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NPTEL ONLINE CERTIFICATION COURSE

Course
on
Analog Communication

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Lecture 39: Random Process (Contd.)

Okay, so we have already started characterizing random process right, we have said how to evaluate mean and autocorrelation function means how to characterize random process first of all, and then why we need at different time instance and there means correlation that is something we have already talked about. And then we have started talking about evaluation of mean and autocorrelation function right. So these two things we have already done.

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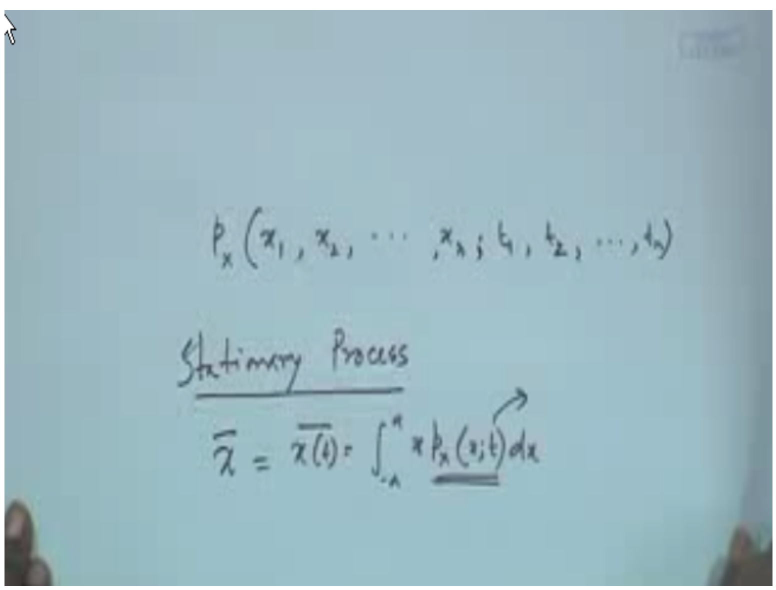

Now for the higher-order what we will have to do is definitely we will have to evaluate this px at let us say n^{th} 1 so we have to do this joint distribution at different time instance. And then multiply this with X1, X2, X3 up to Xn and do n^{th} order integration and you will get the nth order autocorrelation okay or correlation I should say okay. So this is something we can keep on doing for any value of n.

But now what will be interesting for us let us try to see those signals the signals that will be interesting for us for which we can do some analysis. The signal that will be interesting, so suppose a signal is going on okay if the statistical property of the signal basically depends on which time I observe then I have a signal which is very hard to characterize, because the signal statistics is keep on varying it is not just the signal, signals keep on varying that is all true.

But now we are talking about the statistical property of the signal if that also keeps on varying and I have almost nothing to means no hope to at least analyze that signal. So now we will be talking about a class of signal where the signal might vary in time, but the statistical property probably does not vary with time, which is means defined as stationary process. So we will now try to define the stationary process okay.

So this is something we will try to define so I have told a stationary process will be or a stationary random process will be whose statistical property does not change over time okay. What does that means that means all those PDF and joint PDF we are defining they are not really dependent on the time where I start okay. So let us talk about that mean first, so mean was my mean XT bar was something like –NA+∞X px XT dX right.

So this was my mean, now this PDF if irrespective of any time T it is same that means anytime you sample I have talked about that noise signal, I have told that probably the PDF at every sample instants gives me the similar PDF that was the first insight of a stationary process that, okay, this is what happens generally in most of the signals that we study most of the time it will probably be stationary okay.

If there are no other influence, time varying influences in the signal okay. We will see that again we will differ from this and will say most of the realistic signal are not stationary, because it comes through the channel and channel keeps on varying over time. But for the time being let us say that most of the signals that we observe are stationary what does that means that every instance you take associated PDF.

That means like that experiment we are saying that multiple students will be there will sample it and they will be recording the current, will sample it at particular time let us say 12o´clock in the noon and we will say okay give me whatever sample you have got, we plot the histogram get the PDF, normalize it get the PDF. Let us find one part, then we say okay 1o'clock whatever you have observed give me that again we do the histogram plot and normalize it get the PDF.

We will see that those two PDF will be identical and any time we take that probably all of them will be identical okay, that is the time when we would say that mean is irrespective of time it will just X bar. Because if the PDF is same and the signal value, because the PDF is same, so therefore it is mean, because PDF characterizes mean right.

So it is mean also will be same irrespective of whatever time you take if the PDF is stationary over time the mean value also will be stationary overtime it is not going to change. So that is the inside will get that first order it is stationary, if at every time instance the PDF is that single PDF of course we are just taking that at that time instance a sample and talking about this.

So this is happening then we can say first order it is stationary okay, next the second order comes. Now what we do we actually take the joint PDF with two samples or two time instance okay. And then again do the histogram plotting with normalization, try to see what is the joint PDF if the joint PDF again remains stationary for any two time instance we take, but remember the time separation has to be same okay.

So because I am taking a joint PDF and I have just said the statistical property should not be dependent on the first time instance I take or the origin of my calculation okay. But that does not mean any time I take because any time I take the nature of dependency will be varying just a simple low-pass filtering will tell you that, if you take it closer enough the variation that it can take means suppose I have a signal it is a low pass signal.

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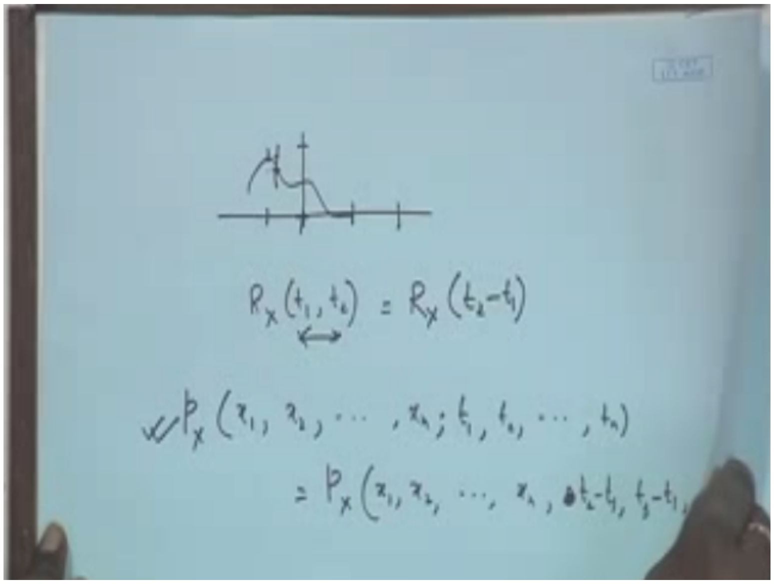

So from here whatever value it has taken if I take a very close at time instance it will have a lower variation, but if I go further away it will have a higher variation okay. So therefore, the time separation between two points I am considering that is very important that has to be there that is the fundamental characteristics of the signal.

But where from I start whether I take this and this or this and this it should be independent of that. So if I can now say that the autocorrelation function which we have defined which is the second-order thing right. So which is that Rx t1 t2 it was earlier a function of t1 and t2, now if I can say that this is only dependent on the separation between them not individual values okay.

So if I can say this is just Rx t2 - t1 it just depends on the separation of them but not where the t1is. So t1 can be anywhere but if you keep the same separation you can put t1 at anywhere what your statistical property will be getting it remains the same. Then we can say at the second order also the function functional or whatever random process we are taking its statistical property remains stationary and so on.

If I go to the suppose in it order then I should specify that px this x1 x2 up to xn the way we have specified in time t1 t2 tN must be equal to, now again the differences right, that should be same. So therefore, I should be able to say that it should be X1 X2 Xn at time I can specify let us say t2-t1 t3-t1…..tn-t1 okay. So wherever the t1 is just as long as t1 to t2, t1 to t3, t1 to t4 are all same I can shift the whole thing take the whole statistics it will remain the same.

Then I can say at the nth order also it is stationary okay, so first order, second order like that nth order up to infinite order whatever order you can go, if you can say all of them are remaining stationary that means it does not depend on the origin then the whole signal has the statistical property which is stationary. And that is a very strong condition we call those kind of signal as stationary or those kind of associated process which is a combination of those signals associated that process is called a stationary random process.

So that is the definition of stationary random process, but as you have seen that stationarity of a random process proving that is going to be a daunting task. So what we will do we will be happy with just two order.

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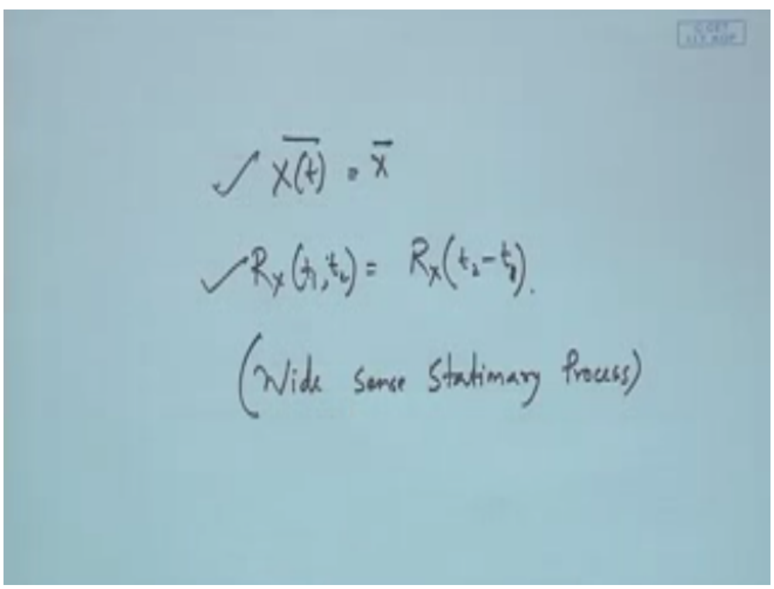

So people are happy with to order we just say that X bar T is just X bar does not depend on time and R X t1, t2 is just dependent on the separation of them, if this happens then in the first order and second order they are stationary all other higher orders we do not test we just say it is a wide sense stationary process.

Of course it is a weaker condition or we say generally say week stationary process okay, so that means first and second order we have tested it is just our limitation of computation okay, so we have tested first and second order we have seen that it is stationary we do not go means wish to go to higher order because that is too complicated to check. So we just declare probably first and second order it is already stationary and you will see all the signal processing that will be done will mostly dependent on this first order statistics it does not go any higher okay.

They are important, but you will see for all our calculation probably it is not going any higher order up to any higher order statistics. So up to this if they are stationary we can say that we can just assume them to be stationary and we call them white sensation area, most of the signal that we will be dealing with will be means we will assume that they are white sense stationary, because later on see that if they are not wide sense stationary there is no analysis.

Because if the statistical property also varies with time I have no hope of any analysis if they are stationary that means statistical properties are fixed, then I have some average sense of analysis for these random signals as well okay, which will be eventually used for our noise analysis and any random reception analysis okay.

So, so far I hope this is clear what we mean by stationary and wide sense stationary okay. Now we will just try to give one example, so just an example where you will be able to see that a particular signal means all these concepts we have told that will be captured okay.

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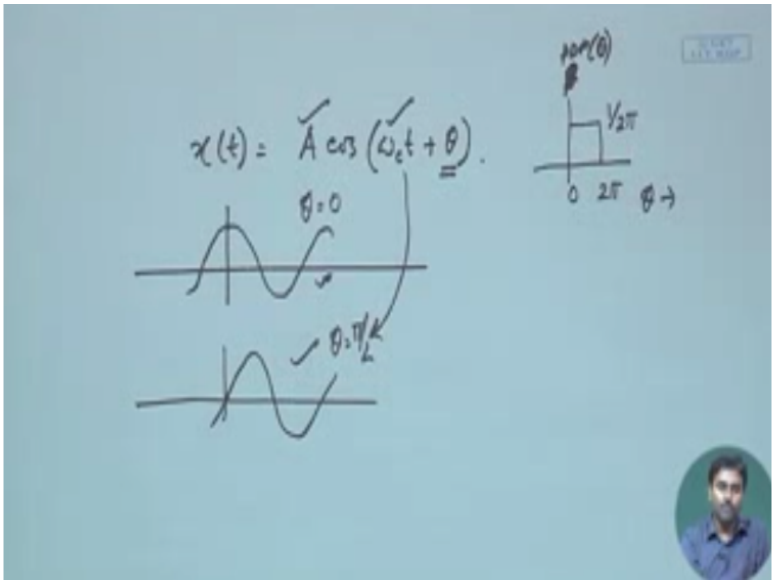

So let us say I have a signal which is actually Acos(ωct+θ) okay, so this is a usual co sinusoidal signal with a phase frequency and amplitude. Now we are just adding some more things to make it random this a is fixed amplitude does not vary with time okay, or it does not vary over signals this ωc is fixed, frequency is fixed, but this θ might vary and vary in a random fashion.

So the θ has a PDF of means it is uniformly distributed between 0 to 2π, so because PDF has to be integrated to one so distance should be 1/2π right, so that is the PDF of sorry PDF of θ that is θ right. So this is something I know, so what will be happy associated with that there will be multiple random signals now will be generated.

So if I just try to see one possibility is it has a zero phase right, so if it is 0 it will just a your co sinusoidal signal the other possibility has it has some phase so that with that lag the co sinusoidal will be starting, so there will be infinite such phases between 0 to 2π and all those signals are those random signals that we were talking about.

Of course their amplitude is not changing but it is the phase they are changing randomly so all those sample signals, so even when θ is probably π/2 then immediately it becomes sine, so it will literally be something like this. So this and this 2 are valid signal generated from the same process same random process only when the θ varies, so θ is 0 over here θ is π/2 over here and so on up to 2π you can.

So it can take any of these, these are all they are actually one, one random signal any of them might be chosen, because when I am transmitting I do not know that θ, what that θ will be at the receiver side. So any of them will be chosen for me, now what I wish to do is I wish to see whether this particular process is that stationary process or is that a wide sense stationary process so this is something I would like to evaluate. So I would like to see that θ with this PDF of θ given will this be in time if I wish to see is this a stationary process so let us first evaluate the average value of that right.

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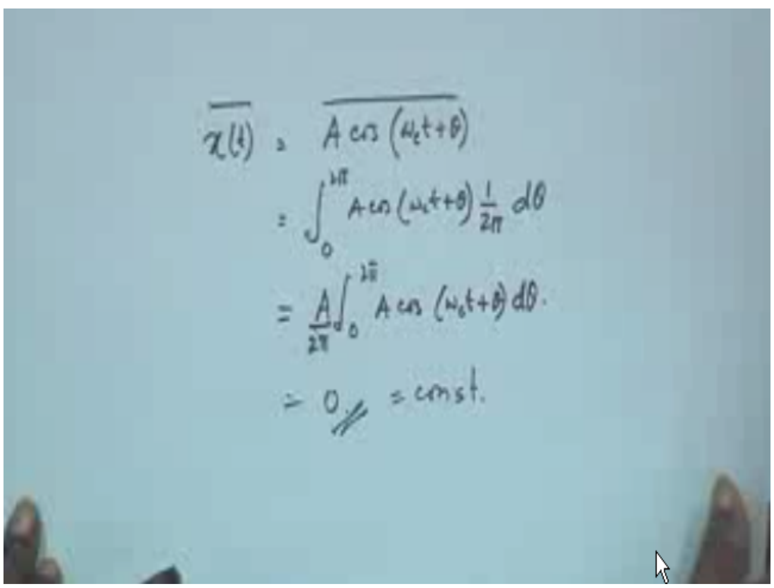

So let us say I wish to do this that should be a $\cos\omega_c t+\theta$ average. Remember whenever they are talking about average so far we have been in signal we are talking about average in time right, this is not average in time this is on that random variable okay. So if I wish to do that average it is again a functional average of that θ.

We have done that already, so what is that, that is actually this function multiplied PDF of that random variable integrated over that range of that PDF right. So PDF is raised I have seen already it is raised over 0 to 2π right, so it should be integrated from 0 to 2π $A\cos\omega_c t+\theta$ as a functional average remember into PDF of θ which is 1/2π within this range Dθ, so that should be the evaluation.

It is not time averaged remember that we will come to time average later on we will be defining another subclass of process, so A is a constant 0 to 2π 1/2π goes out, so it is just Acosωct+θ Dθ. So if I just do this from 0 to 2π you will see that this will be 0 because we are integrating cos okay from 0 to 2π right.

So θ varies from 0 to 2π adding to sinusoidal signal within a particular period if you integrate that should be 0, so this is 0 the good part that has happened is, I have started with any T remember that T was there so it can be any T, but for any T I could now prove that this is 0 this will be 0 for any value of T, so that must be constant over time that is the good part.

So it has a mean value of 0 at every time instance, so that means first order I can say this is stationary this is the statistical property remember I am doing n sample average, and the n sample average is giving me a constant value right, for this signal it does not depend on time. So the average value of this signal does not depend on time, so that is the first step towards telling that it might be a stationary process. Now we have to do for the second one so for the second one what we have to do is something like this.

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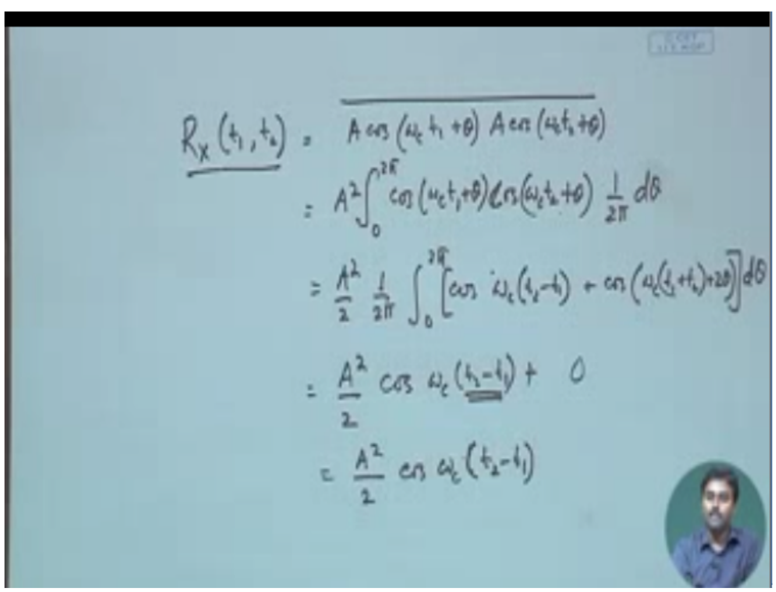

So we have to evaluate rx t1 t2 so at value of t1 I will be taking a value of this particular signal and t2 will be taking a value of that signal and then we have to do n sample average over the variation of whatever random variation we have that θ. So this must be Acos(ωct1+θ) Acosωct2+ average this is n sample average what is the variation it is just θ variation so average again will be a functional average so over θ a random variable.

So this must be A^2 is there integration 0 to 2π PDF of that is 1/2π Dθ and into this function which is cosωct1+θ (cosωct2+θ) right this is the one I can take out, so $A^2/2$ to 1/2π comes out 0 to 2π so now this will be 2 cosA(cosB) so I can write that as cosωc this minus this θ gets cancelled, so that will be t2-t1 and plus cosωc so this plus this so which should be ωct1+t2+2θ right.

So this whole thing D θ alright so we are just doing the ensemble average nothing else. So $A^2/2$ the first term that is not dependent on θ, so therefore the first term if you integrate will be distributed of course first time if you integrate just that thing comes out 0 to 2πDθ will be integrated so that will be 2π this 2π, 2π gets canceled so I just get cosωct2-t1 right.

The second term it will be this cause something which is not a θ variable 2 θ integrated over 0 to 2π again because it is cos and going through 1 period so it will just be cancelled whether it is whatever it is it will be just canceled out, cos positive and negative up will cancel each other so this integration will always be 0, so that part will be 0 whatever happens. So now let us see what has happened to my rx t1 t2 is this separately dependent on t1 t2 no it is just dependent on the separation.

So immediately I can see it is actually just dependent on the separation nothing else, so I can now say in the second order also it is stationary, because my definition of stationary was that in the second order it has to be just dependent on the difference of it nothing else okay. And so on you can keep doing that and you will be able to prove that it is stationary at every order okay but probably will be happy content with this you can test for higher order also.

But we will be happy with this because up to two orders we already have a strong condition we can say this is wide sense stationary already. So that particular function we have chosen fortunately even though θ has a random variability we could see us two means random process which is stationary over time.

So it is autocorrelation function is dependent on just the separation of those two time instants whatever you take okay and it is mean is constant, it does not value overtime okay. So immediately we can declare okay, this is probably a random process which is stationary. Now we will talk about something else which is called the Ergodic.

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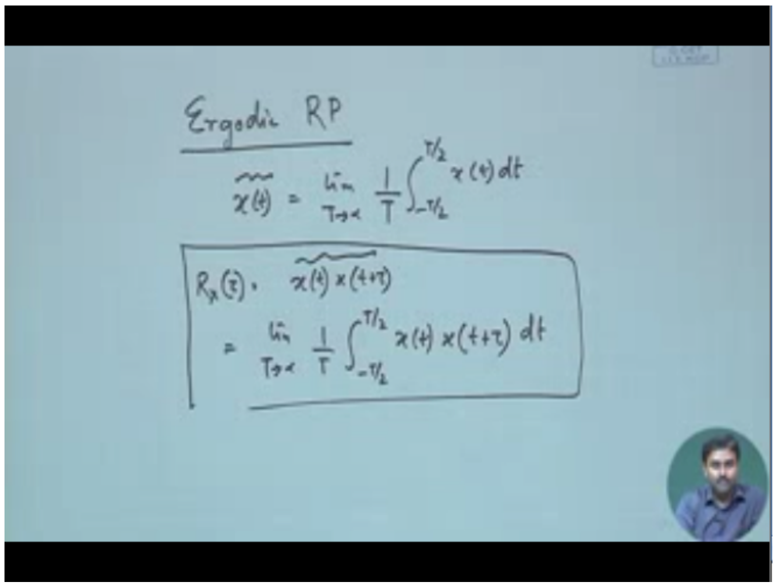

So what do we mean by Ergodic RP now try to talk about that we have already talked about the stationary now what we do we will pick one of the sample instead of taking all of them we will pick randomly one of the sample any one of them is good okay so there is no preference for that any one of the sample will pick and we will try to do the same thing in time the averaging and second order statistics means statistics here it will not be statistics it is the second order autocorrelation function.

The way we have defined auto correlation function okay in the previous means previous few classes right so let us try to do that so first is the average so we say this is that was in sample average this is time average so time average is nothing but this which is limit T tends to infinity because it is a periodic signal so if this is a periodic signal then I can just talk about time average as if I take it for a big period do the average and make the period stretched to infinity so 1 by t - T by 2 ,2 + by 2x T DT.

So this is the time average so over the entire time duration you peak all the samples and then means you cannot pick samples here because it is continuous so you integrate it and divide by T because for T time duration you have integrated if T stretched to infinity then you get for the entire sample or entire signal so that is the average remember what we are doing for the in-sample we are taking a single time and we are taking multiple samples we are doing n sample average everywhere we are picking our value and we are doing average of them.

So that is called the in sample average here what we are doing among all those sample signal we have picked one of the signal no preference for it any signal will do take that signal and do overtime averaging for that entire signal you do not consider any other signals just that signal do a time average you get this okay and the autocorrelation the way we have defined out of corrosion so our X . will be defined as XT and X T + tau and then we do time average.

Which is similar limit T tends to infinity 1 by T integration - T by 2 to+ T by 2X T XT+tau this is how we defined autocorrelation function in time domain right so if I evaluate this one then I get the autocorrelation now if for me or for a particular sample space if this happens that any sample I pick I evaluate these two things and they are exactly equal as the one we have got for the stationary process if this happens.

Then the associated random process is called organic random process okay so Ergodic DCT means that whether you do it in sample or you pick a sample any sample it will do and do it in time both the things will give you same result okay so as long as that is happening then we can say that this process underlying process is probably a gothic process okay what I would request you means I do not do that because it is a very simple one simple exercise I would request you to just test the sample.

We have taken take any θ okay that a cos ωCT +θ here a particular sample means any θ you can pick out of those random 0to 2π any value you can pick any of them it does not have any preference that also will be clear to you so pick any of them either 0 or Π or 2 Π or Π by 2 or Π by 4 whichever value you pick will be able to show that the time averaged if you calculate because here the time averaged will be just if you do it within a single period because it will be just repeated.

So you do not have to do it for the entire thing because it is a periodic signal so you just have to do it over a period and over a period if you just do time average as well as autocorrelation function they will be same as what you have calculated the same value will be coming out okay, so this is something we will be able to prove and then you can show that the signal we have picked that is why we have picked it hat signal that is how I regarding signal as well okay or we should say wide censored guarding.

Because we are not proven for higher order statistics we are not going to order 3 or order 4and so on okay because that would be too so at least up to second order you can prove that it is stationary and then finally you will be able to prove that it is also a θ that is any sample any θ you put pick it will give you same result.

So that is the strong condition for most of the means considered signal probably LC and many things can be proven for these kind of signals either it has to be stationary or it has to be even more stringent that it is air ghatak okay so I will just before ending this class I will just give you one very simple example of Ergodicity probably that will give you some idea of a goddess City this is again an example picked from the book we are following so we have a Manhattan Street kind of configuration okay.

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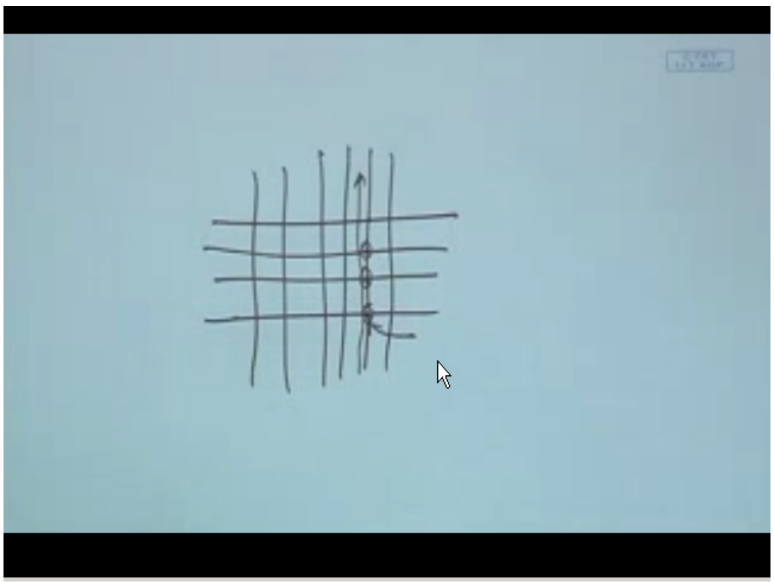

And you have all those southeast and northwest sorry no not south and east west these streets are they are exactly perpendicular to each other okay so every Street has crossing okay now we say that every that crossing whatever signal that are therewith 75% chance that that can be green and 25 percent chance is there that can be red okay now let us try to do this experiment that a particular car going from this to this over a particular street going from south to north okay it will be crossing multiple these things multiple signal signals.

And then what she has to see is and we also assume that all these signals are independent of each other they are not correlated that if this is green that will be also green with this probability and all those things they are completely independent of each other so if I just see one driver who is going from north to south and crossing multiple such signals okay and then try to evaluate how many times he will be or how out of all those signals he has encountered.

How many signals he will be absorbing as red and how many signals he will be observing as green that should be exactly from our definition we know that 75% of the chance or 75% of the signals he has encountered probably will be green and 25% of them will be red so I can immediately see from that drivers point of view and that driver can be any driver now let us say multiple drivers are flocking around okay so they are actually coming to a particular signal how many of them will actually report.

That I have seen green in that particular one and how many of them will say that I have seen red in that because it is 75%the drivers are coming at any time okay so because it is 75% times it is remaining green and 25% times it is remaining red so probably you will see that out of all those drivers 75% will report that at that particular signal point I have seen green and 25% of them will report that I have seen red so what is happening out of all those driver some of them you picked and you actually went through time because whenever he is driving at different time different signal he is observing and he is reporting.

Whatever he is reporting that PDF is exactly true for if you just sample it over a particular signal as if there are single in our definition single time instants okay so signal is single time instants and multiple signals are different time instants so a particular means signal if it is going means particular driver going through it that means it is over time going through and whatever average value he is observing that is becoming same as if you take more means more number of drivers in a particular signal point.

So this particular scenario is actually our Ghatok process because for any driver you pick they will report the same thing okay so this just demonstrate which are the process that can be erotic and what is the meaning of Ergodicity right so with that probably will lend this particular class and then we will try to show you if a signal is stationary what can we do with that I think what is the advantage we get if a stationary signal is there okay in the next class thank you.