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NPTEL ONLINE certification course

**Course
On
Analog Communications**

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Lecture 43: Random Process (Contd.)

Okay so what we have done so far in the last class is that we are trying to characterize the band pass noise right and for that target we have actually devised a circuit and we are trying to evaluate the transfer function of that circuit right so by exciting it with a delta function we first proven that that particular circuit is a linear time-invariant circuit and then we have proven actually derived the transfer function of that circuit and we could prove that it is just nothing but a band pass filter okay.

And then we could show that if a band pass signal exactly matching with its band if it is passed through this it will remain same okay so that is why Xt if that is a band pass noise exactly matching with its band then at the output also it will remain as that and then we could say that okay the Xt can be represented because of that circuitry can be represented as some Xe and excess in-phase and quadrature right. So that is something we have proven.

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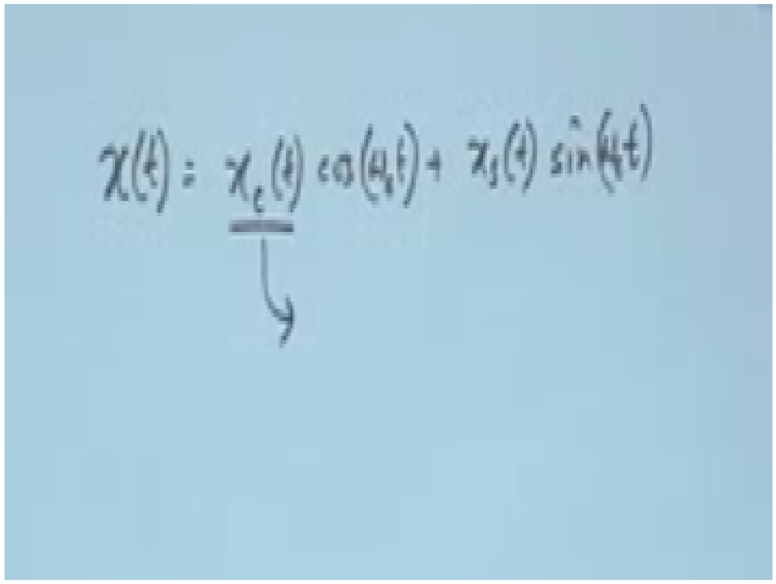

So therefore finally it came to that Xt is Xct cos ωct + xst sin + X st sin ωct where we know that this Xct I can actually trace back them in my circuit the circuit we have shown.

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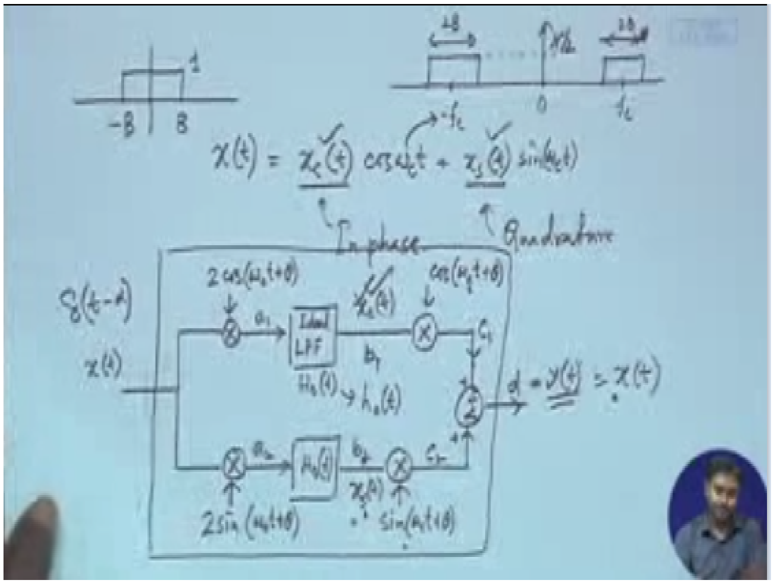

Sso Xct is nothing but equivalent to b1 and Xst is nothing but b2 okay so we already know that what they are okay. So now let us try to see what is the characteristics of this Xct so Xct if we just trace back through this circuit it is nothing but this Xt okay multiplied by this and then pass through a low-pass filter ideal low-pass filter right that is actually be 1.

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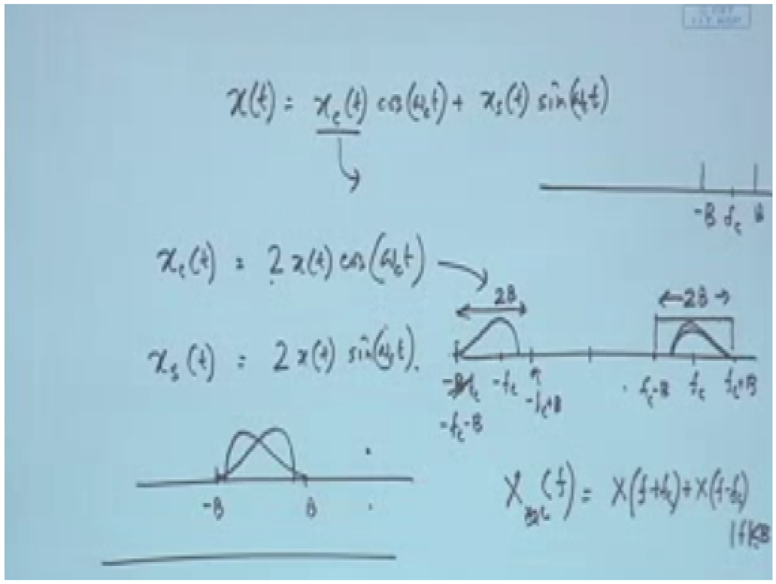

So I can write that as 2xt cos ωct we are again taking θ to be 0 okay because it is valid for any θ so I can always take θ = 0 so this is xct pass through our ideal low-pass filter as long as my Xt is band limited okay so basically the Xtis limited to - B to +B within that fc then if I just multiply it with cause so it will actually come down over here I will just take that low-pass portion of it okay.

So it remains this only we have to make sure that it is within that band - B to +B okay so that is actually Xct which is low-pass equivalent of course it has to be passed through a ideal low-pass filter and Xst similarly should be 2 x Xt sinωct right so let us first try to see this one so what happened my Xt suppose that looks like something like this that is so this is fc it must be symmetric.

So something like this so let us say this is - B - fc or I should say - fc - B and this is- fc +B that point okay so this is actually me to be similarly this is fc +B and this is fc – B this is my 2b okay so now if this is Xt what is Xct the in phase component that is nothing one this xt multiply cause and two is there because whenever I multiply cos in the spectrum if I wish to see there will be a ½ term so that will be cancelled out so it is just whenever I multiply cos this whole thing will be shifted to +fc and - fc right.

Once I shift this this particular thing once I shift to + this will come over here and then this particular term will go to how very high frequency 2fc and when I shift to the other side this will come over here and this term will go to - 2fc those things will be anyway cancelled because there is a low-pass filter so what will remain over here at the center is something like this from - B to +B this particular pattern will be repeated and from this to this particular pattern right.

It is the addition of these two which will be the Xc okay so or if I just take a Fourier transform this so Fourier transform let us say it is X let us say Xc or Xc right so what that should be that should be actually Xf +fc - fc so original f f + f c +Xf –fc but this must be defined for af where mod f ≤ B okay which is just like this all the higher frequency term will be neglected it is just these two part will overlap and whatever I get okay so that is actually my Xct and corresponding Fourier transform of that okay.

So I immediately get the corresponding Fourier transform this one right it is very easy to get directly the Fourier transform as long as I know the Xt means the power spectral density so basically what I can do instead of doing this I can relate them in terms of power spectral density.

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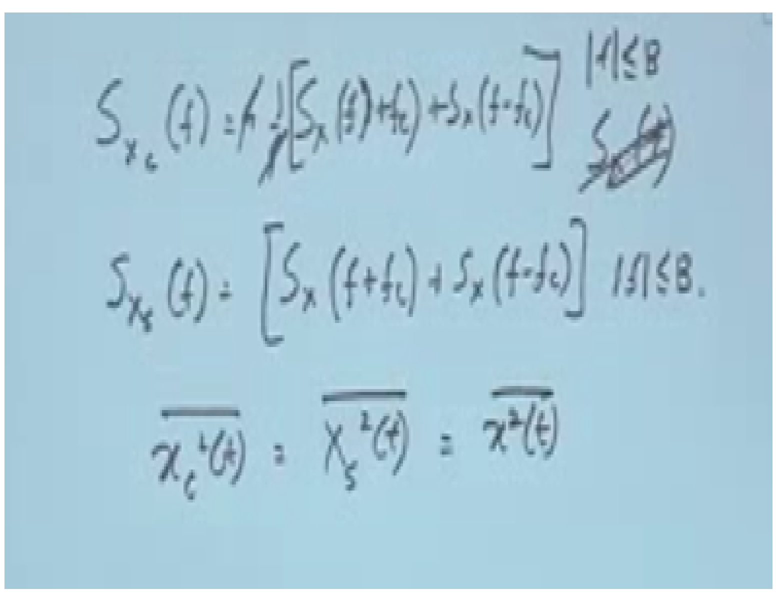

So I can write Sxc f which will be just this whatever Sx suppose Sxf is known that is for the Xt know it is average Fourier means average power spectral density so this should be multiplied by2 cos ωct so therefore it should be that Sxf must be shifted to + fc – fc so I can write Sxf sorry + fc + x f – fc and then I have to put this condition that mod f ≤ b it should be this and then for because it is power spectral density so therefore for cos there will be a ½ term which will be squared.

So there should be a 1 / 4 and there is a 2 term already that should be squared so it would be 4, 4, 4 gets cancelled so this happens to be my power spectral density of Sxc f okay so without going into the autocorrelation or cross correlation function I directly define my power spectral density as long as Xt is defined I can always get the corresponding Xc that is the in phase term his power spectral density similarly Sxsf also will be same because whatever sin is there it does not matter when you take power spectral density it will be squared and all other things will be cancelled it will just be + again f +fc Sx f – fc again defined for this okay.

So immediately I can see the power that we wish to evaluate so if you just try to calculate this power that must be the integration of this one for the whole spectrum okay so this is just defined within B so it should be this + this okay so Sx +fc within this and -fc within this so that should be the power of this similarly Xs^{2} t also have because they are having same similar power spectral density.

So it must be same and not only that this is also same as this because think about these two power spectral density.

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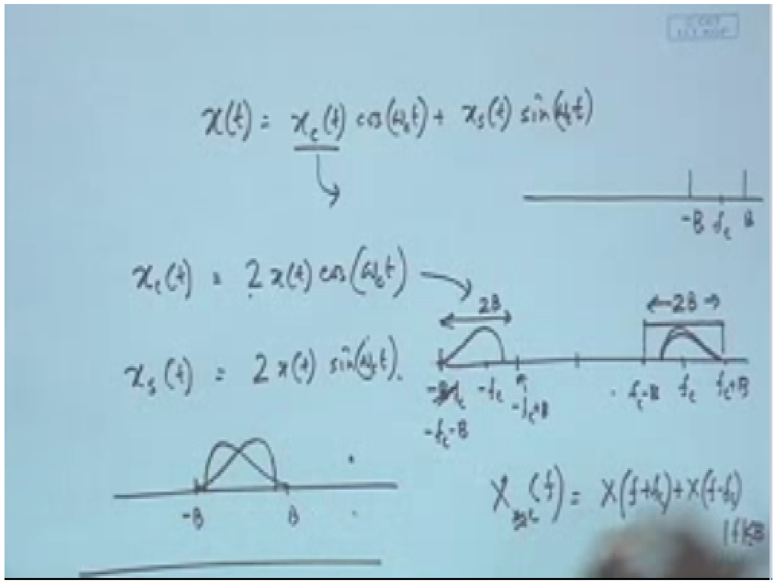

So if I just go back to this example what is happening the power spectral density of this one will be this +this same thing happening over here it is just this portion is being repeated over here this portion is being repeated over here without having any attenuation so the power spectral density will be just addition of these two which is same.

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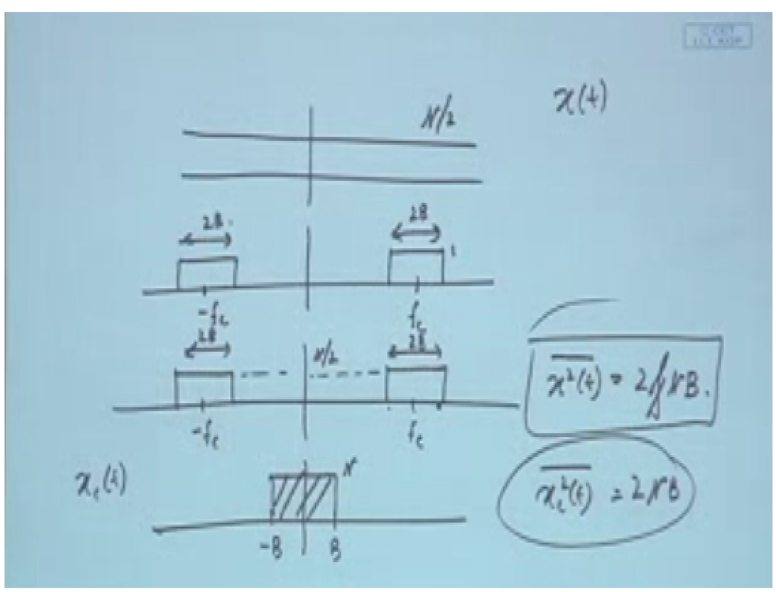

So therefore the overall power remains the same for the in-phase component or the quadrature component as well as the random process that we are talking about this will be always happening as long as the bands are properly adjusted okay so therefore what we have seen so far is we have seen how do we actually evaluate a band pass random process through equivalent to low pass random process which are called in-phase and quadrature component okay so that Xct and Xst and we have also seen the interrelationship between their power spectral density okay.

So that band pass process how the Xct can be related to the corresponding Xt that is something we have seen and we have evaluated their power also okay so this is something we have already done now what we wish to do is something like this that we want to evaluate the cross-correlation of this Xc and Xs now we have to two things so this Xct and Xst + τ which is nothing but Rxc xs okay.

If it will be stationary so what will be able to prove that this will be just 0 okay so for that you just have to take this relationship Xc and Xs and follow through the means you know already the autocorrelation function of Rx just go through that follow the steps you will be able to prove that this is zero so basically this means they are uncorrelated they are incoherent this is something will be always able to prove okay.

So that is another thing which will be probably means heavily required for our analysis of noise so whenever later on you will see whenever we have a band pass noise that is coming out because of the band pass filtering will be always writing it in terms of in-phase and quadrature because once the noise comes in it is the noise + signal which goes through the whole demodulation process.

So we have to actually take the signal as well as noise going through the demodulation process so if that has to be done then I need to have a representation of the noise so that is the representation of the noise that NT suppose it is noise which is a random process that must be Nst sin ω CT + Nct sin a cos ωct so always I will be able to write that way and then I also know they are corresponding power spectral density their cross correlation all those things we know already because we have characterized them already so once though all these things are known you will be able to see that we have a strong tool set which will give us some analysis okay.

So all we are trying to do over this exercise that definition of random process what stationary how do I get past pectoral density what sort of correlation and then if I have a low-pass noise band personalized what are the corresponding representation so all these things we are doing just to add ourselves to do noise analysis of the system that we have already discussed so far all those different kinds of amplitude modulation systems okay.

So we will just give two more things which will complete this entire discussion probably so those two things we will just discuss briefly and after that we will give you an outline of how we actually can do noise analysis so let us just first try to see go back to our noise which was something like this right so we want to characterize that noise so which is having a strength of η/2 righ.

Now this noise is passed through a band pass filter okay centered around fc and band is to be similarly - fc band right how the noise will look like after passing it through that so it will be a band pass process so that should look like this so it should be still the strength should be 2/ 2 because these are all one so mod hf^2 will still remain 1 pass through this we have already proven that it should be mod hf^2 the input noise input noisy η / 2.

So we just multiply that and we should get this right so that strength should be η/ 2 okay defined at - fc +fc and within this band of 2b so what is the noise power so let us say this noise is defined as xt so x^2t that must be integrated noise power is just integrated over this entire band okay so which is nothing but η / 2 so 2 x integration from this to this right so that is nothing 2b x η/2 2 so that is η x b 2 times so that should be 2 x η x B that is my overall noise power right.

Now let us see what is my Xct so what is Xct Xct must be this shifted to this and this part shifted to this so therefore how much I do get it should be band limited from - B to B but this one will be shifted and this one also will be shifted so overall strength will be η/ 2 + η /2 so that must be η what is the power of this one.

So Xct^2 bar there is also 2 η x B if I integrate over this which we have already said that this and this power will be equivalent similar things will be happening for Xst as well right for Xst also same representation will be happening and I get 2η B so that is how we will be evaluating the noise power and will be evaluating the corresponding power spectral density of noise.

Whenever we have a noise okay so in our noise analysis will be taking the noise as this which is a flat spectrum okay and which our filter will be employing be at low pass or band pass will be always taking them to be ideal because if you do not take ideal then probably the analysis will become too much complicated okay you can always do that but just to see the performance probably for both means if I wish to compare performance of two systems I will take ideal filter for both of them and then try to analyze it.

Because it just becomes life becomes much more simpler okay so that is something we will be trying to do now let us try to talk about another thing okay which is called that noise we are characterizing what is that noise you might be hearing this particular term so often that the noise is AWG okay.

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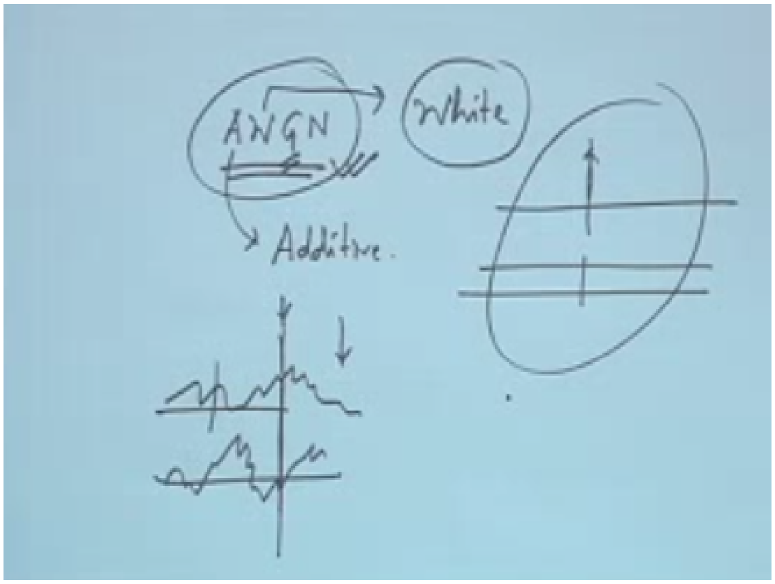

For AWGN or the channel is called as AWGN channel okay what do I mean by this the first term is called the additive it is just defining the noise right that means I have already assumed that noise either is generated in the channel or in the receiver circuit whichever place it is being generated its then passing through the channel will be linear and the circuit also will be linear so as long as they are linear so basically the noise will be just added on top of signal so it is additive noise not multiplicative or any other kind of noise.

So it is just additive noise so you will be always assuming that if I have signal the noise will be just added to it okay so that is the first thing additive then the next term called white so the noise that we have characterized which is probably the most random noise that we know which has autocorrelation function of impulse and corresponding spectral density which is flat this is what characterized it to be white that means it has all the frequency component equivalently with equivalent strength it just comes from that concept of white light.

So what is white light that it has all other lights or all other wavelength that characterizes different light whether it is violet or blue or any other things so all those are equally added to create white light okay so same thing the white noise is being created by equivalent noise component at every frequency as many frequency component you can think of up to infinity so all those frequency components are equivalently present in this particular noise and because the noises like this it just comes from the autocorrelation function of our understanding from there because it is like this that every frequency component or equivalent present we call this as a white noise.

So most of the time will be considered white noise the next part is G which is the Gaussian okay so what does this means this actually says that see this is the power spectral density this is the autocorrelation actual noise will look like something like this right and it will because it is a random process you can have different such samples okay in finite number of such samples which will look different if you take different samples fix any instant it is a stationary process we have already talked about that that whatever that noise is that is already stationary okay.

So anytime you pick I know that the statistical property does not change okay so if I just pick this time instant or this time instants and try to take this samples put a PDF and what I will observe that this is following a Gaussian process we have already discussed about Gaussian process and that is why we have discussed Gaussian process because it is so important that most of the things probably we have not discussed one important property of a random variables that if you have many number of independent random variables it is infinitely many and if you just all of them are independent if you just add them together the PDF that you will be getting that always goes towards Gaussian okay.

That is called the central limit theorem we have not discussed that but in probability theory we will be learning those things so because of central limit theorem we know that always a process which is constructed of many infinite independent processes will tend towards Gaussian process so the noise is generally being generated by infinite number of things which are naturally happening inside the nature okay so because they are all independent and they are all adding up to create noise.

So we can all very safely say that it follows central limit and the noise process will have PDF which is Gaussian so that is why this noise generally is taken to be Gaussian okay so what does that means that any sample time you take they call the in symbol put them in a histogram normalize them whatever PDF you will be getting that will look exactly like Gaussian PDF with a particular σ^2 okay which is actually the power of that right and that $\sigma^2$of that Gaussian should be equated to the power that you are calculating from the from the noise like for this band pass noise we have evaluated this to η B is the noise power that must be the σ^2 of the corresponding Gaussian.

Because the Gaussian process has two things one is mean and variance these two has to be specified so variance is specified and all these Gaussian noise are generally because they are created by random process which can means either create positive voltage or negative voltage equally likely so it is often having zero mean okay so always the signal which is characterized by noise signal will always have zero mean so we can always say that it is always characterized by a Gaussian process which has zero mean and the σ^2 is just equivalent to the power that we are exactly okay.

The noise power we are thinking of so that noise power will depend on that power spectral density of it and the band we are operating at so accordingly we can the way we have calculated noise power will be getting that power and that σ^2will be equivalent to that power because these two are code means exactly equal okay so that is why whenever we talk about a particular channel and a noise process that is being added we always take help of additive white Gaussian noise there are other noise for different system like you know optical system probably it will not be similar things okay.

So because of photon counting and that counting process becoming poison so there will be different kind of noises which can also be added but generally speaking most of the receivers are being analyzed with respect to additive white Gaussian noise so that is why it will be very important that we know this all these terms so whenever will be because of this presence of this noise whenever will be characterizing noise will always say the noise amplitude will be just directly added to the signal amplitude nothing else its additive okay and the noise power spectral density will be flat with the means power spectral density specified as η or it to by to whatever you like and the noise statistics or the PDF if you just take a particular time instants freeze the time instant and take the PDF will always get that to Gaussian.

These are the criteria which will be always fulfilled whenever B means concern ourselves with respect to the noise that we are analyzing okay so this is one thing I wanted to characterize the other thing I wanted to say that this in-phase and quadrature representation that we have done that is not unique.

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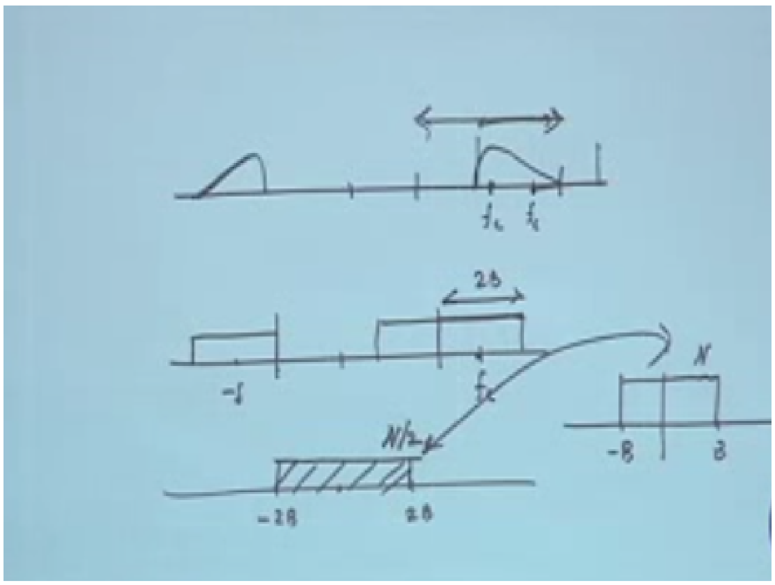

So let us try to appreciate that part you remember we had a means we are just trying to show that suppose I have a band pass signal or random process which is characterized like this okay and this is the band where it is defined now I want to have a quadrature and in phase representation what I will do first I will choose fc.

Now carefully check I can choose the fc over here I can choose the fc over here once I choose the fc over here what I have to make sure that the entire signal means that Xc if I wish to represent that my bandwidth will be represented in such a way that the entire signal comes under that band so accordingly I will be defining my band if my fc is over here then I will have to define my band accordingly once I define a fc and corresponding band then immediately if you try to represent the Xc or Xs that will look different definitely because they will have a different kind of shape which will be translated into the central frequency.

So there is no unique representation of these two things okay so whenever we talk about that in phase and quadrature component it is not by definition unique it just depends on where you put your fcthat central frequency because you will be given a band pass equivalent signal nobody will tell you with that spectrum that this is the central frequency that is according to your choice you can choose a central frequency accordingly you will have to adjust the band and once you adjust this band and central frequency.

There will be a definite Xc and except which will be created and this will keep on changing as you change your Xc and Fs as you have seen also that particular circuit we have drawn that is completely the transfer function of that circuit depends on what cos ω CT we give if we keep changing that ωc and what kind of bandwidth we put for that middle low-pass filter that h0f if we change these two the representation accordingly will change so be very careful whenever you are representing it that you should understand that there is no unique representation it just depends on your fc definition.

And the band definition as you define them accordingly or Xc or Xs will be changed I can give you a very simple example the one we were doing that band past noise that this was my fc by default and this was to be right and this was - fc and I was having this now suppose I do not do that I define fc over here so then the band has to be taken as this that mean before B and then if I translate this what will happen the overall band will be from - 2 B 2 +2 B and the strength will be just η / 2.

Earlier what was the representation because I have defined it at the central so the representation was from – B to +B and the strength was η right so you can see already there are two different representation as I define my fc differently and accordingly corresponding back but the overall power if you wish to calculate it will remain the same so the noise power and the associated in-phase and quadrature component whatever you define their power will remain the same it is just their spectral property will change due to your definition of fc and 2 B.

So I think we have almost finished our discussion of random process and we are now almost ready towards jumping into noise analysis so what we will do in the next class is we will try to see how the means what should be our way of doing noise analysis because whenever we do not do noise analysis we want to compare two system and suppose DSBC an amplitude modulation how they perform in terms of noise can I say this is better in terms of noise okay or in terms of noise cancellation.

So how do we actually benchmark them what is the criteria to do that how we evaluate our things so that we can actually compare two processes so first we will say that with that we will have a process which will be created means for evaluating this particular modulation schemes and then we will go on analyzing those modulations okay thank you.