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NPTEL ONLINE CERTIFICATION COURSE

Course
On
Analog Communication

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Lecture: 37 Probability theory (condt.)

Okay so we have so far characterized what we mean by beat and what we mean by variance or standard deviation for mean we have already said that it if we have two random variable whether they are dependent independent correlated uncorrelated we don't care it's mean is always addition okay so separately you can calculate mean and you just add them you get the overall mean value of that but suppose.

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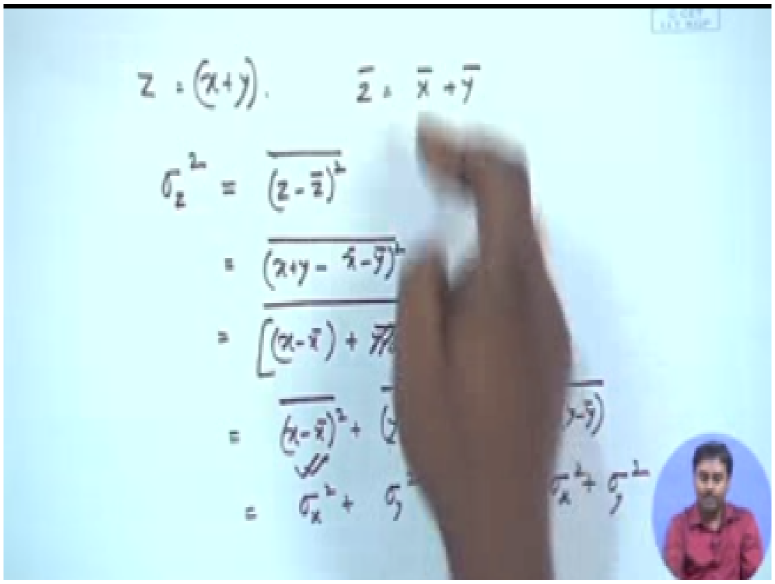

I have some X which is the summation of two random variables right X + y so for that we have said that mean of Z should be this so that is something we have already proved now let us try to see what happens to the variance here we insert or will means say something that and y are independent let us try to see what is the implication of that so I would like to calculate this Σ J square which is the variance of jet that is a new random variable which is constructed by making X + y where x and y are two independent random variable okay.

So Σ J square is nothing but we have already characterized that jet - dead bar whole square bar we have already said that so this is the average of that random variable - mean square so now replace z by this thing z by X + y and Z bar by X bar + y bar I can do that so this is X + y _ X bar _ y bar whole square I can Club this as X _ X bar + y _ y bar right this can be written as X _ X bar whole square + y _ y bar whole square + 2 into X _ X bar into y _ y bar now the overall averaging.

We know that it is just multiplication with PDF joint or marginal whatever it is and then integration so integration gets distributed with respect to additions therefore this must be this bar this bar and this overall bar right now let us see what is this is just Σ Square by definition this is just ΣY square by definition okay by definition of variance let us talk about these things so first of all we can say this let us try to evaluate this X _ bar.

So this will be XY _ X bar Y_ XY bar + X bar Y bar again it will be distributed okay so this should be X Y bar _ now let us see X bar is a constant so it will be just X bar Y by bar this is also same thing Y bar is constant so this should be X bar Y bar + X bar Y bar for independence we have already proven that if X Y are independent then X Y bar is equal to X bar into y bar so this must be 0 once this is 0.

So if you just go back to our previous this one that Σ J square is Σ X s square + Σ Y Square and this is 0that is a very fundamental result that if two events are independent remember four mean independence was not required but four standard deviation or sorry variance the independence is required and then only the variance of the addition of two random variable will be addition of their variance right this only happens when these two random variables are independent if not this factor will always be there and this will not be canceled out okay.

We have seen that this factor is 2 * X y bar_ X bar Y bar so as long as they are not 0 this factor will always be there okay so that is called means that is how you evaluate whenever two random variables are independent okay now from there a new thing comes into picture what we do is something like this that particular thing we have got X _ X bar into y _ y bar we define that as a correlation between two variable x and y.

So we can immediately see if they are independent the Σ X Y will be 0 we call that then X Y are uncorrelated so independence means that they will be uncorrelated but not vice versa okay because actually independence means little bit stronger condition it's just not independence means X Y bar will be X bar Y bar that's alright but if that happens immediately Σ X Y will be 0okay whereas for independence you need something more that the PDF has to be distributed the joint PDF P X Y X Y that must be distributed.

It should be p x Into p y this must be happening which is a stronger condition if that happens we know that there becomes uncorrelated but that does not mean Sun correlations just this must be equal to 0it does not mean that XY must be means having a PDF which gets separated out okay. So always do not get confused with the definition of Independence and uncorrelated unless generally what we should say independence is a stronger condition as long as two variables are independent I know that they are uncorrelated but the reverse is not always true okay they might be uncorrelated still it might not be independent okay.

So this might happen because independence requires stronger condition so let us try to see what this correlation captures it is almost if instead of X I put sorry instead of Y I put X it is actually going back to variance definition because that happens to be X _ X bar whole square bar okay so basically your correlation takes you towards the variance if x and y are same okay so let us try to understand what does this means what is actually correlation so let us try to do some means try to list out some random experiment so let's say suppose the experiments are like this Ian a day or in means in a day what is the temperature that is one random variable for me okay so that I record and also I take another random events which is the selling of soft drinks okay.

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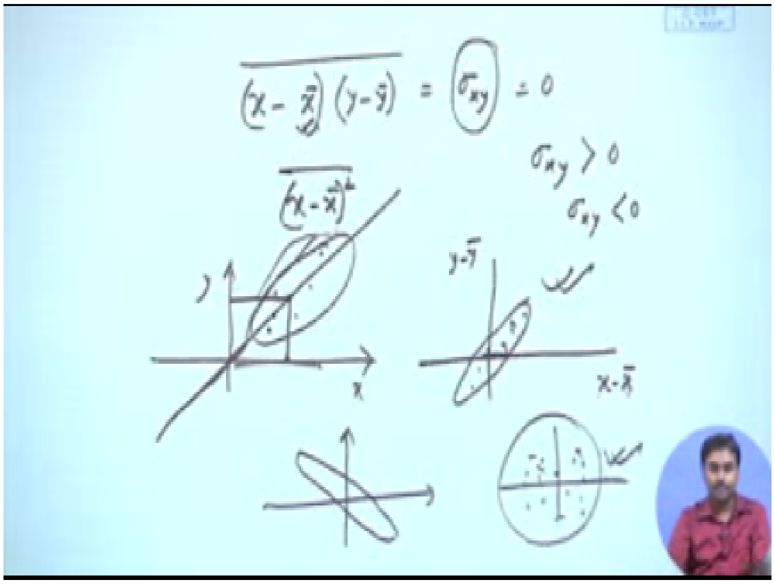

So how much quantity of soft drinks has-been sold in a day let us say these are two random events so temperature can be anything different day I see different values I will be getting same thing will be happening with the soft pink means sold that number now let us try to plot them what does that means we choose multiple number of days okay and this is the x axis and y axis in x axis will be plotting what is the temperature in a day and in y axis will be plotting what is the amount of soft drink that has been sold.

So every this is called scatter diagram where every day will be put somewhere okay so what does that means a particular point corresponds to what was the temperature on that day anyhow many means what is the amount of soft drinks that has been sold so every point represents a particular day it is as ample value of that day okay and sample value of this joint event I am actually measuring two random number and try to plot it over there okay now plot these things it is all fine now if these two events which we could see that as the temperature increases probably more soft drinks or cold drinks will be sold.

So these two events are highly correlate done actually influences the others so what will happen in the scatter diagram also you will see similar things whenever this is high this also must be high whenever this is low this must below so it will all be scattered in that region okay never it will happen that this is higher and this is lower or this is lower and this is higher so it will not be scattered over the entire thing this will never happen okay so if I now so it will all be scattered in this fashion if you just write or draw equal to X it will be around this okay and then if I just take out from each of the samples the X bar so what will happen this will get shifted if I now plot X _ X bar and y _ y bar.

So it will be plotted like this it will be clubbed around this once this kind of plot you are observing then you can say in the scatter diagram that they are highly correlated similar thing might happen when they are just negatively correlated so you could see a plot like this what will happen whenever particular part is high the other part will be low it is it is just negatively correlated that means if this is high then that should be low okay.

And if you have something which is completely uncorrelated the scatter diagram will show you all over the place something everywhere it might be high with low with high both are same so all kinds of things will be getting if they are uncorrelated if you take out this means bar X _ X bar and Y _ y bar they will be all populated around zero everywhere and then immediately if you take these numbers X _ X bar into y_ y bar they will actually tend to cancel out each other.

Because it scattered everywhere right they will not produce similar things here they will always produce positive number here they will always produce negative number okay so if you add them all probably they will produce a huge negative number here they will produce a huge positive number that means they are hugely correlated either positively correlated or negatively correlated okay whereas this one will give you almost 0.

So that is where the scatter diagram can help you to see that if two events are correlated or uncorrelated that is pretty obvious are pretty clear which will be happening so basically this Σ X Y will tend towards zero if two events are completely unfolded that means this kind of scatter diagram you observe okay and if that is not the case probably you'll see that they diverge and they'll have a huge value okay and for positive correlation it should be Σ X Y should be generally greater than 0 and negative correlation.

Must be less than zero and there is a technique to normalize it so what generally peopled the Σ X Y is divided by Σ X into Σ Y that is called the row X Y just to normalize it what will happen whatever the value of Σ X Y this cannot be bigger than this multiplication right because that is the biggest that it can get so if that is the case this will be always less than equal to 1 and greater than equal to _ 1 so at _ 1 it will be highest negatively cold means correlated fully correlated and at + 1 it is positively fully correlated.

And at 0they are completely uncorrelated ok so this is how it goes so Rho X Y if you plot at 0 its uncorrelated this side the positive correlation increases this side the negative correlation increases it sends at _ 1 in that + 1 because you have already normalized it so you cannot get a value bigger than 1okay if it is not normalize it can be even infinity right so that's the concept of correlation and therefore I can relate if suppose I have a random variable jet which is X + y then I can write now which was earlier done Σ Z square that is nothing but ΣX square + Σ Y square + 2Σ X Y so that's the correlation okay.

So as long as this correlation is 0 this would be just Σ X square + ΣY square otherwise this correlation term has to be kept in okay so this is something you will see later on that if they are not independent and not uncorrelated then what is the effect of that what happens in a random variable where you add two random variables so we see most of the time why we are so much bothered means so far in the previous class as well as this class we have been evaluating the addition of random variable.

Why we are doing that let us first try to ask ourselves why this is so much required in a course like another communication that we are so much concerned about addition of random variable so you know in signal what happens we have already talked about channel being linear right so if there is noise and the signal being transmitted most of the time what will happen noise and signal will be added with each other okay.

Now most of the times noise and signal are independentbaath independent and uncorrelated this is the fact most of the time that happens sometimes there are correlation there are noise which depends on what kind of signal has been transmitted like in optical that is what happens if you transmit one the amount of noise will be higher if you transmit 0 the amount of noise will be lower so that happens there is a coalition also.

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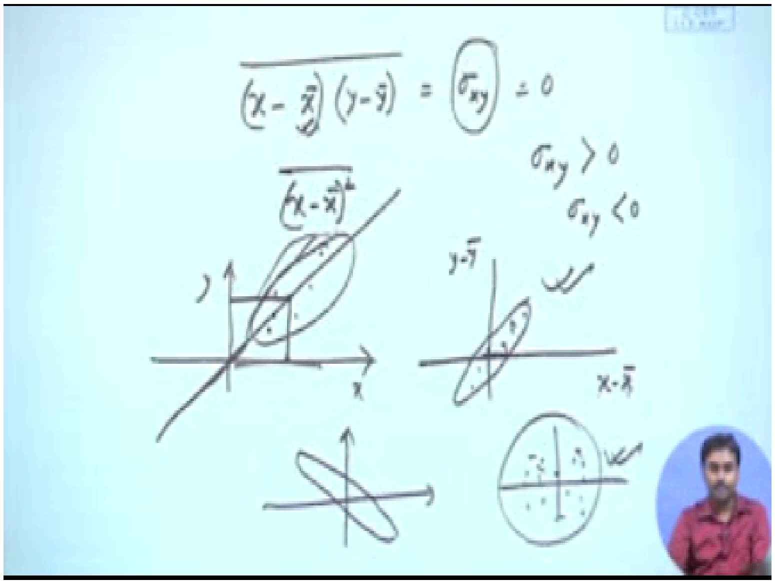

So but whatever it is when signal level you are adding because the channel is already assumed to be linear so there will be just addition of these two signal so your outcome which is the output it’s just addition of your regular signal+ the noise is the regular signal forth receiver it can still be a random thing right if the receiver already knows what is coming in a deterministic manner he doesn't have to he doesn’t bother to receive that okay.

He wants to receive that because he does not know that there is uncertainty over that signal and that is why he knows that there is some information in that so he wants to receive it so that is unknown signal which is also a random thing for him and the noise which is being added at the channel or at the receiver wherever it is it is also unknown to him it is random so these two signals will often be added and that is why we are so much interested in addition of two random variable.

So we are characterizing see you might be thinking that we are Justin the background we are just doing mathematics but not necessarily we are choosing that part of mathematics which will be interesting for us while characterizing a communication channel or means signal processing right so that is why we are saying so much things about addition of two random variable right now they we have already talked about when they are independents if noise and signals are independent then of course whatever theory we have talked about independence that will be applicable if they are not independent like we have told optical noise.

That is not independent of the signal so then of course the correlation has to be taken into account while characterizing them so let us now try to see that suppose and y these are two let's say independent random variable what I knows something like this I know this for What’s the Associated PDF now we are going to PDF so far we were just doing things with respect to measurable quantity that's the variance and mean now let's go towards more concrete things where everything can be characterized.

So the PDF so suppose this and y for both of them I know the individual PDF okay let us also say that these two are independent so for independence only wearer now proving this whatever theorem will be proving okay now let us say my Jed which is a new random variable which is the addition of these two random variable knowing these two PDF is it possible to get the PDF object so that will be our next target how do you get that okay.

So PDF generally comes formed we can always differentiate any CDF to get PDF so let's characterize the CDF of set which is nothing but probability that this random variables it will be less than equal to some defined Z over here okay so from here what do we know we know that I can say Y should be if this is the relationship I know that always Z is x + y then I can always say Y must be Z _ X I can characterize this.

So this means actually probability Z has to be less than Z means probability that my X which is a see once I characterize I can get something on Y right with respect to this set so Z less than Z means actually X for any value of x which is less than infinity I must have a Y which is less than equal to Z _X then J will be less than J this set right always I will be able to ensure this so this and this these two are equivalent definition very carefully see this because what I am saying that I need a probability that my Z random variable must be less than equal to Z but I also know that my l x+ Y is equal to Z.

So now what I am trying to see because I know the joint means I know individual distribution right so that is something I know so let us say this particular event is nothing but that I take any event from X can take any value but I need to ensure that my X + y must be still less than X + y will be that Z that Z must be still less than Z that means Y must have this condition whatever the value of X it must be less than equal to Z _ X then I know that whatever that associated things.

I will be getting there the overall Z it will be always less than this set if I in these two things because Z is always being created by the addition of these two as long as I ensure this I will be having this right so if this is the case b I can always write this so this is again joint PDF from there I can calculate this so what do I have to do for X I have no restriction I can go from _infinity to + infinity but for y I have a restriction I can go from _infinity to this value.
so why should be going from _ infinity to 2 J _ X into P X Y okay.

Inside let us say u W do you write so this is the joint distribution of X and y I still do not know but probably I do not have to we will see that ok but according to my definition this should be equivalent right now let us try to do something so I can write this from _ infinity to+ infinity ok so this is d you and within that my integration _infinity to get _ X P XY u I have-not separated this integration remember it’s still this integration inside this integration I am just writing it in this way okay.

So you can even write D u over here or you can just write D u over here fine now what I will try to do is I will try to differentiate this because I wanted of this right I will try to differentiate this with respect to Z that must be giving me the PDF of Z so P Z said I know that is nothing but DDZ of FZ said which is nothing but D Z of integration _ infinity +infinity I have D u and inside I have integration _ infinity to Z _ XP XY u w DW right this now this integration limit has no Z.

so I can take this into differentiation inside no problem in that only inside of this I would be able to take because limit already has Z so I can write it _infinity to + infinity D DZ now because the limit is Z _ X I will put that as DD Z _ X of course I can put a chain rule so I can write I can write this the problem in that and I will have this integration _ infinity to Z_ X P X Y u w DW right this is just1ok what is this you have this conjugate of integration and differentiation okay.

So this differentiation and this integration they will cancel each other okay because that's just complementary so they will cancel each other and what we finally get _ infinity to +infinity I will have this P XY or youth blue right this is fine seconds that should be with its limit sorry I should not write this I must be hiding -in philosophy see whenever I do this cancellation what will happen this will just be up to that function up to this value this integration and differentiation gets cancelled but after cancelling the value that remains is that value right so I can write this as

P XY this you now becomes X and Becomes up to this yet - X and I will be having if it is X I will be having X okay.

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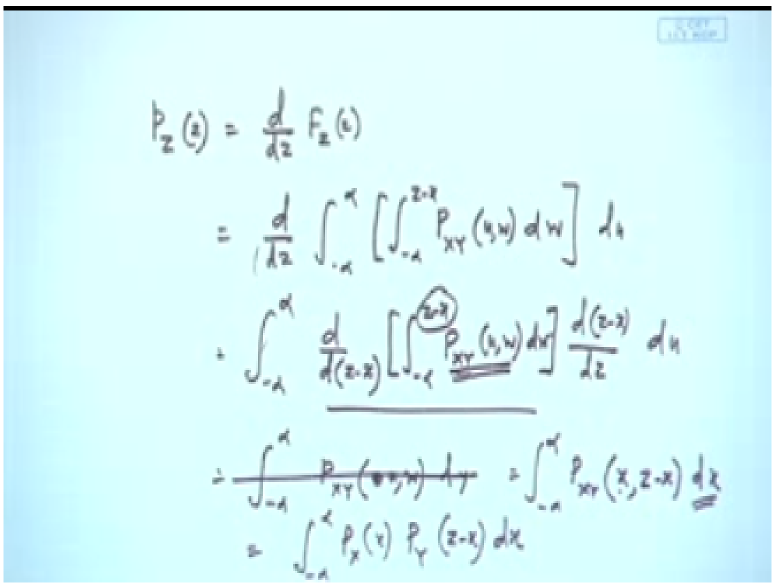

So I did nothing instead of X I have taken a dummy variable DX okay and this automatically goes up to Z _ X so P XY that goes up to the N _ X this gets cancelled out and it is at that functional value right so that is what I get and I have after all these things I have this thing right now if this two are independent I have already told that p x this x and y are independent so immediately I can write_ infinity + infinity this must be p x into j _ X DX right as long as they are independent otherwise no what do I get I get a convolution you can you can identify this function this is just convolution of p x and trance because this variable is Z.

So it takes you to Z so basically that is a very important result that whenever you have addition of two random variable if I know individual PDF and if I know these two random variable areindependentsee in the proof steps all those things were used if they are independent I know that they are that new random variable created through the addition is having a PDF which is just the convolution of the Constituent PDF there is a very strong result as long as they are independents in our case when signals and noise those are characterized by some PDF if I get an additional signal which is the addition of these two as long as.

I know signal and noise are independent I can always say it will be just a convolution of these two PDF right so that is something will be with this with the help of this we will be able to characterize a PDF of the joint signal that so that is a very strong tool which will be required later on so probably what has happened with all these things will probably end my discussion of random variables.

So we have now got enough tool to characterize our signals and characterize our transmitted signals well as noise when they get added and all those things we will be able to do that but now we will go to another step where this random variable will become a function of time which is a very means obvious thing that happens in signal transmission.

So and that particular thing is called random process so now with the help of this random variables and associated understanding we will try to build up our own understanding of random process which will help us actually characterize noise as well as signals okay so then only we will be able to talk about a communication which is contaminated by noise how do we characterize that so from next class onwards we will start discussing about this random process and its characteristics okay thank you.