shifted in the frequency domain to this particular frequency so correspondingly if I start drawing the frequency domain chart.

(Refer Slide Time: 21:10)



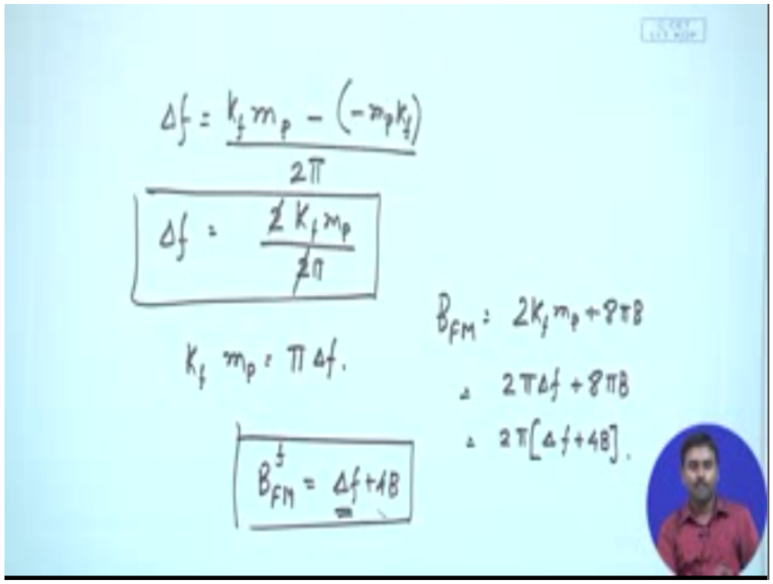
I will see for this m (tk) what will be the frequency that should ωc+k*m(tk) at that centre frequency there will be the pulse sorry there will be a sink function which is having a this zero crossing will be +4πB so ωc+k m(tk) what is the maximum that I can get over this center frequency that should be the maximum voltage that the signal can get.

Let us called that as mp and the maximum of the minimum voltage that I get let us call that –mp so it just seeing the overall signal just seeing the up to the maximum mp and minimum of –mp if that the case this entire frequency domain will be populated with so many half this sink function at every impulse I will be getting corresponding sink function right but the sink function the maximum that I can get will be populated at ωc+k(f)mp it will not go beyond this.

And the minimum that I will get that should be at ωc-k(f)mp it cannot be anything and ten there will be the sink on top of this here it 4πB and there will be the sink on top of this, this is 4πB so overall this variation were the meaningful band will be because beyond this sink will get diminishing all the sink will be diminishing right.

Beyond this point so I can say this is probably my valid band width and what that is that is actually this minus this so that should be 2*kf*mp + in this side 4πB from this side 4πB so it should be 8πB that should be the band width B_{fm} band width of fm right so you can see that in fm band width there is a term called this which sir particularly dependent on the maximum signal strength as well as the kf value that you supply + this 8πB where B is the messaged signal band okay.

(Refer Slide Time: 24:27)



So these two terms actually comes into the picture whenever B try to calculate the fm value okay so I just try to see what this means okay so this is actually what is the maximum deviation in frequency I can get in fm what do I mean by that so by maximum signal is mp kf*mp is the maximum angular frequency that I can create.

At the minimum angular frequency that I can create around ωc that should be –mp*k right because the minimum signal will be –mp so this is the deviation between the maximum frequency and the minimum frequency that I get okay in ω if I wished to represented it in frequency domain that must be divided by 2π.

So this is the frequency deviation that happens due to my signal variation as you can see that is directly dependent on this kf factor so more kf I put probably there will be more frequency deviation less or k if I put there will be less or frequency deviation right and immediately what do I get that should be 2kf mp/2π right.

And fm band width now I can represent with respect to this Δf so kf*mp that is actually π*Δf I can write it this way okay so immediately I can write this kf*mp that is actually π*Δf so therefore this Bfm which was earlier derived as 2kf*mp+8πB that I can write as 2πΔf+8πBor 2π I can take common Δf+4B okay.

So this Bfm was in ω domain if I wish to get that fm frequency band in f domain so I have to divide by 2π so this Bfm in frequency domain let us call as F that should be Δf+4B this is the fundamental relation that we are getting that fm frequency is due to this frequency deviation that is happening okay and the 4B which is particularly band width means relative to band width of the message signal initially you know what happened people were mistaken that in fm the frequency terms are getting populated just because of I am varying the frequency okay.

So that because of that frequency deviation so they were thinking that just frequency terms will capture my band width because every frequency will have deviation sorry every amplitude will have the deviation so maximum amplitude I can calculate from their I can get overall deviation which is the left so if I make Δf times to 0.

Then my fm modulated signals also will have 0 frequency band or times to 0 frequency this was initial understanding of people they were thinking just this Δf I can capture I can capture the amplitude that will be the fm band so initially before understanding all this bands people were thinking that it is fm means for different frequency I actually give a different sinusoidal so if I have if I can capture this entire variation that must be all those sinusoidal will be populated over there.

So that must be my over all band so if I can make my Δf literally very small which is called narrow bend will demonstrate that part so which is called narrow band fm so if I can make this Δf infinite similarly small by just making kf very small whatever no matter whatever the message signal variation it will be bounded.

So that is the case I can always take this very small and the corresponding band width of fm will be infinite so that was the initial understanding if people were thinking and propagating this fm will be the safer it gets the modulation scheme over the band width becomes vanishing this, this is not true that we have understood already that whatever I do I can make Δf to 0 okay.

And then people started means trying to see what will happen if we have a narrow band of this so in the next class what will try to do is will try to see if I do a narrow band version of this what do I get out of this and then will this formula faithful was derived in add up you have seen that already had some intuition that we are not exactly calculating because at every value then how this function wherever the last sink function goes to 0 so we would like to see will more inside into that and try to give some intuitive understanding about this formula the corrected version of this formula and you will see later on.

We will improve that, that was actually the left +2B it is not just 4B will see that correction that comes from modulation so will try to capture that part and try to solve the fm band width means whatever they had okay thank you.