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**Course
On
Analog Communications**

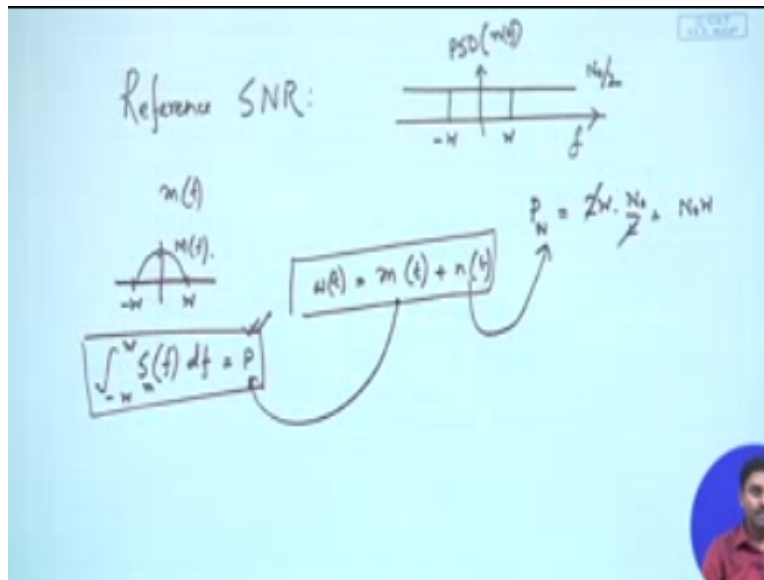
**by
Prof. Goutam Das
G S Sanyal School of Telecommunication
Indian Institute of Technology Kanpur**

Lecture 44: Noise Analysis – DSB- SC

Okay so I think we have gathered enough information regarding the random process and now probably we are almost ready toward utilizing this means understanding of random process towards doing the noise analysis of some of the systems which we have already discussed. So I will first give a outline of the basis for noise analysis means how we wish to do that, what is the measurable quantity? How do we really characterize a particular modulation scheme in terms of noise analysis?

So what we will as whenever you are trying to modulate something we will try to see if we have not employed that modulation that means the original data which is base if you have directly transmitted it and in the receiver hello past filter if we would have received that, so what could have been the signal to noise ratio that we will try to measure 1st. so basically our that we will call as reference signal noised ratio okay.

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So that reference signal noised ratio how would we like to compute, like suppose I have the message signal $m(t)$ which might a voice signal let say. Of course it is a band limited signal so it goes from $-w$ to $+w$, so that is the message signal, frequency response of that message signal. We also know that the power of overall signal that is actually $-w$ to $+w$ if the corresponding let say that is $s(f)$ this if we integrate we get the P , so this something we know.

That is the overall power, so this is already given, so this is the overall power of the message signal and that will be received at the receiver. So I am not saying that it will be transmitted or we are saying in the channel there is no retention let say. So either it is transmitted with this amount of power or there is some attenuation so just at the beginning of the receiver or the antenna of the receiver this is the power that we will be getting.

So either way you can take assumption, so very clear cut assumption is we are not modeling the attenuation that is created or power distribution, that is created by the channel okay so that is something we are not because here we are just interested in channel noise not that invention okay. so we will take that assumption that whatever is transmitted that will be received at the receiver. So this is the power that will be if in the base band directly I transmit the signal without any modulation, so that should be the power that will be launched at the transmitter.

And this corresponding signal is band limited, so what we should employ the best thing we have already talked about AWG and noise so we will say that in the channel AWG and noise will be added okay. So it is the white noise so the spectrum is flat and it looks like this. With the

strength it has $N_0/2$ okay so that is the power spectral density of noise. So there is something that we know already and we also know that the noise follows the Gaussian statistics that means at the particular point you sample.

From the entire signal of noise that is possible and if you plot the histogram with the normalization it will get the Gaussian period okay. And this noise will be in the channel added to the signal that is also something that we know. That is why you have called it white Gaussian noise okay. So in the channel what will happen if I transmit this $m(t)$ at the receiver because we have no attenuation so $m(t)$ will be exact at the receiver + noise signal $m(t)$ will be added.

That should be my signal that is received okay, so let call that as let say $w(t)$ okay. So this is what will be received by the receiver, now what you want do? It is low passed equivalent signal we have already told that if it is a low pass signal I am only interested about the message, if I allow the entire band to come off course my message will come and let assume nothing else were transmitting but the entire noise power will actually come to receiver.

Instead of that we can just employ a low pass filter of bandwidth w , that $-w$ to $+w$, if we do that how much noise will be coming in that will be from $-w$ to $+w$ if you integrate this. So that should $2w \times N_0/2$, so overall noise power should be $N_0 \times w$. so this is the noise power P_N okay, so this what I will be receiving out of which the noise power is this the signal power I already know because we said it is not getting anointed, so that should be the signal power.

So over all we can say signal noised ratio should be corresponding signal power/ corresponding noise power, so this is how we will be characterizing the system, this is just reference or SNR. When we transmit the signal RAW font without any modulation okay so that should be the reference part and then what we will do, we will try to employ some modulation scheme. You do not know what modulation scheme is.

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$s(t) \rightarrow$
 $r(t) = s(t) + n(t)$
 $FOM = \frac{(ENR)_m}{(ENR)_{ref.}}$

There will be some signal $s(t)$ which will be transmitted probably okay, so it will be a modulated of $m(t) \cos \omega_c t$ if it is SSB then it will be $m(t) \cos \omega_c t + m(t) \sin \omega_c t$. so whatever the modulation scheme I will be transmitting this okay. So this is something we will be launching and then at the receiver there will be some procedure because what we wish to calculate is after the entire reception process, whatever signal we get what is the power and some noise that will be also going through that receiver process.

And due to that at the end there will some noise power and we have to take the ratio of these two okay, so that will be the signaled noise ratio of the modulated signal okay. so we modulate we launch $s(t)$ at the receiver we receive it along with noise. So the signal $s(t) + m(t)$ my input at the receiver then the receiver will do all it is processing. If it is the low pass filter, there is a mixture so all those things will be done equally with the signal as well as the noise.

So that is something which we will have to carry through the entire experimentation okay and after that we will try the message signal coming out as usual but there will be also a noise part which will be coming out, which might be now a complicated noise signal okay because it will go through the receiver process and then finally we will try to evaluate the overall power of that noise after the reception and the signal power that will getting.

The ratio of that should be my after modulation and de modulation that should be my signal noised ratio okay, so this is the signal noised ratio which we will be getting for a particular modulation scheme and the signal noised ratio for reference we have got that is we do not any

modulation just base band we transmit and we will be trying to do is, we will try to create a figure of merit of a modulation scheme so we call it as FOM.

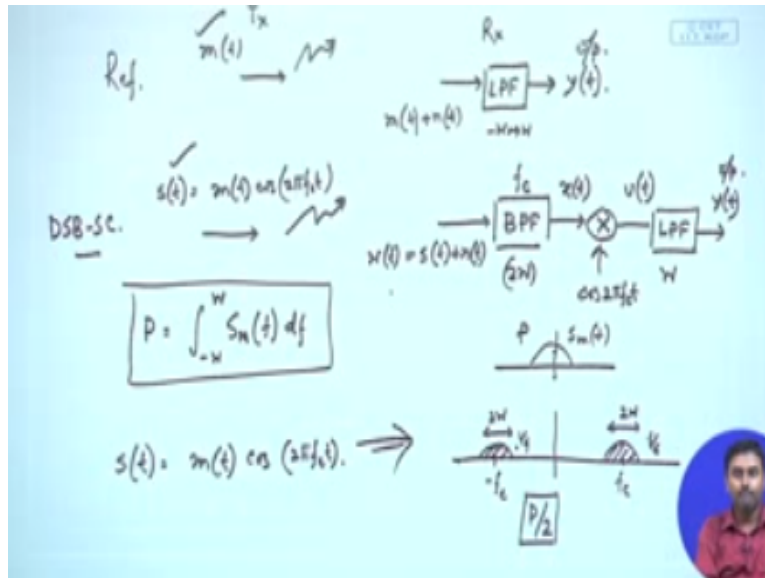
That should be this SNR which is for that modulation scheme so for a particular modulation scheme/ SNR references okay. So whatever value we will be getting if we are doing fair compare for everything we will try to see, whether the SNR improves from it is base band or SNR or decrease. So that is something we want to see, we want to see which modulation actually enhances the SNR performance or which modulation scheme decreases the SNR performance.

So that is why FOM is introduced but here there is something which we have not still talked about, that is the launch power, so if we do not really equate the launch power for this reference signal as well as the modulated signal then we are comparing something to some other thing okay. We are not doing a fair comparison because I can put any power in that particular modulated signal and if you wish to compare them probably that something might be better.

So what we have to also ensure in this process if we really wish to test it we have to ensure that the power $s(t)$ that is been launched that must be = to power that we will be putting in the base band, these two power should be equated. So then we know at the transmitter we are transmitting = power then it is going through the entire receiver process and then finally for both the scheme, which is 1 is base band transmission, because now the power are =. So whatever noise is been added to this process what is the figure of the merit of the modulation.

So probably once we do the calculation it will be very clear but this is the methodology will be applying or adopting towards analyzing this kind of system. So let us try to see what we can do for particular system. So let say DSBSC modulation right.

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So the 1st reference part will be I will put this $m(t)$ right and then this particular this message signal will be transmitted message signal at it is base band I just transmitted at the receiver so this is the Tx part at the receiver what I will do I will take this signal. Now the signal will be contaminated with noise. So it should be this $m(t) + n(t)$ and I know that it will additive, so it is + always, so that should be incident on my receiver.

At the receiver I have a very simple technique we have already talked about that that it will just be a, low pass filter having a band width which is equivalent to that the signal bandwidth, so that should be $-w$ to $+w$. so that low pass filter will be and that should be my output signal $y(t)$, so this is my output okay. So this is for the reference we will be doing. Next I will put a modulated signal which is nothing but $s(t)$ which is $= m(t) \cos 2\pi f_c t$ right.

So this I will transmit, so while transmitting so this is for the modulation, this is for DSB SC so we are now trying to now analyze the whole system for DSB SC okay. so whenever we are modulating we need to ensure this power and this power both of them are launching equivalent power, so this is something that we have to make sure okay. I will receive somewhere so this should be $s(t) + m(t)$. 1st thing we will do is this is modulated so the frequency will be centered around ω_c or f_c okay.

So I have to employ a band pass filter because we have already told that whenever it is modulated I want to take just my signal which is centered on ω_c with band of $2w$ right. so I want also take just minimum amount of noise, so therefore I will put a band pass filter so just the

minimum amount of noise which is centered around f_c and $2W$ bandwidth that will be taken into the account, so I should be putting a band pass filter which is centered around f_c which is exactly the modulated frequency.

And bandwidth is $2W$ okay so that is the 1st thing that we will be putting so this we call as let say $x(t)$ okay after passing through the band pass filter. Now there will be 1st thing for DSB – SC what do we do? We first put through a mixture we will be multiplying by $\cos 2\omega_c t$ or $2\pi f_c t$ so we just multiply by this, sum of we will do carrier synchronization we are not talking about that part okay. So this is just the simplest part where the carrier synchronization is differently done. Where separately we carrier synchronization we are not bother about that right now.

So with that I will get $v(t)$ which will be generated this we are calling as $w(t)$ and then of course whenever we multiply by $2\pi f_c t$ there will be higher frequency term that will be created which we have to filter out, so we will be putting a low pass filter that is actually the DSB-SC which is again centered and that must be my $y(t)$ or output okay. Now what is happening? This entire process your noise also will go through this process.

So you have to characterize it properly that whole thing, the 1st process of this is equating these 2 powers, so let us try to see how we do that. So already we have said that my P is we have already told that, this is something we are assuming, that the overall signal if we integrate over the band of interest that is the power we gets okay. Now whenever we modulate let us try to see what kind of power we will be getting because we have to also launch similar amount of power okay, so that is something we will have to do.

So let us try to see when we modulate what we will get? If we modulate I will be getting this, so that is the $s(t)$ which is $m(t) \cos 2\pi f_c t$ right. What will be corresponding perspective to density? So if this was SMF after modulation we already know from the pass that this will be just, if this was my SMF this will be going towards $+f_c - f_c$ with the same band of $2W$ and the strength will just half right.

So basically sorry the strength will not half it is $\frac{1}{4}$ because $m f$ becomes $\frac{1}{2}$ and square of that because, so it will be $\frac{1}{4}$, so this will be $\frac{1}{4}$ th of that whole thing or overall if I integrate this must be $\frac{1}{4}$ th of this thing okay. so over all power of this one will be $\frac{1}{4}$ th of this, so if this was P that

should be $P/2$ this integration will give me another $P/4$. So $P/4$ over $P/4$ over here so if you add that so over all power should be $P/2$.

So this is something we know any were you multiply with a cos sinusoidal or sinusoidal you will be Halfling the power. So immediately we know that the launch power should be $P/2$ so this is something we should be aware of okay. Now let say for the base band can be analyzed, so to be more specific or generalized.

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$$s(t) = c A_c m(t) \cos(2\pi f_c t)$$

$$P_s = \frac{c^2 A_c^2 P}{2}$$

$$P_n = N_0 W$$

$$SNR_{\text{avg}} = \frac{c^2 A_c^2 P}{2 N_0 W}$$

Let us try putting this instead of just putting $m(t) \cos \omega_c t$ let us try to put a factor into it okay. So where there is the amplitude of this cause and there is the modulation factor let say that is c , that is within our hand and we will be equating the power so it does not matter right. So let us say instead of $s(t)$ writing as $m(t) \cos \omega_c t$. We write it as $c s_e m(t) \cos 2\pi f_c t$. So immediately what will be the power of it?

That must be $m(t) \cos \omega_c t$ that must be $P/2$ and this because it should be square of this, power should be $e(s)$ should $c^2 f_c^2 / P/2$. So that is happens to be if I try to transmit this particular signal that happens to be the power. What we said already that this power must be also launched when we do base band that reference signal because we have equate these two power we need to actually put the same power. So we need to equate these two powers so this is P_s the same power as to be launched for base band.

We will be launching again instead of this launching this P power in the base band will be launching this amount of power, so which is $c^2 s_c^2 P/2$ this amount of power, so accordingly will amplify the signal and whatever need to do we will do that okay, so that the overall power will become this. So if this the power noise remains the same right so what will be noise power? Noise power will be whatever we have calculated so which was $N_0 W$ so that is the noise power.

So therefore what is the SNR reference that should be this power / by this noise power already passing through low pass filter, this noised power as been already evaluated passing it through the low pass filter of band width w okay. So therefore it should be $c^2 s_c^2 P/2 N_0 W$ so that is one part of our calculation. SNR reference has been evaluated, so now let us try to do for the actual modulated signal right.

We have drawn it so this must be my overall de modulation process where $s(t)$ is given by this of course $c \times s_c$ should be there okay so let us try to write that.

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$$\begin{aligned}
 x(t) &= C A_c \cos(2\pi f_c t) m(t) + n_1(t) \cos(2\pi f_c t) \\
 &\quad + n_2(t) \sin(2\pi f_c t) \\
 v(t) &= x(t) \cos(2\pi f_c t) \\
 &= C A_c \cos^2(2\pi f_c t) m(t) + n_1(t) \cos^2(2\pi f_c t) \\
 &\quad + n_2(t) \cos(2\pi f_c t) \sin(2\pi f_c t) \\
 &= \frac{1}{2} C A_c [1 + \cos(4\pi f_c t)] m(t) + \frac{1}{2} n_1(t) [1 + \cos(4\pi f_c t)] \\
 &\quad + \frac{1}{2} n_2(t) \sin(4\pi f_c t)
 \end{aligned}$$

$x(t)$ so we will try to see that $x(t)$ after passing the signal + noise through a band pass filter so that is $x(t)$. What is $x(t)$? $x(t)$ must have that $s(t)$ and the noise which is pass through the band pass filter. In the last class we have already characterized a band pass noise right we have told that, a band pass noise must have an in face component, we have already characterized those things.

Those are also random process which as we have also evaluated that they do not have any cross relation right, this is something that we have already and they are generally orthogonal to each other okay this is something that we have already stated. So we know that the noise must be now represented as in face component, so I can write like this. so that is something we have learnt from the random process. So $x(t)$ must be my signal because signal through that band pass filter remains in intact.

Because it is a band pass signal of that same band which were I am putting, so signal remains unchanged, so that should be $C A_c \cos 2\pi f_c t \times m(t)$ that must be my signal + noise which was n now it is pass through the band pass filter. So I can write as $n_1(t)$ which is the in faced part $\times \cos \omega_c t$ which is central frequency or $2\pi f_c t + n_2(t) \times \sin \omega_c t$. We have already told that is the representation we can get whenever we pass a white noise through a band limit.

Now what will happen? If I again go back to circuit $x(t)$ gets multiplied $\cos 2\pi f_c t$ right, so what eventually is happening by signal not only the signal is getting multiplied by $\cos 2\pi f_c t$ the noise will also be multiplied by that. So we are actually in the process that additive noise we are taking

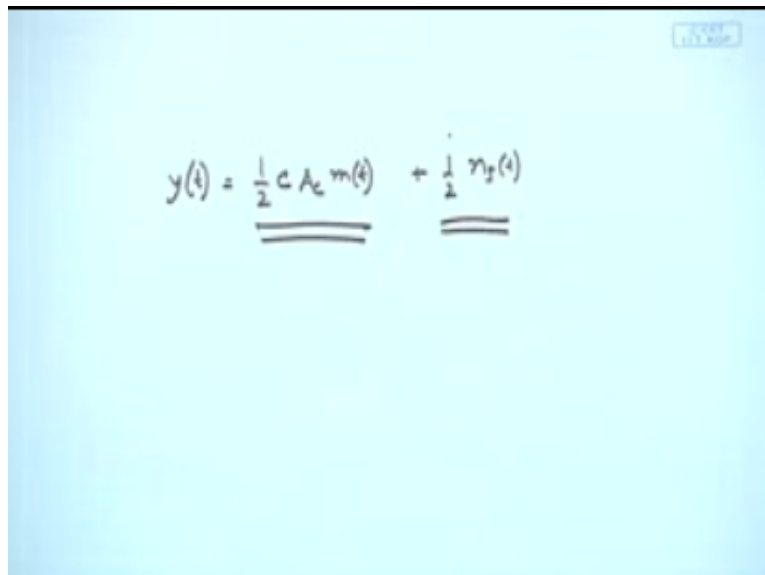
through the receiver and every step we are trying to see what is happening to the noise after doing that step.

So let say that is $v(t)$ which is nothing but $x(t) \cos \omega_c t$ or $2\pi f_c t$ that must be this multiplied by $\cos 2\pi f_c t$, so $c A_c \cos^2 2\pi f_c t \times m(t) + n_1 t$ again $\cos^2 2\pi f_c t + n_1 t \cos 2\pi f_c t \sin 2\pi f_c t$ right so this is something what that we get. So we get the overall expression after the multiplier okay. Now what we will do? We pass it through the low pass filter; first of all let us try to see which are the higher frequency term, so this $\cos^2 \frac{1}{2} C A_c$.

So \cos^2 will become $2 \cos^2$ so I can write as $1 + \cos 4\pi f_c t$ similarly I can write for this one also, $n_1 t (1 + \cos 4\pi f_c t)$ this is again $\frac{1}{2}$ this is $2 \cos \times \sin$ so that should be \sin , $n_1 t \sin 4\pi f_c t$. Now let us pass it through the last part of the receiver circuit is this low pass filter or band with w . So which are the components of this one, so I have this component out of them which are higher frequency term, which the low pass filter will reject?

That must be rejected because f_c is much bigger than w therefore $2f_c$ must even $> w$ so that must be rejected. So this must be rejected this also = must be rejected, so finally after low pass filter let say this.

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$$y(t) = \frac{1}{2} c A_c m(t) + \frac{1}{2} n_1(t)$$

$Y(t)$ it remains to $\frac{1}{2} C A_c \times m(t)$ that is related to message signal + noise term which is $\frac{1}{2} n_1 t$ okay. so we have left this two terms which is very good for us because the signal and the noise

term are clearly separated, so all we have to now do is try to evaluate the power of this one and try to evaluate the power of this one and take the ratio of these two and then try to see how this is compared to the reference. So what we will do in the next class we will just try to see this figure of merit of ASB SC okay thank you.