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NPTEL ONLINE CERTIFICATION COURSE

**Course
On
Analog Communication**

**By
Prof. Goutam Das
G S Sanyal School of Telecommunication
Indian Institute of Technology Kharagpur**

Lecture 47: Frequency Modulation

Okay so we have already discussed about the noise in AM so we have almost finished whatever we wanted to do in and we have given a brief introduction in the last class about FM so today what we will do we try to explore FM more there were a few controversy around FM so initially people thought probably with FM there is a huge bandwidth that advantage so they, they thought okay probably FM will give us the smallest bandwidth that is possible.

And it was the initiative of bandwidth means you know that DSP was there then SSB came or cram came so people were thinking of a more bandwidth efficient modulation scheme and then people came up with FM and they thought okay this is probably the most bandwidth efficient I can actually vary my bandwidth as I wish and I can make the bandwidth almost infinitely many small well.

We will demonstrate that okay why people have thought that but that was a mistake probably so initially FM become very popular that they can actually save band width by this then finally when people started doing FM they could realize that probably that is also something will prove that FM has infinite bandwidth instead of means controlling the bandwidth to the extent that it will be almost 10 to 0.

And we can make really multiplexing hugely efficient by putting so many FM carriers people realize that one single FM is almost taking in finite bandwidth so immediately what happens the faith of FM was very bleak people could understand that probably this is not the modulation scheme that we should try for and then the bandwidth problem was in a way solved people could realize that okay.

It is neither 0 or tending towards 0 and nor tending towards infinity it is somewhere in between and we will try to prove that part because FM bandwidth has bothered people researcher demonstrator and implementer alike for long time so we will do that, that is one of the major thing and then people thought okay it is, it is a bandwidth not bandwidth efficient scheme and they could prove that it is much lesser efficient compared to DSPs.

So it was even worse than DSPs see of course it is not that worse that it takes in finite bandwidth but it was worse than DSPs C people could realize that, that the boundary I can get is not zero bandwidth so it will be at least DSPs C and it can be even more so immediately the idea of FM was having a very means negative response from the entire research community and entire implementers and then Armstrong thanks to Armstrong actually FM become so popular.

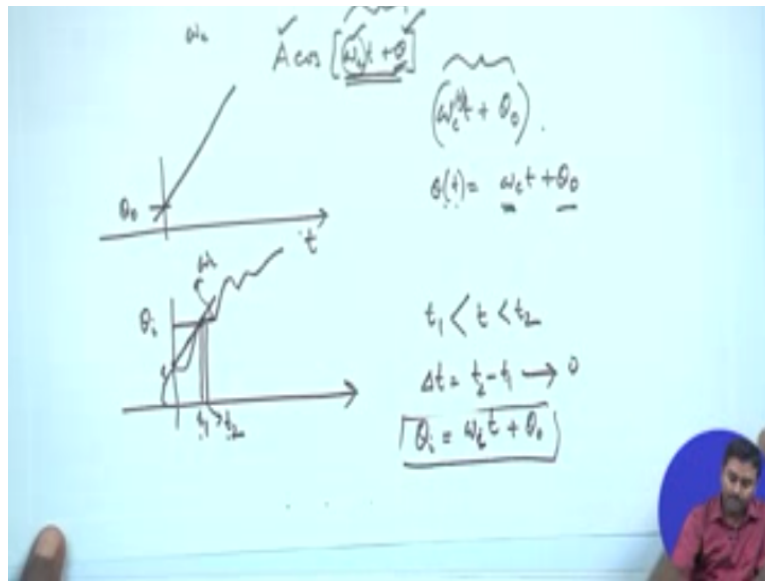
So he started inquiring other thing that okay communication we generally try to see bandwidth and all those things that is all fine bandwidth efficient whether it is energy efficient and all those things but there is also another thing that you put a signal for communication and then, then there is noise there are interference there are all those impairments that we have discussed which comes from the channel, channel non-linearity and then he started understanding whether FM has a good response in terms of those things.

And then he could realize that that is also something we'll be proving that FM has in potentially very good response against all these impure impairments channel non-linearity interference and then noise in particular so that is the point where FM started becoming again popular but Armstrong had to really fight hard but that was legal battle and nobody was trying to accept his technique and then he had to fight a very long legal battle and then finally he committed suicide also because of that because that took a toll from him but anyway because of him probably and all his battle if FM become the winner and then people could see that all the radio actually started becoming FM.

Because you might have also experienced if you have heard about maybe the Marathi in earlier days and then all FM channels FM has much better clarity so that is because it is more prone to noise interference channels a non-linearity and all those things so it always transmits data with much bigger clarity it is easier to receive the data in its original form so all those things actually helped FM to become a much, much more popular broadcasting channel.

So what we will try to do this is just a summary of what we are targeting in FM but we will try to see all these points that we have discussed that the bandwidth issue then the channel non-linearity how FM is better than better in that aspect all those other channel impairments like noise and interference so we will try to prove all those things but before that the first task is how do we really realize fm.

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What is the basis or genesis of FM so let us try to see that part so we have already told that it all started with the parameters of a sinusoidal right it is all carrier modulation so we need to see what are the things we can modulate so we have said that a sinusoidal or co sinusoidal it is defined by three parameter right so one is the amplitude the other one is the frequency and other one is the phase okay so whenever you do amplitude modulation that is something we have covered already.

When you start doing modulating these two parameters will also prove that they are probably similar so if you start doing that modulation probably that is when the angle modulation comes into picture that means either the ω_c will be linearly varying with respect to the message signal or the theta that is there will be linearly varying with respect to the modulating signal like the amplitude was earlier varying with respect to the modulating signal right.

If you just think about DSBs see so this was the target so then people started thinking that is FM and a means p.m. and that phase modulation or frequency modulation are there almost two similar things so for that we need to establish the relationship between phase and frequency so if we start varying frequency or phase what we will see is overall this whole thing inside will be varying with respect to time okay so let us just call this as a overall phase okay which is like Co sinusoidal what value it takes the angular value that it takes inside.

So that is the phase it is a co sinusoidal of that particular value for a signal which is having this varying this parameter with respect to time let us plot that okay so with respect to let us say time we start plotting this so what will happen this particular phase will of course we can see with respect to time it will increase okay because there is a function T and there will be some variation because this ωc might vary with time theta might vary with time.

So there will be some variation so let us say probably this particular part is right now not very okay so let us say it is like this $\omega c t + \text{some } \theta$ zero okay so entire variation is over here so we are trying to capture the FM part okay so because this $\omega c t$ that is also a function of T this whole thing will now become a random variation of this particular thing okay so therefore if Omega C also was constant I could have expected something like this a linear curve because then that theta T would be $\omega c t + \theta$ where θ is a constant ωC is a constant.

If that is the case then θT must be a linear function of T so it should be like this where it cuts at θ zero and the slope is ωC right this is something what we know okay now what will happen if ωC start varying with respect to time then there will be a phase response which will be something like this okay so let us concentrate just for the time being a particular time T1 and a particular time which is close enough which is called T2 and we are trying to concentrate on this interval that is this t between this T 1 and T 2 okay.

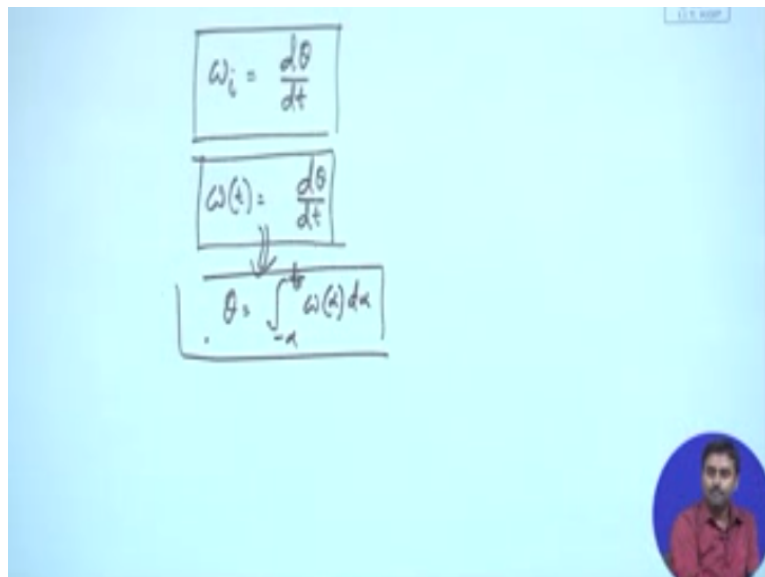

And what we also do that separation we call that ΔT that is T 2 -T 1 we make it infinitesimally small so almost tending towards 0 if you just do that then add that at a particular instant let us say at T 1 what will be the instantaneous frequency okay so that is something we are trying to evaluate so at that point this must be the phase let us say that is θI so I can say that theta I at that instance should be at this point if I see from t1to t2 if this θI does not vary too much okay.

So then between t_1 to t_2 for all these values of T it must be remaining constant okay so that θ I can just write at that point whatever ωC I will be getting into T plus θ_0 suppose I can write this way then what will happen this particular part if I just put a tangent over here that must be the linear curve so if I just say this is that ωC okay.

Because of that tangent whatever slope it will be creating I say that is the means that is ωC so, so for this instant I can write that ωC into $t + \theta_0$ wherever that tangent will cut this particular part I can write it this way right so or instead of writing this as ωC I can write it as instantaneous frequency at that instance okay similarly at every instance I can start putting tangent where ever it will cut that will become my theta initial phase and then accordingly this particular thing will be coming out right.

So what I can see over here immediately that basically the instantaneous frequency is becoming the slope of the tangent I draw over that theta icons this is something very clear almost like when you start talking about velocity and acceleration okay so if the velocity was not constant then the acceleration was similarly defined right it is the rate of change of velocity and if the velocity goes in any extent then you start defining it with respect to the tangent now the tangent from the differential calculus.

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$$\omega_i = \frac{d\theta}{dt}$$
$$\omega(t) = \frac{d\theta}{dt}$$
$$\theta = \int_{-\infty}^t \omega(\tau) d\tau$$


We know that that is the differentiation of this particular part so immediately what we can say this ω I which is the instantaneous frequency that must be the differentiation of the phase which is happening over there at time T okay so this relationship I can always write or I can write ω_{CT} must be $D \theta / DT$ so time varying relationship of ω I can get which is our differentiation of the overall phase of the co sinusoidal okay.

So this is something I can write now from here I can get the definition of θ we know that differentiation and integration are actually conjugate okay so immediately I can write theta as from minus infinity to T $\omega \alpha D \alpha$ so this relationship also I can easily write right so these two relationships fundamentally comes from our understanding of frequency and phase okay so if I just take overall phase and then try to define what is frequency and frequency immediately that angular frequency immediately becomes the instantaneous differentiation at that particular point okay.

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The image shows a handwritten derivation on a light blue background. At the top, the phase $\theta(t)$ is defined as $\omega_c t + K_p m(t)$. Below this, the modulated signal is given as $A \cos[\theta(t)] = A \cos[\omega_c t + K_p m(t)]$. A box contains the derivative of the phase: $\omega(t) = \frac{d}{dt} \theta(t) = \omega_c + K_p \dot{m}(t)$. Below this, a block diagram shows an input message signal $m(t)$ entering a block labeled $\frac{d}{dt}$, followed by a block labeled FM , with the output labeled PM .

So knowing this relationship let us try to do you see what will be happening if I try to modulate a signal that angular things we wish to modulate okay so let us say I have a θ T okay so now this θ TI want to vary instantaneously with respect to the input message signal right so this is

something I can do so mean immediately what I can do it must, must be having a constant part $\omega C T$ plus it should be a linear variation with respect to that so I can put some $K_P m_p$ okay.

So what I am doing the overall phase that is being created that has a constant angular this one due to an angular frequency constant this plus means a constant angular frequency due to that whatever phase is being created plus there is a variation due to the message signal itself so this I can call a phase modulation because the overall phase how we define phase it is should be $\omega c t$ plus a θ now I am varying that θ linearly with respect to this m_p .

So whichever way M_T is varying I have just a linear means coefficient constant which is constant K_P I multiply with that I get the phase and that phase I will be inserting in the signal so therefore if my θT becomes like this the phase modulated signal must look like a $\cos \theta T$ which is nothing but a $\cos \omega c t$ right this is what happens no problem in this now what we will try to see that what happens in the frequency of this part.

So what do we know about frequency so frequency must be if this is that phase θT my frequency must be $D T$ of this so that should be my ωT right so if I differentiate this that should be ωC plus K_P differentiation of that okay so the differentiation I represent with the dot so this becomes my frequency now what was my idea of FM modulation the idea was that the frequency must now vary linearly with respect to a particular signal which is almost happening as you can see over here.

The frequency instantaneous frequency which has a constant term that you can forget and this is where the $m \dot{t}$ is actually model modulating the frequency so what I can say if I do a phase modulation that is nothing but also a frequency modulation but in the phase modulation I modify the phase with respect to $M T$ whereas if I have a frequency modulator similar phase modulation I can create if first I differentiate my signal and then do a FM so correspondingly what will be created that should be my p.m. right so phase modulation is nothing but I first do a frequency modulation.

But when I do frequency modulation I also remember that if I do the frequency modulation with respect to the differentiation of that particular signal then automatically that defeats the phase modulation right so this is what we are trying to do so therefore I can say that suppose I have a frequency modulator and I want to generate a phase modulator so what do I do I take my $M T$

first I pass it through a differentiator circuit okay and then give it to the FM so FM will modify the frequency accordingly and that, that is nothing but this if the frequency gets modulated like this automatically the phase will be this.

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$$\omega_c(t) = \omega_c + K_f m(t)$$

$$\theta(t) = \int_{-\infty}^t \omega_c(\alpha) d\alpha = \int_{-\infty}^t [\omega_c \alpha + K_f m(\alpha)] d\alpha$$

$$= \omega_c t + K_f \int_{-\infty}^t m(\alpha) d\alpha$$

$m(t) \rightarrow \left[\int \quad \text{PM} \right] \text{ FM}$

So that becomes this whole module becomes the PA right this is just by seeing the relationship between phase and frequency we could see that okay similarly what we can write suppose I do a FM modulation now so what will happen my $\omega_c t$ should be some constant frequency plus some K_f another constant into $M(t)$ because now I wish to do frequency modulation so frequency must be linearly means dependent with respect to the input signal that is what is happening if this is the frequency.

Then what should be the phase, phase must be integration we have already derived that integration from minus infinity $\omega_c t$ or $\alpha D \alpha$ which is nothing but integration minus infinity to T $\omega_c \alpha + K_f m(\alpha) D \alpha$ this happens to be just $\omega_c t + K_f \int_{-\infty}^t m(\alpha) d\alpha$ so this is $\omega_c t + K_f \int_{-\infty}^t m(\alpha) d\alpha$ and this is just K_f

comes out integration minus infinity to $T M \alpha D$ also right so that is my phase so what I can do from ωc .

So basically I need suppose I have a phase modulator now you can see it very clearly I have a phase modulator so what I do first I pass it through my message signal pass it through our integrator circuit okay so I will get this and then I actually modulate the means I do a phase modulation with I get actually the frequency modulation so suppose I have a phase modulator then what do I do the message signal I pass it through an integrator then I put a phase modulation correspondingly whatever I get that must be a frequency modulated.

So from this relationship between phase and frequency what we can see that these two are almost equivalent if I have one modulator I can always construct another modulator just by using a linear circuit in front of that either a differentiator or an integrator so if I have a PM I use an integrator to get FM if I have a FM modulator then I just use a differentiator circuit in front of that to get PM so this is something we will be able to always do and that is why probably all our discussion.

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Generalized Angle Modulation

$$\phi_{GAM}(t) = A \cos \left[\omega_c t + \int_{-\infty}^t m(\alpha) h(t-\alpha) d\alpha \right]$$

$$h(t) = K_f \delta(t) \rightarrow = A \cos [\omega_c t + K_f m(t)]$$

$h(t) = K_f \delta(t)$

$$\rightarrow \phi_{GAM}(t) = A \cos \left[\omega_c t + K_f \int_{-\infty}^t m(\alpha) d\alpha \right]$$

I can just do it for one the other one will follow automatically because the modulation as well as the modulation all these things you can do very similarly okay so with that can we just give a unified representation so that something will just say so we call it our generalized so this is just

the more means most generalized FM or PM modulation that we can give or any other angular modulation that we can give so we can see if I just write it in this expression.

So it is a $\cos \omega_c t$ plus integration from minus infinity to T $M \alpha h t - \alpha D \alpha$ I will just explain what why what I have just written okay so we are saying that this is probably the most generalized representation of either PM or FM or any other variety of angle modulation why is that so I will just give you two example let us say $H T$ is equals to $K P \Delta T$ then what happens to this so if I just take this as Δ and if I just integrate a function multiplied by a delta function it will just give me return me back the function itself right.

So immediately what I will get if I put this, this will become a $\cos \omega_c t$ plus I will get $K P M p$ which is nothing but F M sorry PM so that is the phase modulated signal because the phase is now that phase portion is now linearly modulated with respect to the message signal if instead of that I start putting $H T$ equal to $K F U T$ now let us see what will happen so then this Φ generalized angle modulation T will have a form which is nothing but $\omega_c T$ plus now I multiplied with $U T$ or $U T - \alpha$ that means it is actually just remains as I am α because that will be from, from T equals to α to infinity this will be one okay.

So wherever I take that α it will be one over there so it should be just rest of the things will be vanished so if I just do this integration you just look like minus infinity T that $K F$ should be over here $M \alpha D \alpha$ can you again identify this form this is actually the FM form okay so the frequency modulation can be realized by realizing at different $H T$ so it's just almost like a means this $H T$ you can already see it is just like a impulse, impulse response of a particular transfer function.

That I am putting in front of the modulator okay so that is what you are trying to do so you just maybe you are having a FM modulator where the phase modulation you are creating by that transfer function okay so whichever transforms on you choose accordingly either PM or FM will be generated and not only that we can also see a plethora of other things which are generated which you do not know because you can now take H anything and this will be one version of angle modulation okay.

So this generalized representation just tells us that we do not have just one option or two option that FM or PM there are in between multiple other options we put will probably get one kind of angle modulation okay whichever way that is but it will have one angle modulation because it is

just with respect to my input signal it is just modulating the angle okay so it just depends on what kind of transfer function you are putting so we have given two examples those two examples actually takes us to PM or FM and that is why when we are talking this as generalized angle modulation probably that is true.

Because we can we can even get multiple other angle modulation using just this particular form so once this is being done what we can now start talking about is a bandwidth of FM okay so this was something which has been means which was bothering people over the year as I have already told in the introduction part of this particular class and that FM was initially thought that it has means I can, I can vary the bandwidth according to my wish so I can have even zero band width.

So that was the initial assumption then people started seeing probably no that is not true if I actually have infinite band width okay so it went from one extreme to another extreme and why does that has happened I probably will try to examine that portion so what we'll try to do in the next class is try to see this why people had so many different version of FM bandwidth so this is something we will try to explore in a better way that that lead us to actual bandwidth calculation which is known as Carson's formula.

So we will first go through all this jungle of different bandwidth proposition for FM and then we will probably try to hit the right thing where the actual FM bandwidth is being evaluated it is neither infinity not zero it is somewhere in between and then people could realize which I have already told that it is initially people thought it is bandwidth efficient so it was not bandwidth efficient then people started thinking it might have infinite bandwidth and people could realize that it is at least as watts as DSPs see it is not better than that.

So if you have to consider FM of course it is not infinite but it will be as worse as DSP sC you cannot have anything lesser than a DSP sC so it cannot be any way band width efficient compared to any other amplitude modulation schemes so whichever we have discussed from DSB to cram to VSB SSB it is not better than any of those things so that something will try to prove first and then we will see the other benefit of it okay thank you.