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NPTEL ONLINE CERTIFICATION COURSE

Course
On
Analog Communication

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Lecture 04: Fourier Series (Contd.)

Okay so now what we will do we have already discussed about how to represent a particular vector with respect to another vector now what we will do on this staple which is a two dimension we already know we will just take a two vectors one is x.

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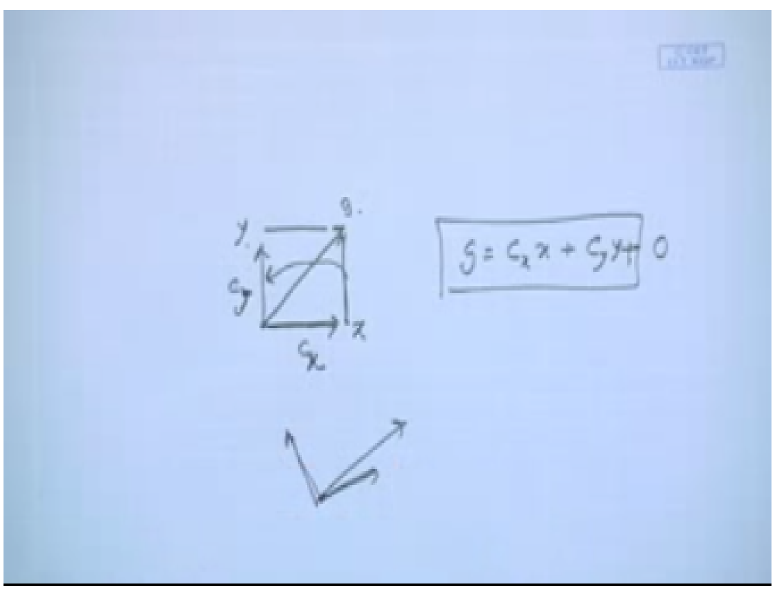

The other one is y now we just do the same exercise we will try to represent a particular vector let say any vector let say this is g okay again what we need to do first g needs to be represented by x so we will again putting a perpendicular an accordingly will you calculate c or let us called that cx okay, g also can be represented with respect to y is I will again put a perpendicular an I will get another value C2 or Cy let say so two constant we have got, now I am trying to represent it with respect to these two vectors so what I do I actually take both the vectors so basically I am taking a liner combination of these two.

Vectorial representation okay so g idea id just where representing with respect to some c into x now I will do a linear combination so some cx into x + some Cy into y and if we can properly minimize so right now we have just shown how to minimize that but if you just put it this way and try to minimize it you will see that as long as x and y or orthogonal to each other, will always be able to completely represent this without any error because what will happen if you represent.

By this, this will be your error now this can be directly with represented by this one so error will be further represented by the other vector and then the residual error that is left that become 0, okay so the lesson that we have learn that if you can so two things one is if you can whatever dimension you have the suppose vectorial space so this is a two dimensional vectorial space, in that vectorial space whatever dimension you have, so it is a two dimension so two orthogonal vector to each other.

And find out those any two so it does not have to be this x and y I can represent g even by this x and this y as long as these two are orthogonal to each other okay so if you can take any two mutually orthogonal vector because the dimension is to with those two vector you can always perfect they represent any vector in that particular steps, if I wish to represent it in three dimension, I need to have three vectors so that is why they actually represent it with respect to x y and c.

So 3 point that you need or three vectors you need okay as long as this always you know that xy and z are orthogonal to each other all you have to do is we have to all mean worry about the dimension of the space you are talking about vector space you are talking about and we have worry about or you have to find out, if the dimension is n, n number of mutually orthogonal vector as long as you can do that then you get a complete we call that position, and with those basis said any other vector.

In that particular vector space you can represent them without the error so and eventually what happens it is just a linear combination of all this vectors okay so this is the fantastic thing which has been already understood and which also you are also familiar about this okay so you already know that this is how it should be, what will now do will try to see if a signal any general signal can be represent it by some constituent basis this is what will be targeting and that is the basis of fouriers so let us try to do that let say I have a signal gt.

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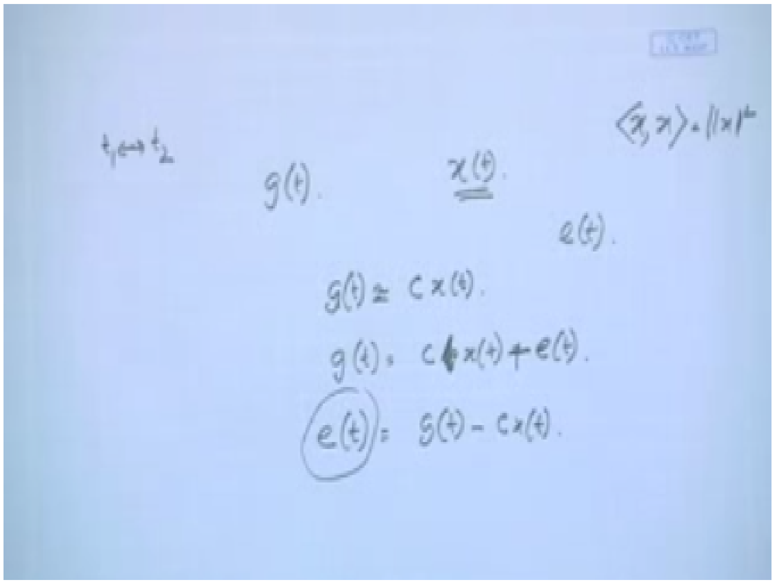

I wish to represent it with respect to some non signal xt so xt is the non signal okay so this is what I wish to do okay, so I am saying that if only xt is there like vector I will be just multiplying it with the scalar quantity okay so I will say it is almost represent it by this c xt now all we have to do similar thing if I do this representation of course there is a error which is called that et, so therefore I can write gt must be equal to cxt + the error signal et so et must be al we have to do is now minimize the error.

That is what we have learned earlier that in the vector we have minimizing it there it was easier because we are just seeing where it becomes projection they are the error vector will be of minimum length okay that means a measurable quantity vector we are talking about length and length is nothing but plus second non that is the length square of vector so that is how we where minimizing the error basically we are doing error length minimizing that is what are doing here also.

We will try to or target to minimize similar things so what we are trying to do is let say this gt and xt they are defined over a time let say t1 to t2 okay so it is initially for a simplicity we are saying that this signals are time bounded it is only defined from t1 to t2 both tt and xt so we want to now minimize the overall error, so what do you minimize we have already characterized a signal that is actually the energy of the signal so we will target to minimize the energy of this error signal so our target should be let us calculate the.

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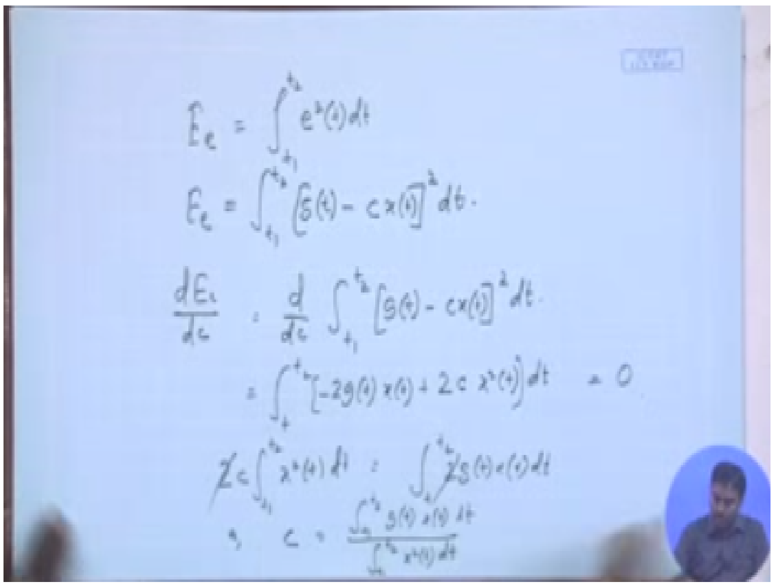

Energy of error signal that should be integration is square t dt from t1 to t2 right now et we can replace t1 to t2 gt – c xt whole square t okay so this is what we are targeting to minimize, gt is a non parameter ort known signal or this is the target signal that we are trying to minimize xt is a known signal with which we want to approximate the other signal here with respect to what we are trying to by minimize it is to see think about that vector also there also we are doing the same thing.

With we are trying to get a optimal c for which the error will be minimized same thing we will be doing over here so here we should minimize with respect c that means we will differentiate it with respect to c and make it 0 find out that value of c that should minimize so we juts device this now force integration is over t and differentiation is over c they have no depended is so I can interchange these two right, so I can take this thing inside differentiate g suppose I open up this square.

So it should be g^2t differentiating with respect to c that will be 0 then – 2c g txt so if I differentiate it with just – gt xt right and differentiating with respect to c remember, so it was c gt and xt the dc of c because gt and xt does not depends on c, so that will be coming out it is just 1 okay the next is $+C^2$ x^2t right so C^2 x^2t should give me 2C x^2t dt is must be 0, this is alright so I have just take dt c inside and I started differentiating it I know that t g txt they do not depend on C.

So those downs I have juts open the square and they started differentiating it okay so I have got this values, now if this should be 0 then immediately I can find out C because C does not depend on the integration variable, that is t so I can write this as 2c integration x^2 t from t1 to t2 dt = integration t1 to t2 gt xt dt I can write this or C becomes this is fine oh so there should be a 2 because it is 2gt xt so there should be at least two cancelled so C becomes t1 to t2 dt xt dt / right g txt dt integration x^2dt so basically this is what I do get okay that is a very important criteria. If now think about that vector an logic when I was putting the vector thing what was my C.

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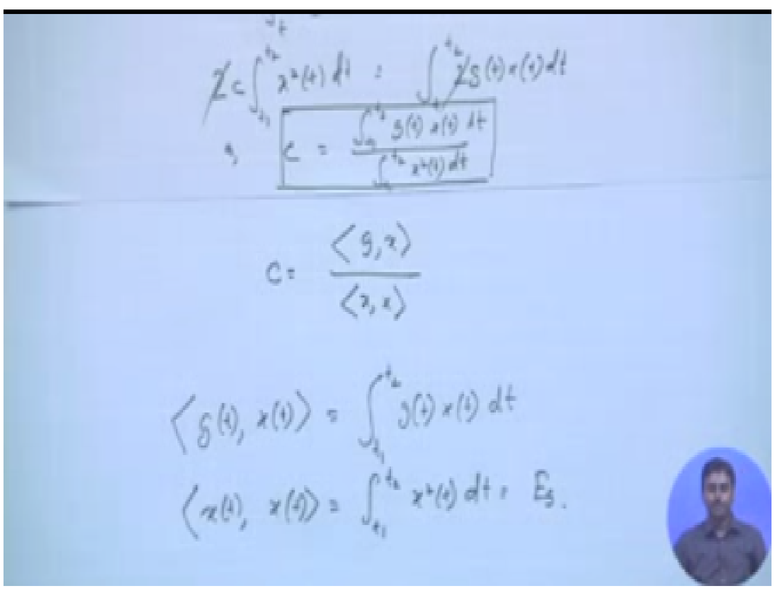

C was dot product gx / dot product x x right now I can define y. product over this signal space so that dot product now gt and xt if I defined my dot product as this one wherever the signal is defined from t1 to t2 and then g xt dt okay so if defined this as dot product what happens immediately you can see, see all the things are getting connected now if I now put xt the dot product between x and x that happens to be the energy of x there also characterizes by ideal proportion that the way I was putting dot product.

Dot product is eventually in a vectorial space giving the, the measurement that is the distance of that vector here also the dot product is giving the, the measurement that I have characterized so far, that is the energy of the signal so x, xt. Product will be t1 to t2 x^2t dt this is nothing but the energy that is very good the dot product in vector also gives the measurable quantity that is the length of the vector dot product defined by me over here also gives the energy which is the measurable quantity of a particular signal.

Okay from here again we can talk about orthogonal so when a signal will be orthogonal to another signal when that similarly in vector if I cannot represent a vector by another vector then we say that these two are orthogonal that means this corresponding C must be 0 here also same thing I have defined my C as this okay, or if I just write it one more time the C is.

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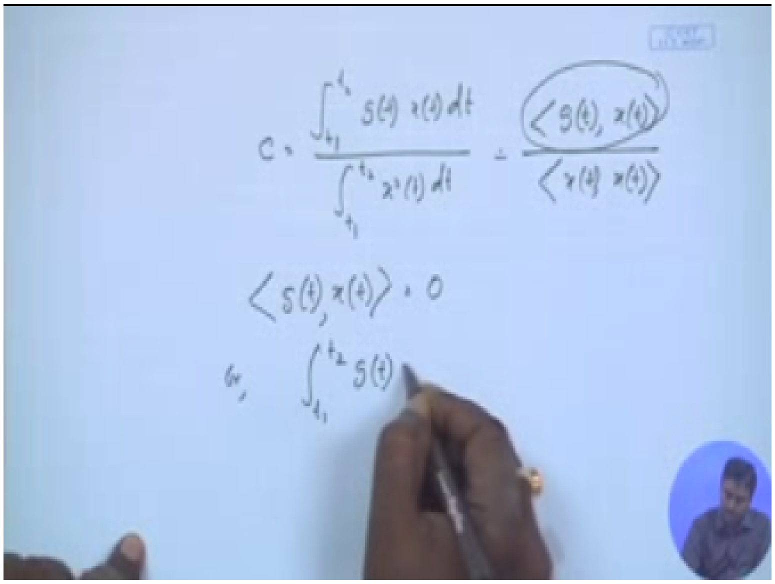

Integration t1 to t2 gt, xt, dt / t1 to t2 x^2t dt which is nothing but dot product of g txt the way I have defined dot product and dot product of xt, xt right so if C has to be 0 that means x gt cannot be represented by xt and gt and xt are orthogonal and C must be 0 immediately the condition comes at this must be 0, so therefore gt xt must be 0 or we can write g txt dt = 0 so that is the famous orthogonality condition for a signal right, so this whenever we talk about the signal two signals xt and gt will be orthogonal to each other.

As long as this condition is satisfied if the signal is defined from t1 to t2 then the multiplication and integration within that limit must be giving the 0, okay so this is the criteria for orthogonality and similarly we have seen the representation okay now we will do some more things, so whatever we have talked about right now this was actually good or a real signal we have talked about all real signal and that is why energy calculation was easier because energy is a measurable quantity right.

So in a measurable quantity I want something real and I was a doing squaring and integrating it now if I have supposed xt.

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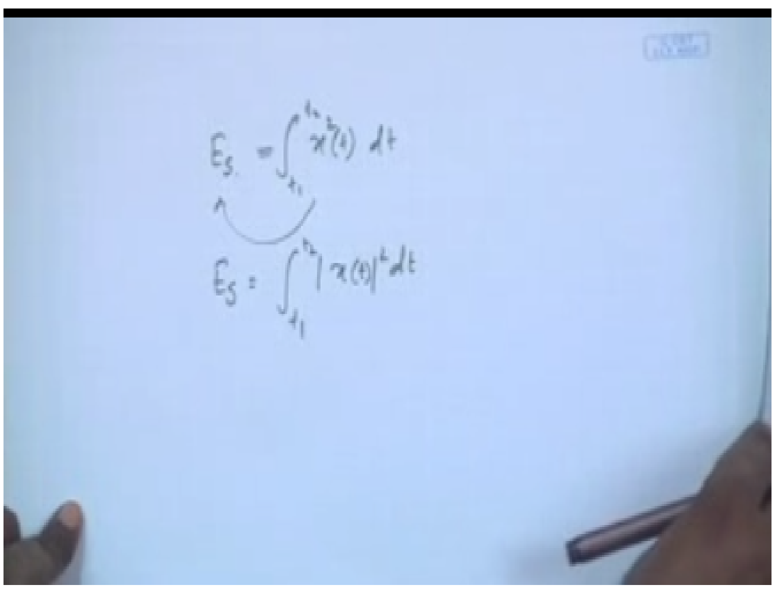

Okay I wish to do this integration that you want t2 dt this is my energy now if the xt happen to be a complex say which has a real path and imaginary path if it is a complex signals then I do not have this evaluation because then immediately I can see that eg will not be a real number, so to do that there is a way to do that so eg I can define in this fashion, so basically the definition of eg the happens to be that if it is a complex is real it will be just x2t if it is complex then I actually take that complex.

And take the complex constitute of that and multiply okay, and then I integrate okay so that is just a slide difference in definition the a definition for a real signal in slightly different from the energy definition of the complex signal but the thing is that because complex signal also captures the essence of I mean a real signal so t is happening is here, if it is a real then it will just become a complex conjugate is the signal itself and if I just multiply that will be just where right so if it just captures the other definition.

Also it is not away from that other definition so this is my energy immediately what will happen if suppose I have a now the representation changes slightly.

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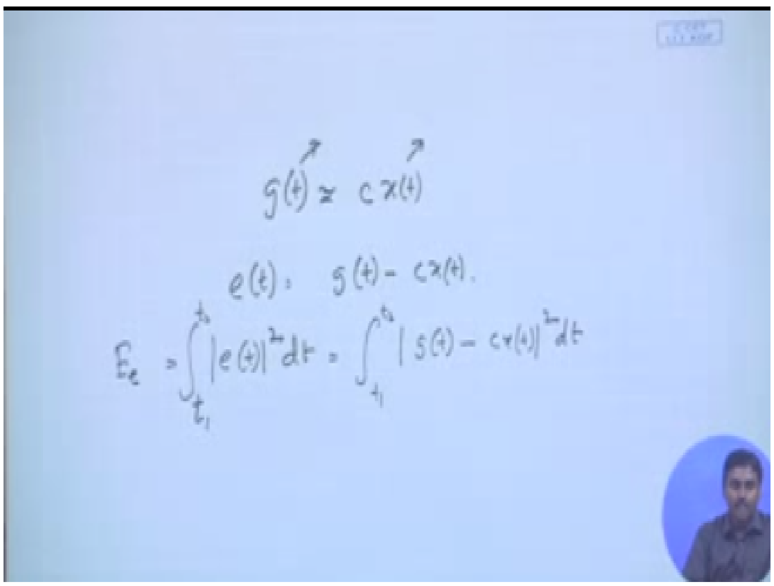

I have signal gt and I wish to represent it with respect to a singal xt but the only difference is now that this gt and xt are no longer real nobody has said me that these two has to be real this can be now complex okay, so now I will try to do the same thing same derivation little bit complex it will be but I wish to again derive the similar definition for orthogonality for the representation of those C value optimal C value okay so that is something we wish to do okay so now let us try to see.

What will be error that is actually same thing again gt – Cxt now we have to minimize the energy of this error signal so the energy of the error signal now because it is a complex signal, and here even C can be complex the positions were also can be complex that we will see later on okay so if that is the case what we need to is we need to take that complex conjugate multiplied with error signal and then integrate over the limit whatever limit it has let say the limit is or just t1 t2 to whatever okay.

So immediately happens to be gt – cxt and now all I have to do is I have to actually evaluate these things and I have to this is my energy I have to minimize this with respect to C but this is not going to be as easy as we have done, okay so direct differentiation will not give me because there will be C and it is complex conjugate so I cannot differentiating with respect to C so I have to device another tool to minimizing okay, so this is something I will discuss now this ee.

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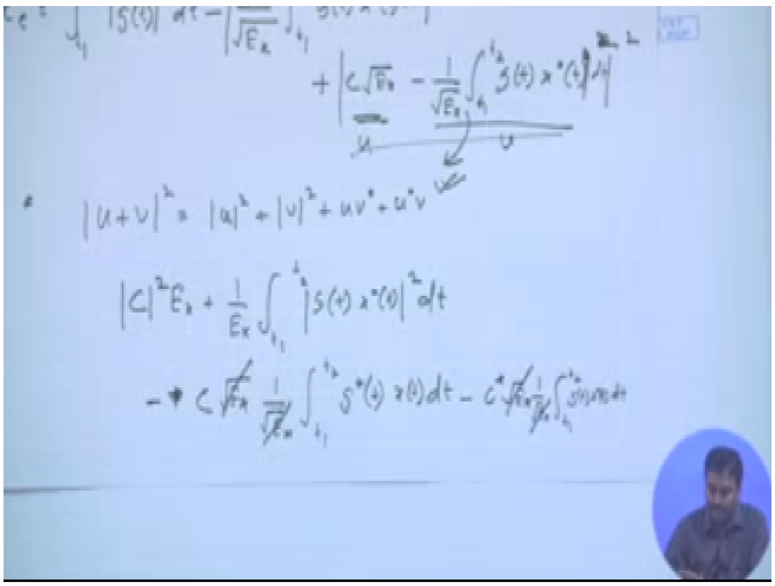

The way we have defined I just write it in this fashion let see whether this is correct so t1 to t2 will prove that this is the correct representation – ex is the energy of signal xt see why I have represent it this in this fashion it is because I want to minimize it with respect c right these two part if you see these two part are no longer depended on C okay, so all I have to do is somehow choose C so that this become 0 okay then only I can say that this will be minimized with respect C so I have just device a separate rule.

For evaluating it because direct differentiation was not possible I was getting CC complex conjugate and then differentiating with respect C that was difficult okay but we have to first prove that this is the actually the error square or error modular square integration we just keep calculating that you will see that this is the actually the same, so basically if you just evaluate this part okay so this you keep it as it is if we just calculate this part so you have a this – this modular square.

Now I will just write how to evaluate suppose I have a variable u and v which are complex number I need to evaluate this generally we can write it in this fashion modules square mod way square plus instead of 2uv it is written generally u is * + u* okay so this is how the complex numbers mean square of mod of complex numbers are represented okay, so if I just evaluate this portion with respect this what we get so this is actually u + v so this is becoming my uv and this is becoming mu v.

So just evaluate it what do we get here we get C mod C^2 ex right because √Ex is not a complex let say energy it is must be a real number okay then mod of this okay so mod this square so that should be 1 / Ex and then and modulus and integration that has nothing to do, does not have anything to do with the integrations so it will go inside so I can write t1 to t2 gt x*t mod square t right so here I think we have forgetting that and then it is just uv* + vu means u*v so uv* if you put so what we get.

We get C√Ex 1/√Ex of course there should be a – okay and we have that same integration t1 to t2 but we have to put a * of that right, uv* so * of that so that must be g*t xt x*t and it is complex conjugate a and will give you that xt or dt okay so we get this – we get then C* √Ex into 1/√Ex and then we have this integration t1 to t2 gt x*t ut right, find now let us try to see if this is exactly equivalent to our gt – cxt mod square it is eventually because what is happening we can see this Ex, Ex gets cancelled okay so what do we get so we had let see we had t1 to t2.

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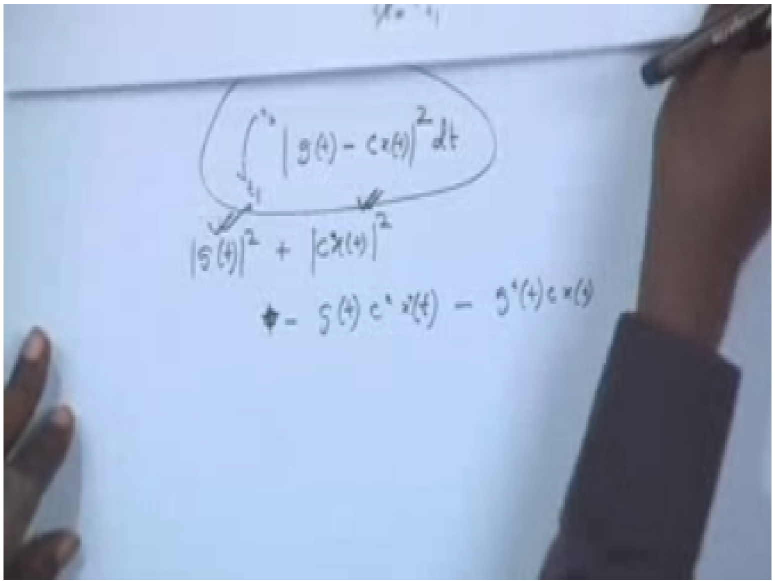

Integration gt – cxt mod so t1 to t2 right so we have if we start evaluating it so again we can this square we can actually evaluate it with respect to uv mod square so this will be mod gt^2 and then +Cxt mod square these two things will be coming and then gt C* x*t right and there should be a – and then – g*t Cxt right and if we just see the previous one it is just giving me the same thing okay so if you just see everywhere I have this C okay so if just match it so what do we get we have got this square so this square term.

If you have evaluated we have got this C C^2 Ex right which is actually x mod x^2t right so this term and this term so this particular term and this term exactly matches then our gt exactly matches gt^2 and that term is also present over right, so which is this term that is already there so this term is matching the next term is matching this particular terms gets cancelled and the other two terms are already there right so this representation exactly id the representation if we just do it yourself.

We will see that it is exactly the representation of this one so therefore all we are trying to say that the representation we have put over here for Ee that is exactly the representation of that error mod square dt, okay so now harm in doing that representation but by doing this representation immediately we can see that these two part are free of C so therefore all w have to do is we have to minimize this or we have to make it 0, so to make it 0 this inside term if I just equate it to 0 I get my C and that should minimize the whole E because whatever else I do this is mod square.

Term so any other thing other than 0 will actually increase the value of Ee because this term is free of C this particular term to both the terms are free of C is this terms is anything other than 0 because there is mod square and addition so it will always increase the value, so the minimum value that I can get where this particular part is 0 so whenever I put this particular part 0 immediately, what do I get for my C calculation so C happens to be.

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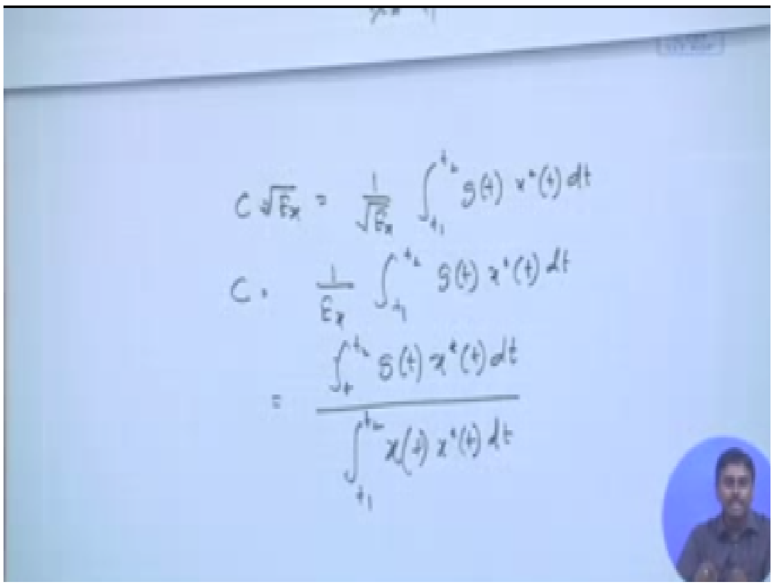

So C √Ex must be equal to 1/√Ex t1 to t2 gt x*t dt right so that must be the case immediately I can evaluate C that is actually 1√Ex integration t1 to t2 gt x*t dt so for a complex singal now you can see we have t1 to t2 only difference is earlier it was gt into xt now I have a x complex conjugate t dt this integration and Ex can be written again as mod xt2 or xt into x*t dt, so now I can define just redefine my for a complex signal by dot product as this that suppose I have a signal.

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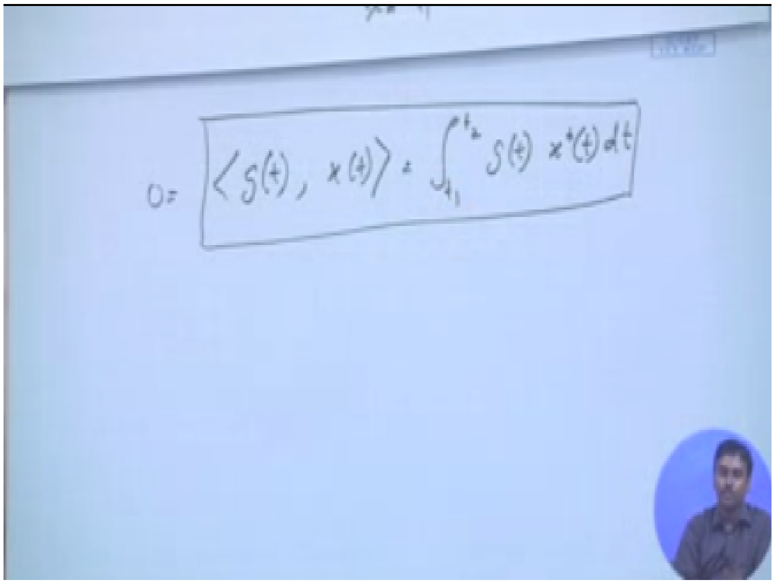

gt and xt and I wish to do the dot product so dot product can be defined as this gt x*t dt so this should be my new definition of dot product in a complex signal and immediately if I take the dot product with itself I get back the energy no problem in that and the orthogonality condition is the C must be 0 that means gt must not be represent it with respect to xt so if C is 0 them immediately this dot product must be 0 or this particular integration must be 0 so this is what we can see from our derivation next what we will do will apply this to actually get back the Fourier represent.