

**NPTEL**

**NPTEL ONLINE certification course**

**Course  
On  
Analog Communications**

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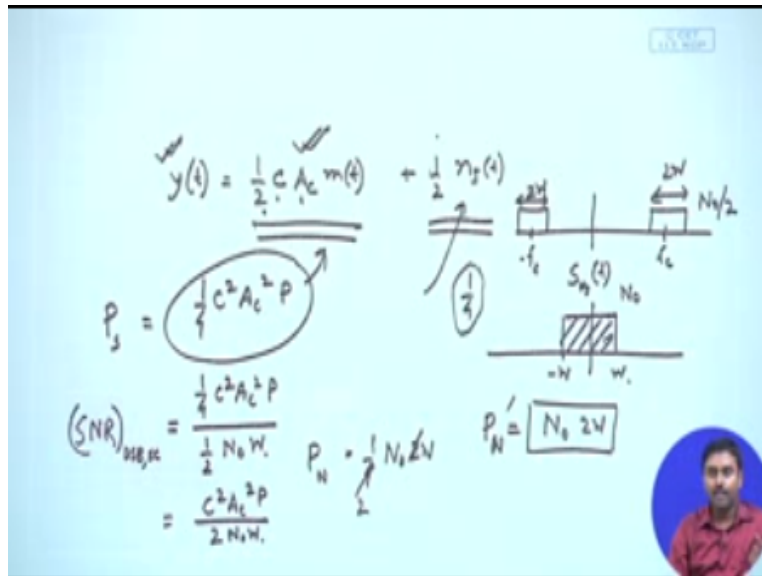
**Lecture 45: Noise Analysis - AM**

Okay so in the previous class what we have tried is we were trying to actually get a methodology or propose a methodology where we can actually compare in terms of noise analysis different modulation schemes, so we have already proposed one where we have told that okay will take base band transmission and we will take the corresponding modulated transmission and for this purpose we will try to equate these two power base band whatever fall we are transmitting we should be also transmitting the same power and the modulation part, so basically our methodology was for the modulated.

First calculate the power the same power you put in the base band okay and then base band we could evaluate because base band whenever you transmit at the receiver side it is very easy it is just a low pass filter now we will be added in the channel and low pass filter, so we put actually evaluate the a sign SNR see the noise issue and we call that means our reference SNR okay and then for the modulate it we have actually done the entire modulation process for DSB-SC we have already analyzed it.

We have told that we have the receiver front and we have band pass filter that is of course for rejection of noise followed by the entire demodulation okay, so after doing that entire demodulation we just have taken 1/2 of this noise representation which is the band pass noise representation okay or band limited noise representation so that particular in phase then quadrature we have written after doing the entire demodulation process we could see that I ultimate output.

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Of a DSB SC signal should be this is something we have already prove that okay, so now let us try to see how do we calculate the signal power so the signal power must be coming from this cmt gives the power of p and that is multiplied by c AC and pass so pass filter density if you just go and then integrate I should be getting one fourth C<sup>2</sup> AC<sup>2</sup> and P right, so that must be the signal power after demodulation now I have get the noise power that should this power okay which is nothing but 1/4 because there is a 1/2.

And the integration of prospective density of ni this something we have already characterized in our band pass random process analysis, so we have told that if I have a corresponding n which is going from that center frequency minus fc + fc and this by this 2w we have told this ni or nq will be nothing but this shifted to the side plus shifted to this right okay, so if this strength was it has 1/2 both of them will be shifted and the overall strength will be it has 1/2 or n0 so this be n0 define from - w to + w right.

So that is my ni or the noise spectral density so overall noise power then it is just the integration of the noise spectral density so that must be N<sub>0</sub> into 2w right, so that is my noise power there is a factor of 1/4 so it should be 1/4 into N<sub>0</sub> 2w so that is actual so this if I write as this so that is actual noise power right, so what is my signal to noise ratio for this modulated scheme, so if I just say SNR for this DSB SC that process with this power 1/4 C<sup>2</sup> AC<sup>2</sup> P / this noise power which is nothing but 1/2 N<sub>0</sub>w.

So that 2 gets cancelled I get  $C^2 AC^2 P / 2 N_0$  so that is the SNR for DSBSC if you just go back and try to check the reference SNR was exactly the same okay the expression was exactly the same so therefore the figure of merit must be 1.

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$$FOM = \frac{(SNR)_{DSBSC}}{(SNR)_{Ref}} = 1$$

Noise Analysis of AM :

$$s(t) = A_c [1 + K_a m(t)] \cos(2\pi f_c t)$$

$$r(t) = s(t) + n(t) \rightarrow \text{BPF} \rightarrow \text{ED}$$

Must be this SNR for DSBSC / SNR of the reference because they are exactly the same so it should be 1 so for DSBSC in presence of noise it gives me no degradation or no advantage compared to base band transmission this is something we have understood whatever the SNR I get when I do base band transmission the same amount of power if they transmit whatever advantage I will be getting DSBSC does not give me any degradation or any advantage okay, so this is something we have now figured out for DSBSC that something which happens okay.

So next what we will try do is we will try to see if the am version of it where you do not suppress the carrier is it having similar thing okay or is it something else then only we will be able to compare whether noise performance wise these two modulations schemes are equivalent or one has some advantage over the other, so let us try to do the noise analysis or amplitude modulation process will be same so the process must have a base band equivalent transmission which we have already seen okay and there should be a modulation.

Now the modulation our signal is something like this you already know this okay so where this  $K_A$  is related to the modulation index they define this when we did our  $m$  and this is the amplitude of the carrier signal okay so basically what we do nothing but we actually add a DC to

the signal and then modulate or do our DSBSC or otherwise in a way you can see it as the DSBSC + carrier part if you just multiply this that should be the carrier part, so the carrier has been also added.

Over here so this our modulation corresponding the modulation the demodulation of AM was easier what we did is something like this you have this  $S_d + \text{noise}$  which let say  $w_t$  coming through the channel at the receiver what you do you first put any receiver will have band pass filter that is guaranteed because you want to reject the noise as much you want if it is a low pass signal you will have a low pass filter if it is band means signal having some band with that particular band.

Then we have to put a band pass filter so you put a band pass filter followed by just envelope detection this was our amplitude demodulation right, so we will not change that circuitry so that must be our  $y_t$  okay, so let us now try to see what kind of power we are talking first a fall see anywhere the techniques are same we need to launch same power for the base band transmission as well as the modulated transmission okay, so that is why because the modulation transmission is will be completed to calculate first.

We will calculate the modulator transmission power and that same power will be put Interviewer: the base band already here also we done same thing, so here also we will do the same thing so what is the power.

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$$P_{\text{mod}} = \frac{A_c^2}{2} + \frac{A_c^2 K_d^2 P}{2}$$

$$(SNR)_{\text{ref}} = \frac{A_c^2 [1 + K_d^2 P]}{2 N_0 W}$$

$$P_n = \frac{N_0}{2} 2W = N_0 W$$

$$v(t) = [A_c + A_c K_d m(t)] \cos(2\pi f_c t) + n_z(t) \cos(2\pi f_c t) + n_q(t) \sin(2\pi f_c t)$$

$$y(t) = \sqrt{[A_c + A_c K_d m(t) + n_z(t)]^2 + n_q^2(t)}$$

This modulation okay so the power that will be launch see this AC Cos  $\omega C t$  that is the Cos sinusoidal signal with amplitude AC we know that the power of that is  $AC^2 / 2$  right this is something we have already done, so that must be  $AC^2 / 2 +$  this AC into KA that is already there right already here also we had done this C into AC right, so instead of C it is KA so that must be  $AC^2$  into KA and this mt is that has a power of P then it should be because it is multiplied by Cos so it should be  $P / 2$ .

So I can write as  $AC^2 K^2 P / 2$  this is the power if I modulate will be launched so what I have to do I have to equate the power so therefore this is the power I will be also putting in the bas band so base band power is this where is the base band noise that is equivalent to the previous case because it is just  $N_0 / 2$  from  $-w$  to  $= w$  so overall power noise power is  $N_0 / 2$  into  $2w$  which is  $N_0$  into  $w$  so therefore our SNR reference must be this divided by this so it must be  $AC^2$  that it is common  $1 + K^2 P / 2$ .

And  $0w$  right so that is the reference SNR that we get now we have to do the envelope detection so we will again employ the same thing, so I have this signal plus noise it should be pass through a band pass filter this band pass filter because it exactly matches with the band of this one so ST will remain in tech the noise will be passing through the band pass filters so it will have a equivalent band pass representation okay with in phase and quadrature so therefore that after the band pas filter if I call that signal as  $V_t$ .

That must be our  $AC + AC K_A m_t \cos 2\pi f_c t$  right so this is the message signal or modulated signal plus  $n_t \cos f_c t + n_Q t \sin 2\pi f_c t$  just the noise gets our in phase quadrature representation right now next what will happen this particular  $V_t$  will go through envelope detection that means whatever I have this is a sinusoidal or this can be termed as a sinusoidal its envelope will be tracked so I will all I have to do is I have to now see what is the envelope of this one which has a frequency of  $f_c$ .

Because this as this carrier which is a very high frequency so it is actually the underline frequency term is  $\cos \omega_c t$  we just need to see the envelope of that okay there is a  $\cos$  term and there is a  $\sin$  terms we know how to calculate the envelope that should be the coefficient of  $\cos^2 + \text{coefficient of } \sin^2$ , square root that must be the envelope we have already derived this path okay so let see what is the coefficient of  $\cos$  coefficient of  $\cos$  is  $AC, AC K_A m_t + n_t$  right so that is the coefficient of  $\cos$ .

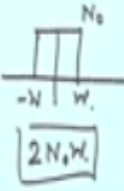
So this square plus coefficient of  $\sin$  is  $\sin 2\pi f_c t$  is  $n_Q$  so  $n_Q t^2$  to the square root of that, that must be the envelope so this is my  $y_t$  then because the envelope detector will exactly detect the envelope of this carrier okay so that must be my overall law now we will insert something okay because this is the complex this one now you can see the noise square, square root all of kind of things ahs come okay so it is no longer or very nice linear thing like we have assumed or we have we could see for the DSBSC, so it going to be harder to analyze okay but we will take a very simplified assumption over here we will say that this noise term generally that should be the case this.

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$n_i(t), n_q(t) \ll A_c$

$$y(t) = \cancel{A_c} + \underline{A_c K_A m(t)} + \underline{n_i(t)}$$

$$\frac{A_c^2 K_A^2 P}{2N_0 W}$$

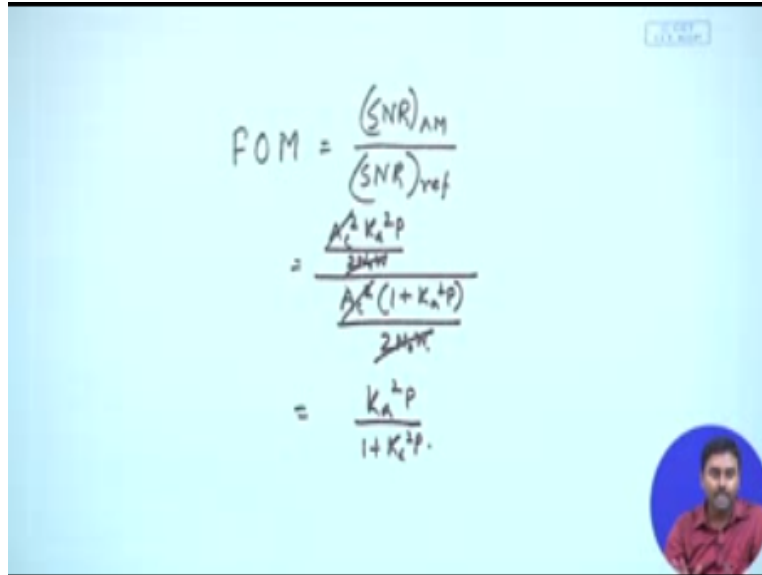
$$SNR_{Am} = \frac{A_c^2 K_A^2 P}{2N_0 W}$$


This  $n_i$  both  $n_i$  or  $n_q$  which is the in phase or quadrature component that must be much less than this  $A_c$  okay so this is one assumption we have taken with that assumption generally you will have to put a high area, so if you are doing that it does not as this is valid you can do some analysis so let us try to see what that would be so immediately  $y(t)$  I can write so we have this square plus this square this particular term as  $A_c^2$  so this will be much lesser compared to this particular part.

Okay because this is already lesser than square of that will be much lesser, so I can neglect that part so immediately if I neglect what will I have I will have a square and a square root that we cancel out each other so I will get this  $A_c + A_c K_A m(t) + m(t)$  so again we get back similar things where this is probably the signal part and this is the noise part this is then DC part we have already seen that after envelope generally you will be have a DC blocker which will block this part.

So this could be already gone will have just this power and this power so now we have to again evaluate the signal noise ratio okay the signal power is how much it is actually  $A_c^2 K_A^2$  into  $P/2$  so that is the signal power and the noise power is this  $N_0$  whatever that is okay, so  $N_0$  we have already evaluated that, that must be its strength it as 0 going from  $-W$  to  $+W$  so that must be  $2N_0W$  right so this if I put I will be getting  $A_c^2 K_A^2$  oh sorry this two while I have put there should not be any 2 because there is no modulation term it is just directly  $m(t)$  so it should be this  $P/2$  and  $0W$  right, so that is the SNR of  $A_m$  now if you wish to calculate the overall figure of merit okay.

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$$\begin{aligned} \text{FOM} &= \frac{(\text{SNR})_{\text{AM}}}{(\text{SNR})_{\text{ref}}} \\ &= \frac{\frac{A_c^2 K_a^2 P}{2W}}{\frac{A_c^2 (1 + K_a^2 P)}{2W}} \\ &= \frac{K_a^2 P}{1 + K_a^2 P} \end{aligned}$$

Of this modulation so figure of merit will be first we have to put SNR of Am / SNR of the reference SNR of Am we have got  $AC^2 K^2 P / 2$  and  $0$  w and SNR reference we have got as what was it  $AC^2 1 + K^2 P / 2$  and  $0$  and w  $AC^2$  gets cancelled to now gets cancelled so you are that with  $AC^2 P / 1 + K^2$  right which is for sure less than 1 because I already have a  $K^2 P$  I have something added to it, so definitely I know the figure of merit of this one is less than 1 okay, so what does that means that actually says.

That after doing this analysis we could understand that probably DSBSC is better noise resilient compared to m because m to get similar amount of signal to noise ratio will have to launch a huge amount of power okay because that career power are putting that is wasted and that is not really giving me health in terms of noise cancellation right, it is not boost in the signal power which will be the actual signal power that will be demodulated, so because of that we have a degraded signal to noise ration performance okay so if we just give on example probably.

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$$\begin{aligned}
 \text{if } m(t) &= A_m \cos(2\pi f_m t) \\
 S(t) &= A_c [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t) \\
 \mu &= K_A A_m \\
 P &= \frac{1}{2} A_m^2 \\
 \text{FOM} &= \frac{K_A^2 P}{1 + K_A^2 P} = \frac{\frac{K_A^2 A_m^2}{2}}{1 + \frac{K_A^2 A_m^2}{2}} = \frac{\frac{\mu^2}{2}}{1 + \frac{\mu^2}{2}} = \frac{\mu^2}{2 + \mu^2}
 \end{aligned}$$

So if we take our  $m(t)$  to be some  $m$  just a tone modulation okay so if we just take that so  $m(t)$  is also a sinusoidal so basically we are modulating our slow varying sinusoidal with the faster varying sinusoidal which we have also done for demonstration purpose right, so this is usual practice we just want to see what happens okay and let say our modulation index in that case is something like this  $1 + \mu$  so this is the modulation we employ this is the modulation index okay, so if this is the case now what is  $\mu$  this is actually  $K_A$  into  $A_m$ .

I can write in this way and my power is  $\frac{1}{2} A_m^2$  because it is a sinusoidal so the power should be amplitude square divided by 2 right, so this is my overall power okay so therefore the figure of merit that we have already calculated that is  $K^2 P$  divided by  $1 + K^2 P$  which if we just put so  $K_A$  okay we can put it in terms of  $\mu$  so  $K^2 P$  if I just put  $\frac{1}{2} A_m^2$  so I get  $\frac{K_A^2 A_m^2}{2}$  divided by  $1 + \frac{K_A^2 A_m^2}{2}$  that is nothing but this is actually  $\frac{\mu^2}{2} + 1$  or I can write as  $\frac{\mu^2}{2} + 1$  what is the highest value of  $\mu$  that I can take.

1 we have already seen that for proper  $A_m$  modulation so that we have enough DC shift it does not do any 0 crossing so to do that so that envelope detection is proper we need to maximum  $\mu$  I can take is 1 so if I just put that maximum  $\mu$  I can immediately get figure of merit to be one third so that is the maximum I can achieve so it will be whatever happens for a tone modulation we can already show that this is the best figure of merit we can get it can become even worse than this.

This is the best we can get so this is the best we can get so this one third time more than the corresponding DSBSC modulation okay, so that something we wanted to say at now with the noise analyses we can actually insert that okay one of them is better and the other one is probably not that it what will do next is means similarly you can start doing analysis for everything I will just demonstrate one of them because it is little more critical so that is why I will demonstrate the other one which is the SSB.

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SSB-SC

$$s(t) = m(t) \cos(2\pi f_c t) + \underline{m_h(t)} \sin(2\pi f_c t)$$

$$M_h(f) = jM(f) \operatorname{sgn}(f)$$

$$= \begin{cases} -jM(f) & f > 0 \\ jM(f) & f < 0 \end{cases}$$

$$m(t) \leftrightarrow M(f)$$

$$\downarrow S_m(f)$$

$$\int_{-W}^W S_m(f) df = P$$

$$m_h(t) \leftrightarrow M_h(f)$$

$$\int_{-W}^W S_{m_h}(f) df = P$$

Okay so single side band of course suppress carrier so this is something I will try to demonstrate next and what will try to do is this will almost give us most of the tools that are required while doing this analysis if you wish to do for VSB it is almost similar all three is that for the filtering you have to take a proper care of that because VSB has a different kind of filtering it is no longer a sharp filtering okay, so accordingly the noise and all those things will be accordingly at just it we will have to do that okay.

So I will not demonstrate that VSB part but probably the SSB I will demonstrate because we will see something more critical will come up with the noise processing okay, so for SSB what we need to do is again the similar thing okay so we have to actually define a signal for SSB which is nothing but  $m(t) \cos(2\pi f_c t) + m_h(t) \sin(2\pi f_c t)$  this can be + or - depending on upper side band or lower side band which we are transmitting okay and  $m_h(t)$  is something we know already it is the Hilbert transform.

Of our signal right so this is something we already know so the characterization of that is if we write the frequency response of  $m_h(t)$  is  $m_h(f)$  or we should not say frequency response the frequency representation of that or the amplitude spectrum of that so that is the  $m_h(f)$  that must be we already know that should be  $jMf$  if  $f > 0$  or  $-jMf$  if  $f < 0$  right one of them you can make equal okay so this is something we already know.

Why where telling all these things because we will have to finally calculate the power to evaluate the power I already know the power of this  $m_h(t)$  I have to now calculate okay what is the corresponding power of that because otherwise I want to be able to evaluate the overall power that will be transmitted right because whenever I see we transmit about that signal I have to first get the power and that power I will be putting in the base band so that something we have to first characterize okay.

So to evaluate the power let us try to see how do you do that we know that  $m_h(t)$  corresponding Fourier transform has  $m_h(f)$  and it as a corresponding prospectal density which is  $S_{m_h}(f)$  right and you also know that  $-w$  to  $+w$  it is a band limited signal and  $-w$  to  $+w$   $S_{m_h}(f)$  this is something we have we are keep on telling that this is  $P$  that something we know.

Okay now let us try to see what will happen to  $m_h(t)$  so immediately we have already told that  $m_h(t)$  that must have a Fourier transform which is  $m_h(f)$  okay which looks like this now let us go to the prospective density so whenever you will be evaluating pros spectral density it is a modulus of this square you do the more or less this  $+j$  or  $-j$  has no effect so you know that the pros spectral density whatever happens it will be equivalent to whatever we get through  $m_h(f)$  okay so both the pros spectral density will be equivalent where.

Amplitude spectrum or I should say means amplitude and phase spectrum will be different amplitude spectrum will still remain the same only phase spectrum will be different but if I go to pros spectral density they have exacted no difference okay, so therefore for this one also if I just write that as  $S_{m_h}(f)$  so that also I can insert that is band limited and  $-w$  to  $w$  if I integrate that first  $P$  also  $P$  this is the extra information I get okay, so what will try do in the next class is taking these two information.

We will try to first evaluate the power of this okay and then we will try to see what is the corresponding overall noise analysis can be okay thank you.