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NPTEL ONLINE CERTIFICATION COURSE

Course
On
Analog Communication

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Lecture 38: Random Process

Okay so we have already discussed about random variables right so that is something we have already done in a previous class so we have told that will be probably starting discussing about random process and that will be in next few class probably will be occupying ourselves with the discussion of random process how it is different from random variable why it is important so all those things and then how to manipulate a random process so that in our case where either the signal is random or the noise is random we can actually analyze things okay.

So will be in the process we will also see if a random process passes through a system because most of the time we will be passing signals through our system so how it interact with the system also that will be something we will be discussing in next few classes okay so let us try to understand the basic fundamental difference between random variable at random process so I will start with an example which is probably a widely accepted example to understand random process so what we do is something like this.

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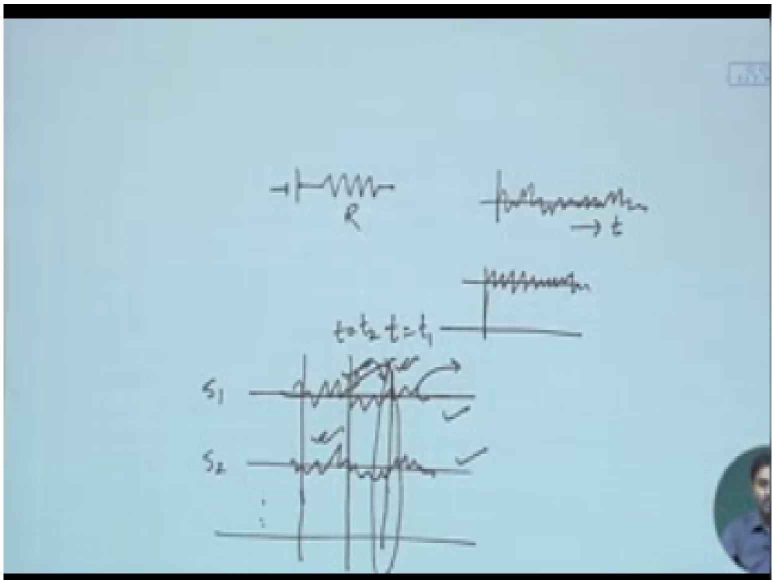

We take a resistor okay and we put a ammeter against it okay ,so no voltage source nothing just a meter against a resistor so what will happen it will try to measure the current through the resistor okay so initially we will be wondering why there should not be any I have not put any voltage so there should not be any current but what happens inside resistor this is just apiece of wire probably and inside that there are free electrons and all those free electrons or we should say charged particle which are free to move so what will happen they will be because they mobile so they will be also doing Brownian motion.

Okay so they will they will go here and there they will get collided again deflected back in a random fashion due to that Brownian motion there will be some net current which should be recorded okay and that net current is completely random because you never knowhow that net flow of overall electron density will be accounted because it is all random motion so what will happen if you just see against that the current against that resistance.

So what we will see is something like this very low amplitude of current which is randomly varying and this is this variation is over time so you are recording more time and you just through the emitter whatever you are getting you are recording those values of current through that resistor in time so this is the nature you will be probably seeing and it will just keep continuing of course the amplitude will be very low because it's just due to the Brownian motion okay so this random amount of current if you.

If you now start giving a voltage source across this so what will happen due to that voltage source there will be a current constant current okay across the resistance so immediately you will see that there is a DC SH shift to this particular random current and on top of that DC this DC shift will be due to this voltage source and whatever the values are resistance due to that whatever current you can calculate Ohm's law you put ammeter also if you have something on that you take that resistance also and you calculate overall current.

You will see that current will be there but on top of that there will be a random variation almost similar like that okay so that is due to that Brownian motion okay however small that amplitude is of that current there will be some amount of current which will be randomly varying so what we do now suppose we prepare this same kind of resistance so we try to this is a hypothetical experiment probably we try to prepare say resistance multiple number and at the same time instance.

We say in the experimental this one laboratory so we say multiple students to actually measure with a similar ammeter across that resistance and at every time instance they should record this for currents right so what we will see, we will see every student recording different kind of current pattern so if suppose this is for student 1 this is for student 2 and so on and you will see that similar kind of current pattern but which will have different random variation which will be recorded by different students okay.

So this is what will be happening because it is though they are identically prepared at the same time they are measuring because these are two different resistance they are Brownian motion they are independent so there will be means differently creating net current through the register and you will be seeing something and it is guaranteed that because net means Brownian motion over a time if you see that should be 0 so overall average current.

If you wish to see that we all zero here also it will be 0 here also it will be 0 and so on as many you can take everywhere the average current will be 0but there will be a random variation and that random variation reported by student 1 will be completely different from student 2 and so on that is what will be happening now let us say we take this different signals.

For a sufficient amount of time they have measured it they recorded it they have given it to you and now I fix a time right let us say time T equal to t1 okay so at that time instants every student will be reporting some amount of current okay, so take that value now these values which are reported if I have enough number of students let us say almost infinite number of students if I have that then these things at a particular time instants that means I have almost in a snap shot of all these signals that has been produced by the resistor identically prepared resistors by an experimented or measured current through it by different students.

If I just take a snapshot at a particular time we get different voltage reported and then whatever we get those will be random values at a particular instance so this is actually a random variable at a particular time instants if I take from different students different reading whatever they are they are just like random variable and over there we can construct whatever we have constructed so far that means we take those random variables we can actually try to produce what is a PDF Associated PDF of that we can also try to see what is the mean what is the standard deviation variance and all those things higher moments.

We can produce all those things okay so we will be getting that suppose I decide okay no T 1 I am NOT happy with hell I will go for another time let us say that is T 2 and again I do that okay so I take all those samples again it is a random variable at a different time instants right so there also I can again do all those things okay so I can I can get mean standard deviation variance PDF everything that I wish okay what might surprise you is something like this you go from this time to this time you try to see the overall PDF of the random variable that you have recorded.

You will see that that will be same that is very surprising because this has been experimentally demonstrated that happens okay why that happens and what kind of class of signal those are will classify them later on but this is something which happens that means there is something underlying which tells us that a particular signal you take okay which is random in nature.

So that means I can take different kind of those signals which are identically prepared and then I take samples at a particular time instants from all the sample function we should call okay so remember samples value and sample function there is a difference between these two so this particular function that entire function in time domain that is called the sample function whereas this at a particular time instance whatever value.

I am recording that is actually sample value and those values we are saying that forms a random variable okay right and then I can get a PDF but what we are now seeing is something more interesting that the sample if you take sample function if you take samples from all those sample functions and then try to evaluate the statistical property they remains almost the same so that tells us that maybe something more has to be said about these things okay.

So in that means with that observation only people started defining something else which is called random process.

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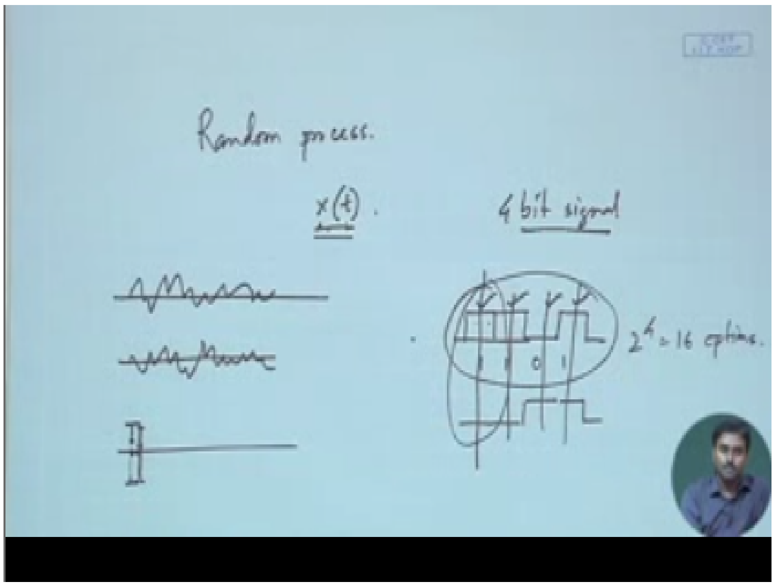

Which is I should say nothing but a random variable but it is a function of time all is so whenever we say XT at a particular time instant this X is just a random variable okay but this random variable keeps on taking values at different instants of time okay and so whenever it takes if I just go through the time and takes different values and I construct a function that becomes one sample function of a random process from which it is drawn so the associated random process is just that experiment of putting identical resistor.

And then measuring current through it which is completely generated due to our inside randomness happening which is called Brownian motion okay due to that whatever current is happening for the entire time duration I if I record the overall sample function those sample functions are now random because as you have seen one particular sample function is quite different from the other sample function okay so we can even say that as if like random variable suppose head and tail so you have two outcomes and whenever you do experiment.

One of them are chosen right here also I can say that as if there are different kind of sample signals and when you do a experiment one of them are chosen okay which one will be chosen it completely depends okay so I will give you one very simple example probably that will take out the doubt from you so let us say we have a 4-bitsignal which is being transmitted now I know that it's 4-bit each bit of duration T okay so T might be anything.

It might be 1 second or it might be 1millisecond 1 microsecond whatever it is not bothered about that but it is just a 4-bit representation that means four bit pattern which is completely binary it can only take 1 or 0 value right but whether it will be a particular bit will be 0 or 1 that is something I do not know before I am means when we are at the receiver beforehand I do not know whether this is 1 or 0 it can be anything okay, now suppose the first bit I receive.

I see that that is probably 1 so I can construct the first bit that is 1 then I receive the second bit I see that that is also 1 then I receive the third bit it is 0 and the fourth bit is 1 so immediately I can see after receiving the whole sequence I have a pattern which is being received but this is actually one of the random pattern that could have been generated at the other side this is just one of them this is the sample function that I have received.

But I could have received equally likely other sample function also like it might be 0 ,0 followed by 1 and then followed by 1 it could have been this sample function or there might be multiple such things how many of them will be there because each bit has two options so it is 2 to the power 4 so there will be 16 options just like you are rolling a die or tossing a coin right it has two options or this one rolling a die has six options but there we do not have the notion of time.

It is just single thing that you observe right you do not have that notion of time here we have the notion of time it is random what is coming I am still not knowing it is just coming to me and after I detect the whole sequence probably I will know which sample function among those 16 possible things has received by me but whatever it is it has a randomness associated at every bit so that is what we are trying to characterize that every time instance probably here we just have four distinct time instants okay, so there are many other but we do not have to specify them because it remains the same if it is 1it will be remaining same as one for the entire bit duration.

T right so there are 4 time instants where it might change okay so it is a function of that time instance where things might change and that randomness is already there if I would have talked about just a random variable then I could have sampled it over here and it is just 2 values 1 or 0 whereas because I am talking about the whole signal that is why we have for sample value and each of them characterizes a particular sample function okay.

So you have to see them in conjugate and then only we will be getting an idea about what will be the overall sample function in digital it is little bit easier because we have accountable numbers of options right so 4 bit we already have 16 options not more than that not less than that this many options are there but if I just say it is a continuous signal like that signal generated by the register ok so it can take infinite options so there might be associated with it there might be infinite number of sample function which can take any value at any time instants right.

So I can this way I can construct any other sample signal and all of them are basically a particular sample function any of them might happen so this is something which is actually characterized by a random process so whenever we talk about random process associated with it a signal comes into picture and that signal we talk about is a random signal ok that will be any of those possible signal that can be constructed so if you start talking about possibility we know that at every time instance there is upper level and lower level that can be created by this particular resistor okay so of upper level and lower level of current.

If I know that among this any voltage level at this time instants again next time instants which is infinitely small distance away from that time instance you have considered there also any random value it can take so you can now see there will be infinitely large number of possibilities of sample signal that you can produce because immediately over here only it can take infinite amount of values and then the next one depending on what value it has taken it can take again another infinite number of values and so on.

So like that there will be some means there are infinite number of sample signal and probably one of them will be coming to you right and it is not guaranteed that always that signal will be coming to you so you experiment it today probably one particular signal next day probably it will be different right so like that whatever it is we are saying that particular thing that signals are represented by XT because it is a it has a variation of time whereas at every time instant it is a random variable.

So that is what random process is now let us talk about characterization of a random process right so whenever we say characterization of a random process we have already talked about two things one is you take a sample time okay so or you take a snapshot and then try to see the statistical property of it will be immediately getting over all those sample functions some values at that time and you try to construct a random variable out of that and then try to see the PDF of it okay and all other moments right.

So this is one way of characterizing it that is all good but probably this is not all that we can do okay there is something more.

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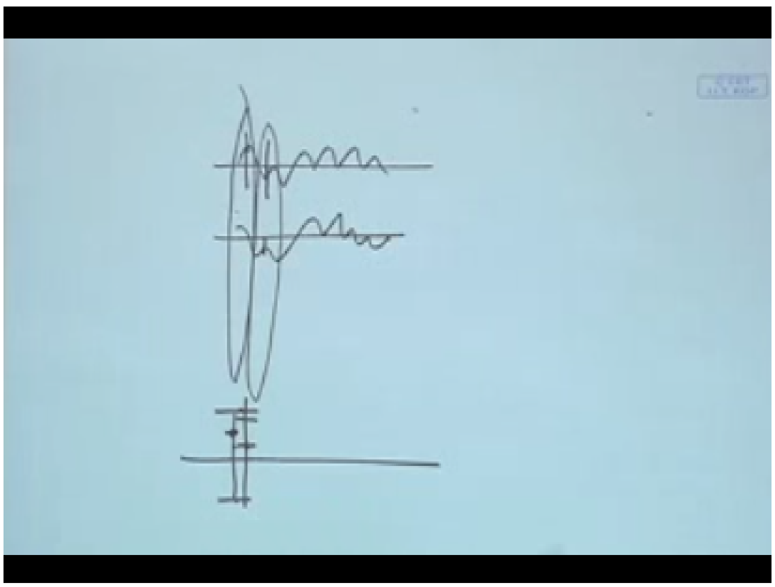

So I have a signal like this I have another signal like this now what I wish to also see that at a particular time instants if I have this value in another time instants whatever value I will begetting or whatever statistical property I will be getting if I take the snaps hot among all the signals will that have dependency on the statistical property that I have observed earlier or later time instants so this is also very important that is there a statistical we have talked about dependency correlation and all those things.

So now we are trying to see that that in time I am taking snapshot and immediately I am generating random variable another time I take a snapshot immediately I generate another set of random variable means another random variable with different set of values these two things are they dependent or are they not dependent because that also comes out from the signal quality of signal specification and that characterizes the signal as well because if there is a dependency.

I need to know what kind of dependency okay so it might happen that the signal even though it is random at a particular time it is suppose it has picked one random number but the next time instants because of the low past nature that means the signal does not vary too much okay, so no past nature means what within some amount of time it has only low frequency values high frequency values are not there so within a certain amount of time it cannot really vary beyond something that is called no past nature because the slope will be lesser than means it has to be less than something right.

So if that is the case immediately see I have already spoken that within this it can take from this value residual but then I can specify that whenever I from this to this I go and if I know what is the maximum frequency it can take immediately from here if it has already taken a value this much over here there will be a bound up to which the value can vary this bound can only be calculated.

If I know the dependency so that is why it is very important that I also have some understanding about the dependency of a sample signal with respect to the previous or previous to previous all those sample signal so what do I have to do that we have already talked about random variable and this is why we have talked so much about random variable dependency joint PDF and all those things so right now you can only immediately see that two things which might be dependent okay.

So if I wish to characterize what do I have to do I need to go towards the joint PDF of these two random variable so a random process I know that it is an infinite collection of random variables and there should be associated dependency among them so if I need to characterize the entire random process I need to actually capture all those dependency or rather I should say The Associated joint PDF of all of them so that is where the next part comes where I will be characterizing a random process.

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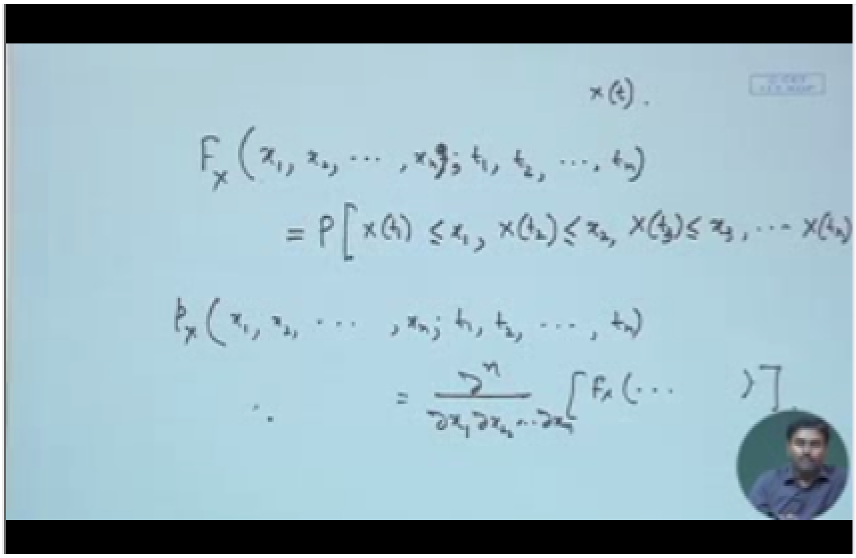

So what do I wish to suppose X (t) is that random process so I wish to do a characterization that means CDF I will be calculating so in CDF I will talk about X 1 X 2 X n what are these bracket semicolon again do you want T to I will just explain what are this T n what does this means actually it is actually the sample we were having we define a time T1 T 2 and so on up to TN at every instance we say the associated random variable is here X 1 here it is X 2 and so on up to X n okay so these are the associated random variable and what.

We are trying to do is at those time instants we wish to see the joint CDF of those random variables I have told you that they might have dependency so I want to capture that full dependency so only way to capture that full dependency is get a joint PDF or joint CDF among them so this is what you will have to do and remember it has to be done overall sample time that means this T will go towards infinity so every sample time you take try to capture all those samples which will be the random variable.

And then take the joint CDF of all of them right so this will just specify that my probability that X at T1 is less than equal to X 1 comma X at T2 is less than equal to X 2 and X at T 23 3 is less than equal to X 3 and so on X at TN less than equal to xn so if I can characterize this entire probability then only I know that that entire random process I have captured it with which is a daunting task you can already see that if I have a signal it is a continuous time means and then immediately you have infinite number of time instants and at every time instants.

You need to actually get joint PDF that means or joint CDF that means that random variable at T 1must be less than equal to some specified value X 1 and so on that will give you the joint CDF x1 x2 and xn right and if you just do a net order differentiation you will get the CDF so px this x1 x2 up to xn that should be just an eighth order differentiation with respect to all these variables right DX 1 DX 2 and the CDF effects right so once I have this probably I know that I have characterized the whole signal okay.

Which is already we have we could see that it is a daunting task okay so any random process characterization of that fully is really beyond our capability most of the time so therefore what we will try to do is we will try to restrict ourselves okay try to see something and try to give some definition to it so what we will start doing is something like this we will go to first order okay so what do we mean by first order so we will talk about mean.

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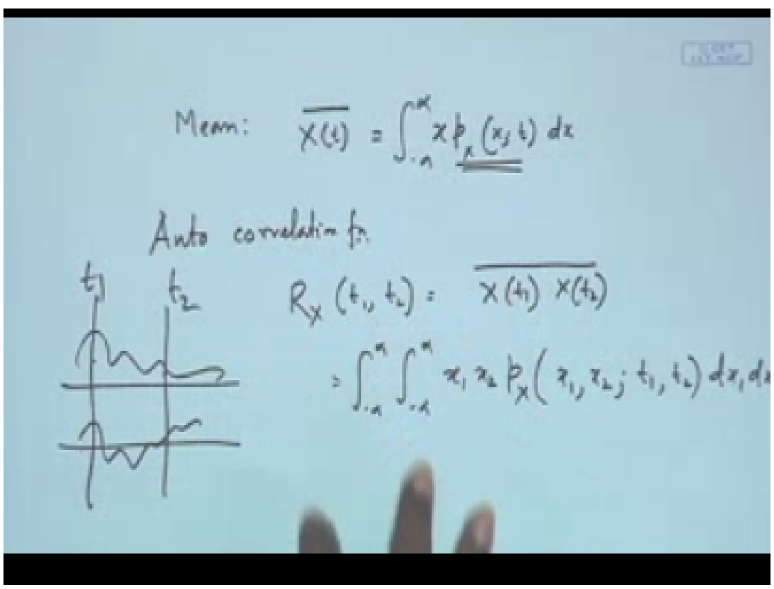

What do you mean by mean that is defined as this so what we are doing over here is we just take one sample time that is T and then try to get the mean of it so a particular sample time and just calculate the mean so this is called the first order immediately what will happen so I have two mean means it is the value multiplied by the PDF so X and in PDF because it is a one sample so I do not need that Joint Distribution I only need one of them right so at time T so I will be calculating this px X atT X so if I can do that I will get money right so this is the first order we'll talk about okay.

So this just we are trying to get the mean or we are trying to characterize the PDF at a particular point so in time so in time just take a snapshot try to evaluate the mean or you try to evaluate the PDF once you get the PDF you will begetting the mean as well okay next we will try talk about the second order that means at least two time instants we will take okay so that is called the autocorrelation function you might have already seen out of correlation function.

But that was a different kind that was in time we are just shifting it and multiplying the same signal so it was the same signal here it is not say you have multiple signals okay but we will try to evaluate the autocorrelation function what is that we call that our Xt1 t2 so it's two time so therefore it must be at t1 I get the X at T 2 I get.

The same from that same random means sample signal I take that and I take the average overall average this is remember there must be a joint PDF and it is average so it should be a double integral okay so it must be - infinity to plus infinity minus infinity - infinity I take suppose I pick value x1 at T 1 and X 2 at t2 and then I need to get this joint distribution which is x1 x2 at t1 and t2 so now I have to get at two time instants what is the Joint Distribution among them okay and DX 1 DX 2 this is all that I will have to do so basically.

I take 2 or infinite number of sample signal at t1at t2 I just try to get the multiplication of these two ok and get the joint PDF of them and then try to evaluate the overall average value okay so which is the autocorrelation function now from this autocorrelation function you can see also if I just make t1equals 2 T 2 it will just go back to the same time and then I will get the second order so that is why over here with the single this one.

I have not defined the second order or third order because that will be just a outcome of this thing where you just put u 1 equals 2t 2 you will get back that ok so we have defined two soon we can just keep on defining things so what we will do we will just get an idea of how to define them at higher order and then we will try to talk about the minimum requirement that we need to talk about so that the signal is almost characterized so in the next class we will do that.