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NPTEL ONLINE CERTIFICATON COURSE

Course
On
Analog Communication

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Lecture 32: Probability Theory (Contd.)

Okay so in the previous class we have already defined means we have given one example of rolling a die we have said that there are two events which is called the event where the means stop surface of the die will be will be showing me a even number and that is called or odd number okay so that AO is called that it will be showing odd number and then another event we define that is B that the number it will be showing that is <= 4 okay so we started evaluating the joint probability that $P(A_0)$ or AO and B both are same okay.

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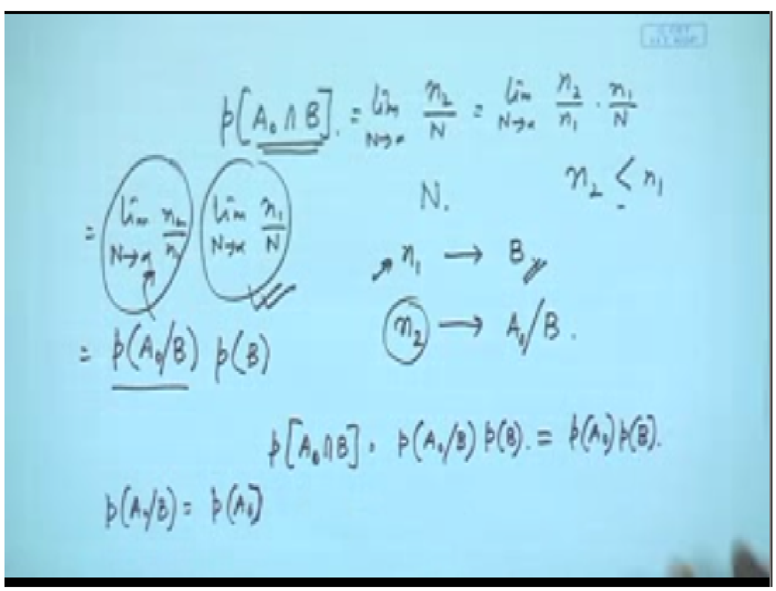

And then from the frequency definition we could see that it should be this where we have defined a conditional probability that a particular event has already occurred means given that what is the probability of the other event happening and P B okay x PB is something we could see this is another axiom of probability theory p[A o ∩B] so that is that is pretty much what mathematical theory does the initial part you take it from experimentation okay.

You build up the first few forms of axioms and from there in a deductive logic which we are doing now okay so you have given that definition frequency definition to probability and after that it is just deductive logic with which we are actually going in forward okay so this is one axiom which is being derived from that frequency definition of probability theory okay so this is something we have already understood.

Now let us say that this a A the event is independent of B what do I mean by that that is a very important definition in probability theory that two events are independent okay so what do I mean by that that means that occurrence of a oh does not really depend on whether B has occurred or not okay so whenever we are trying to evaluate this n2 how many number of times Ao will occur given B has occurred even if I do not consider B I will probably get the same number.

If this is happening that means I have no dependency for the probability calculation of this particular event on the previous event I will give some example so basically in that case what I can say is p AO given B is independent of B whether B has occurred or not I will get the same number so I can write this as P[Ao] and immediately this formula turns out to be that is a very important theorem of probability if two of the events for which you are calculating the joint probability or the intersection.

If both of them are independent their probability just gets multiplied probability of individual event gets multiplied we will just give you some example and it will be very clear.

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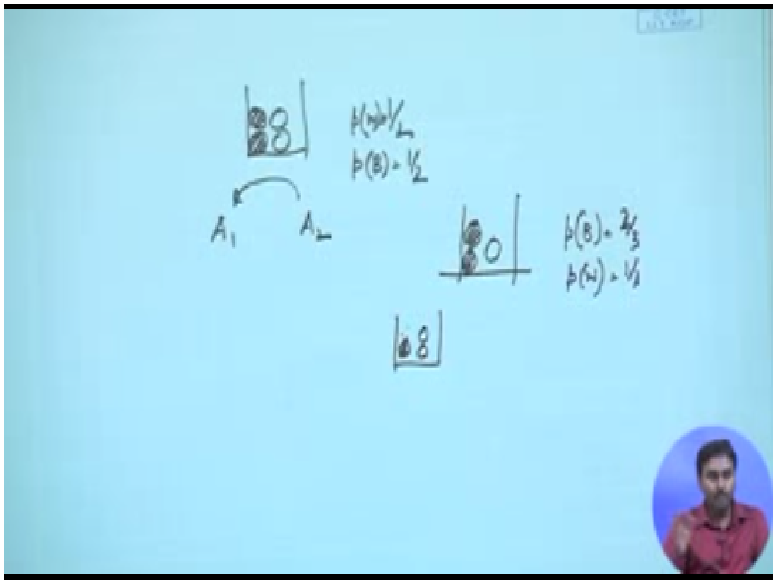

So let us take this let us first give one example one very simple example or this dependency so let us say I have a particular arm with two red ball and two black balls so I have this thing where I have two black ball and two let us say two white wall now if I cannot see what I am picking up and randomly I pick a thing there is a probability that will be either red or it will be black okay if me picking a ball is any of them is equally likely then what should be the probability that I will be picking up means let say white ball that should be half because there are two favorable and there are four overall things okay.

What is the probability that I will be so this is probability of picking white ball probability of picking black ball should be also half okay so now I say I define two events one is first time I will take out a ball okay and I see what it is and then I put the ball back inside darn and next time again I takeout about okay so the first time I am taking out that is a random event and second time I am taking out that is also a random event let us call θ1 and a2 the outcome of a2 does it depend on the outcome of a1.

Absolutely no because I have replaced the ball so the experimental scenario goes back to the previous case okay so whether in a1 I have taken white or red I have replaced the same ball the overall means sample space remains the same so whatever again I will be taking out if things remains equally probable I will be getting same probability so this a1and a2 these two events are independent of each other but suppose I do not do that in the first experiment I take out something and I do not replace it okay.

so immediately what will happen the probability will be now dependent on the outcome of the first event because if I in the first event I have picked a white ball then this balance will be little bit changed for the second experiment there will be two black and one white so automatically pee black will become1 sorry 2/3 and pee white will become 1/3 and if I in the first experiment I pick a black ball then there verse will be happening so now I can see my probability calculation or the overall event is dependent on the first event okay so this is where I say two events are dependent or independent

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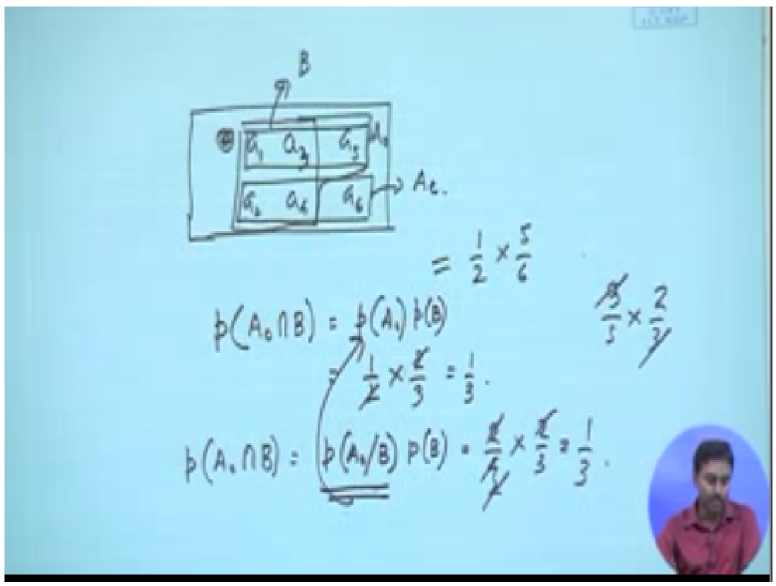

So let us just for fun we go back to the same example we have taken sorry now lets define this as my B this as you know this as a lets try to see if I wish to calculate P a or intersection BI first assume that they are probably independent I still do not know whether Ao and B are independent or not we will try to derive this okay try to see that and you will see a very interesting case so suppose I assume that so immediately what will happen because they are independent the probability theory according to our derivation it says it should be this .

So what is probability of a oh that is actually three events / 6 events so that should be 3/6 or 1 / 2 x What is probability of B that is actually 4/6 so that is 2/3 so it is 1/3 right if I take them to be independent I might be wrong I will try to vary verify that now I will do the right calculation if I do not have knowledge of independency I will do a right calculation so which says probability of Ao given it is B x Probability of B this was the actual definition if they are not independent.

So I still do not know whether they are independent or not so this should be the right calculation this is more generic so let us now try to see what is the probability that if given B has happened what is the probability that Ao will be happening so given B has happened how many cases are there I will be restricted already okay so given B has happened I have only four events out of them what is the probability that Ao that is my favorable case will be happening.

That is actually to say it is 2 x 2/4 x Probability of B how much is that is again same thing probability of B remains the same 2 / 3 that is 1/ 2 so I get the same result which says yes these two events are independent I just change this definition I change the definition of B I say my B is this you will see a fantastic thing happening if my B is this that means B that event B is actually less than 5 <= 5 happening okay.

So let us try to do that same calculation if they are independent what should be this PAo that is actually 3 / 6 so that is again 1/2 and P B that is actually 5/6 right so if they are independent this is the case right now let us see if they are not independent so what do I get I do get over here B has already occurred okay out of them how many are A o that should be 3 /5 x PB which is 2/3 C the probabilities are not safe so the event whether it will be independent or not it all depends on whether this is equal to this probability A o given B is equal to probability you know that I can evaluate.

So if that is not happening that means the events are not independent see whenever I was taking 4 because what was happening actually because it was taking same number of odd and even so eventually or half of them were even so eventually my independency was coming out whenever that has not happened because I have taken now 5 number so there is a bias towards odd even once that has happened immediately my independence goes away.

So why I have given this example a slight definition of the event or slight variation in the definition of the event can change your dependency so you have to be very careful whenever you are writing or joint event to be the multiplication of two event so whenever you do that that will be very careful you have to ensure that is the case otherwise you will be doing a mistake this is more generic definition with a conditional probability so always you should take that into account.

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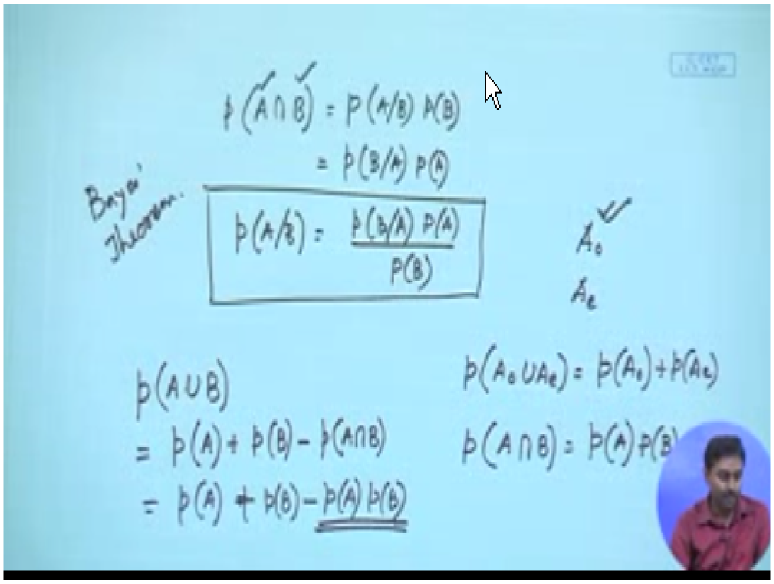

So now let us see we have already defined P let us just go back from that example let us just take two things two events a and B so a intersection B we have said it is P A given B x P B okay these two event I have no preference so I can even write this as P B given A x P A there is no problem in that because I can just replace A / B and B /A I get the same formula right so these two are equal and immediately I can see that P a given B can be calculated because these two are equal I can always calculate P given A PA /PB that is the famous bias theorem.

So what we are doing we are just building up the probability theory entire probability theory just from that initial frequency postulate with that only we defined all these things and now we are getting one after another theorem so these are all linked and they have some importance also you later on see why this is very important okay so we will talk about that later on when we give more generalized thing so now I would like to touch on two things okay we have now talked about two particular kind of event one is independent event one is one is mutually exclusive events are the same thing or are they not okay.

Basically it is not completely different do not confuse independent event with mutual independence or sorry mutual exclusiveness in mutual exclusiveness what we are saying just take that example of Ao and Ae odd and even in that case if Ao happens that means the odd number I get immediately I can say that event will not be happening so basically I am almost saying there is a strong dependent instead of becoming independent mutually exclusive events are strongly dependent if I say this event has occurred that actually means the other event has not occurred.

So I have some already some understanding of the other event by defining this event okay so they must be strongly dependent so mutually exclusive events are never independent mutually exclusiveness has nothing to do with independence people often get confused with these two definition they start thinking that mutually exclusiveness is probably also defining independence no it is not that is the mistake people do often when they are just starting to learn probability theory.

So that is not the case in mutually exclusiveness remember the probability axiom that if two events are mutually exclusive then I can write this and if two events are mutually independent suppose A and B then I can write this so always remember this and if two events are not mutually exclusive then I can write suppose a union B I wish to write so that must be set theory we know PA + PB we have already talked about that it will actually count two times so I have to take this out.

If they are independent but not mutually exclusive then I get this theorem this happens so the two events A and B are independent but they are not mutually exclusive then this should be happening if they are mutually exclusive this should be 0 okay so whether they are independent they are not independent means actually mutually exclusive means they are not independent so and this immediately happens to be 0 okay. So that is something which we should be keeping in mind whenever we are defining probability now let us do something more okay.

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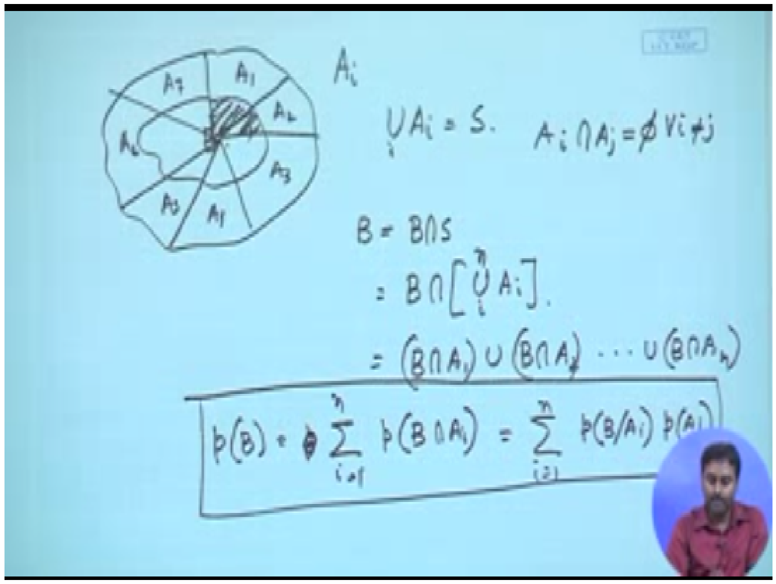

Just take the bias theorem towards more generic one so lets say I have a event B okay so this is the sample space of B lets say in between all those events are actually or all those elements are included and this sample space I now actually include it within a particular sample space called A and I divide that a into some mutually exclusive partition okay I call that A1 A2 A3 A4 and so on so what I have done is I have taken this B I have taken a superset of B which is A and then I have actually partitioned a in mutually exclusive parts okay.

And I am calling them a I and what I am saying that this a actually happens to cover the entire sample space that I am experimenting with so that means all the events that can happen that is included in the entire A okay so a include every everything in that sample space B might not because B is a subset of a so B might not include that but A include everything so I can always write UAI because they are mutually exclusive so they will be just added one after another or their elements will be just added over I if I add them so that is that must give me the overall some sample space that is according to definition okay.

And I also have this definition AI ∩ AJ is null set for all I not equal to J so this is the definition of my a that is how I have created my A know what I can write about BCB because B is a means proper subset of A or that sample space S I can always write B equal to this because it will have C when I take if it is a proper subset the elements of that will be actually elements of that and intersection of the bigger set because it will have the same elements right so I can write this now yes I can substitute in this one so it should be B∩[UAI] okay.

So now because they are mutually exclusive I can actually just flip these two so I can write this as which is very obvious from the Venn diagram also the diagram I have drawn so B I can write as this Plus this where this is actually B ∩ AI or A1 this is actually B ∩A 2 if I just add all of them I must be getting the back so this is what is happening so U (B ∩ A) suppose it goes up to N and so I have made n partition here it is n=7 right.

So I can write this now because AI are all mutually exclusive so the B ∩ AI are mutually exclusive that is very obvious from here also they are all mutually exclusive because AI are mutually exclusive so if I take B intersection AI that must be a subset of that now B intersection A1 and B ∩ A2 if A1 A2 are already mutually exclusive they have no common element therefore that small subset must not have any common element so I can always write these are all mutually exclusive.

So now probability of P what can I write that should be probability of this whole thing because B is this now because they are mutually exclusive now I put that probability theorem of mutually exclusiveness so that should be sum of all those probabilities so I can just write summation over I going from 1 to N it is P B ∩ AI, I can write this no problem in that now I can put Bias theorem the way I have learned it so I=1 to n this intersection is I can write it in terms of conditional probability so I can write P B given AI x P AI that is something which I can write now okay. PB that event we were talking about which is a subset of that whole thing must be this.

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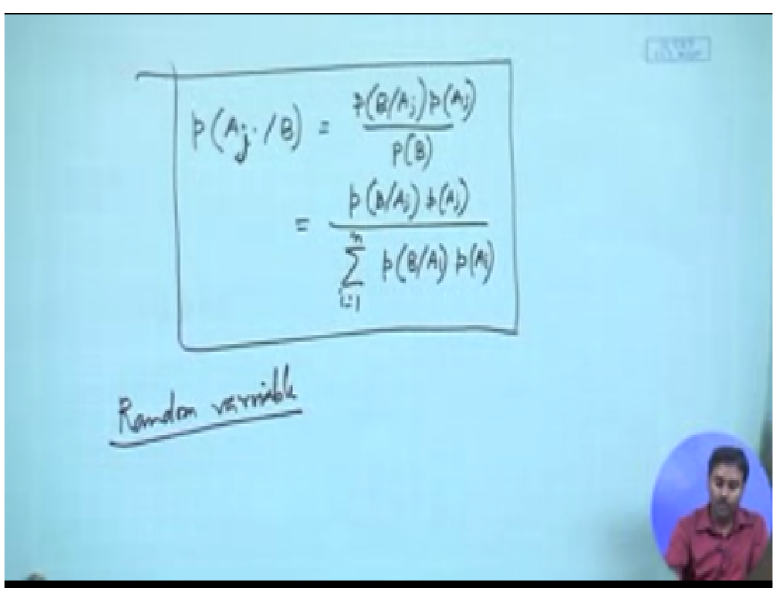

So eventually what happens I have earlier proven that P some AI given B or lets say lets just put J because I will be putting a dummy variable so this is something I have already proven that that should be P be given AJ PA J / P B now just now I have proven PB should be a summation term so I can just write this as given AJ PAJ / ∑ I =1 to n P B given a I that is the most generic form of Bias theorem okay you might be asking why we are doing this why Bias theorem is so important okay.

So what we should do is we should give some example of these things okay that whatever we are doing that is that is very important but before giving that example probably we will have to now go into another definition once we get means our hands very clear on that definition probably it will be easier to give that example will give well we will pick on every important communication example to say why this is very important okay why we are actually learning Bias theorem as a communication expert why this is very important for us okay.

But before that let us try to do something let us try to give another definition which is very important to probability theory which is called random variable so what do I mean by random variable or RV.

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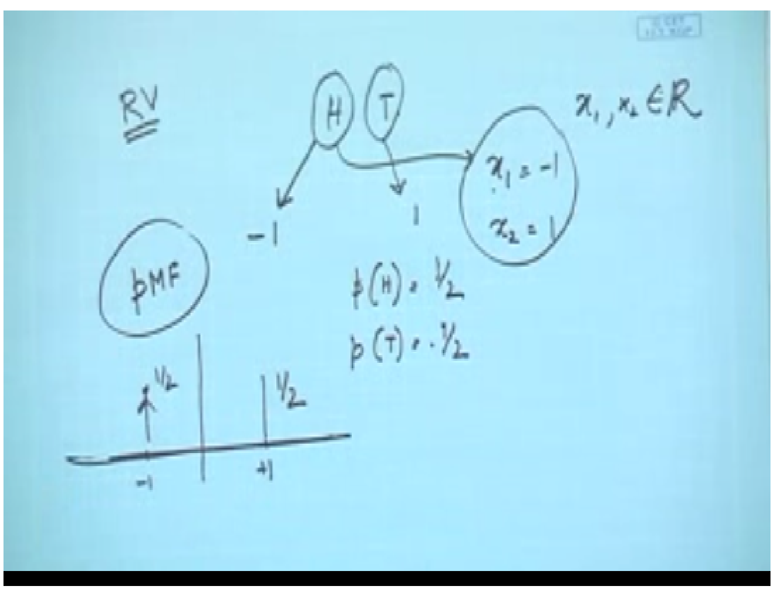

See so far we have been talking about set elements events are like if I roll a die it is just a description right it is the top surface getting one or top surface getting two and all those things mathematically those are not that description is not very sound right I cannot represent them mathematically I cannot write anything over there okay in terms of mathematical equation and all those things so it is very important that we define those things into a sound mathematical background okay.

And that is where random variable comes into picture so basically random variable is nothing but mapping those abstract things of definition of events into the real axis so suppose I have head and tail these were two outcomes of tossing a coin okay so this was a abstract thing it is just a word that head or tail it has no mathematical notion it does not go into some number and all those things so I cannot really manipulate them.

So now what I do I do a mapping I say that head gets mapped to -1 in the real axis and tail gets mapped to + 1 or vice versa whichever way you wish to put it okay so if I just do that then I get a definition of random variable so because this head and tail we are actually random outcome the event is defined but I do not know if I do the experiment whether I will be getting head or tail that is why I call it random because before hand before doing it I have no notion of or no understanding what will be the outcome of that experiment.

So I cannot say that but what I can actually define is probability of them that means what is the chance of getting head I can I can define that that is what we have defined so far so probability of that event I have already defined so I have said probability of head is actually 1/2 and I have said probability of tail is actually 1/2 this is something I have told now what I will do instead of saying probability of a event which is abstractly defined I will say probability of this +1 and -1 and I will put some number.

Immediately what do I get I get a functional representation of this probability so what I do this head I put it to some discrete value which I have done X sorry this -1 to the real axis so I call it X1 so X1 =-1 and tell I put it into the real axis I call this X 2 okay which is a real number so X1 and X2 are taken from the real number okay set of real number so X 2 + 1 it can be anything either you give -1 +1 I can also give 0 and 2, 0 and 5 whatever you wish you can give that but as long as your definition is clear you know that it will be mapped as a function okay.

So at minus 1 this will have a value of 1/2 and at +1 it will have a value of 1/2 and the Associated function is defined as PMF or probability mass function it is actually defining that if I map uniquely this is this must be one-to-one mapping remember every event should be uniquely mapped to the real axis there should not be any ambiguity if head and tail I map to the same point I will just get ½ + ½ one and I will not have any distinct means distinguishing feature between head and tail whereas actually they are two different events ok ay.

So I need to have a one-to-one mapping to the real axis and then I plot the probability with respect to them whatever I get that I call as probability mass function that means in the real axis as if that is representing the random variable that I am describing by doing this mapping okay and here because I have countable number of events so that is why the overall mapping will be discrete mapping remember if a random experiment gives me uncountable number of events then I can actually map it to a continuous variable.

Whereas if I have countable number of events I will only be mapping them to a finite number of or if not finite at least countable events and there will be all discrete so I get discrete probability values and the Associated function I call it probability mass function okay we will also see the other definition of it whenever I will have continuous one will have probability density function so this is called probability mass function okay fine so this definition is there is no problem in that okay so how do I define this.

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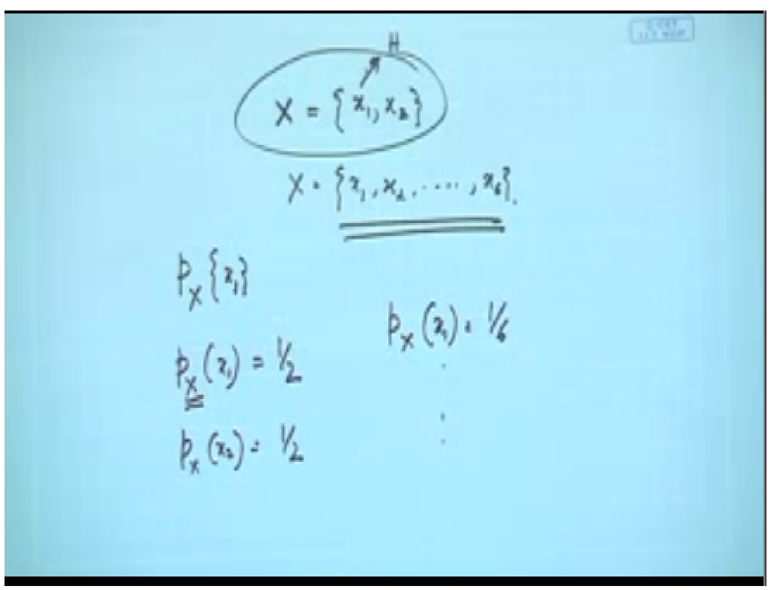

So I say The Associated random variable is X and this X might take value now we are defining earlier we are saying just head or tail now we can say this takes value X1 or X2 right if it is head tail if it is a die I can write it is X1 X2 up to X6 right if it is any other experiment suppose its card so there will be 52 such elements right so it depends on what kind of experiment you are doing accordingly how many events are there or elements are there will be all mapped to a real axis and those becomes the random elements.

So I define a random variable X and those have elements X 1 X 2 X 3 and then the PMF I define it this way P probability of this random variable taking a value X1 okay or you can put first bracket as well this actually says that your random variable which is this X defined by this X this is a random variable it takes a value x1 what is the Associated probability so if I am doing this experiment this must be half if x1 is defined as head and px X 2 that becomes half that is all.

If this is the random experiment then I call P X x1 that should be 1 / 6 and so on all of them will be 1 / 6 okay so that is how I define it and that is how I get the probability mass function similarly I can start defining the joint event okay.

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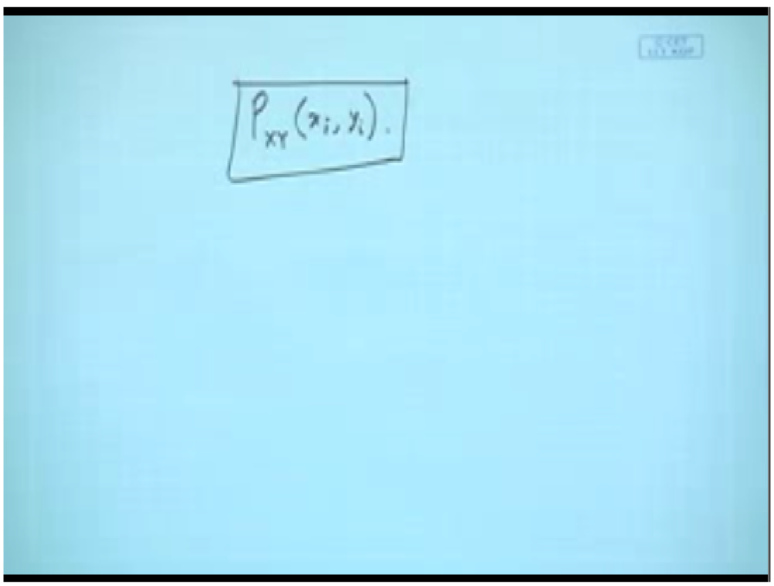

So the joint event will be it is like this so let us say I put a value X I and Y I so this is the definition of joint event that means basically now I am trying to see a probability that two random events or two random variables taking corresponding the value of X 1 or X I from the random variable X and y1from the random variable Y this is the Associated joint probability of x and y simultaneously happening which are taking simultaneously value X I and Y I.

So this is the definition of the joint probability what we will try to do next is probably we will try to give the axioms of these things now in terms of random variables so far we are defined in terms of set theory and events only but we are not mapped it properly to the random variable now we will actually properly map it to the random variable which we have just started we will do all those axioms we will actually map it to proper random variable and then try to see what is the outcome of them ok and then from there we will go towards the definition of continuous random variable okay thank you.