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NPTEL ONLINE CERTIFICATION COURSE

Course
On
Analog Communication

By
Prof. Goutam Das
G S Sanyal School of Telecommunications
Indian Institute of Technology Kharagpur

Lecture 31: Probability Theory

Okay, so far we have discussed about signals very bit of system and then try to describe about the modulation scheme which are employed for amplitude modulation mostly and then we started characterizing different kind of amplitude modulation that are there I have done a comparison and finally we discussed about little bit about the channel okay. And then we said that one of the most detrimental part of channel which will be probably either channel or receiver or I should say rather impairment is noise this is something we have already said.

But to understand noise probably we have to really understand random process, so what we will try to do probably give you a brief overview of random process but I do not assume any prerequisite. So what I will do I will give a first a brief overview of probability theory basic probability theory and then from there we will go to random variable and then from there we will develop the theory of random process probably which will be all targeted towards communication or characterization of noise.

So let us start with the concept of probability, so we all know there are events which are means eventually looks to be random to us, so let us just discuss one event which is almost known by everybody like either tossing a coin or rolling a die right so whenever we do that we say that it is random but most probably it is not random as we state that means whenever we toss a coin if we can calculate all the forces that that is being exerted on that coin say which angle I put it on my nail and then when I toss how much force I give in which direction and the air friction and all other things if I just gravitational force how it change changes with time so everything if I calculate and then probably elasticity of the ground where it is hitting.

So if I can calculate all these things probably there will be no randomness associated with it I can definitely calculate and tell you what will be the outcome of this event okay but as I have started describing you might have seen that there are lot of process a complicated process involved and associated physics involved in it. So if I really wish to means take a journey into that and try to calculate this will be a tedious job which as in any engineering or scientific study we do not want to do that we really wish to get some extract or some meaningful information from there and then try to see whether that can be used further.

So this is something where probability theory came into picture what people started doing they started looking into that particular random event I have talked about that tossing a coin instead of going through the entire physics of bins tossing the coin means exerting the force and then coin flipping in the air and dropping in the ground, so instead of going through all this physics they started thinking that okay given all those things let us try to see what is the outcome of that okay that experiment.

So if it is a coin and we assume that the coin whenever it is landing it is not landing on its side or edges so it will be probably landing on either of its face so there are only two outcome of this entire process.

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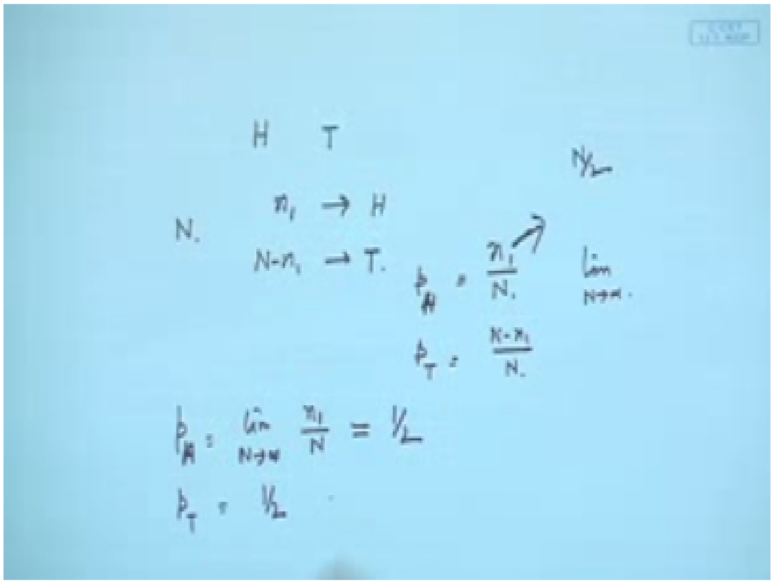

So one is we call it head or tail okay so these are the only two outcome of this whole experiment right then people started saying okay if this is the only two outcome why do I have to go through all these details let us try to see into can we give some understanding of this events okay. So because finally I need to see whatever happens what is the overall outcome and what is the associated means mathematical understanding of our physical understanding that we can derive from this process instead of going through all these details people started saying okay if this is the case let us try to do it this way that I will take either that single coin and I will keep flipping it okay.

So I will do this I will keep repeating this experiment okay let us say n number of times and I after every time I do this experiment I tend to record what is outcome okay and I start counting it so out of this n how many times let us say n1 times I have observed that head has occurred and definitely n - n1 times because we have already assumed there is no other outcome. So it will not land on a edge of the coin and all those things so n - n times there will be 10 okay and then we people started questioning that what is the relative occurrence of these things okay.

So or let us say frequency of each of those events which will be happening so the frequency is if I just take this experiment this simple experiment so I say from that experiment allowed outcome I can say n1 by n that is the relative occurrence of I say head, so let us call that as P H okay and P T is some n - n 1 / n okay and if I now start doing this experiment for a very large number of time okay.

So what does that means mathematically that I will take limit and tends to infinity okay and people started doing that and then they could observe that if the coin is means we should say unbiased coin or the coin we see that it has no preference for any one of the surface head or tail whenever it is landing see it might happen sometimes that the edge is designed in such a way that it has a preference towards a particular service it might happen that it is little tilted over there so if that is not the case people have seen that this n1 almost tends to become n /2 okay and finally what we get so we get this Pa which is limit n tends to infinity n 1/ n that goes towards 1/2 and same thing happens for sorry pH and something happens for PT that also goes to up.

So this was probably the initial definition of probability okay, so people started saying that probability of this particular event head happening is ½ that means if I just do this experiment for n number of times I will almost see n /2 times head is occurring and n by 2times tail will be occurring so this is the frequency definition of probability. So I associate with every event that are the outcome of that experiment whatever random experiment I am doing. So those events I associate with a value which is called probability.

So this is just percentage I should say means or I should not say percentage it is just a fraction that defines how much means if I just multiply that with 100 probably that will give me how much percentage of time it will favor a particular outcome okay. So that is actually the definition very definition of probability this is called the frequency definition of probability and all these numbers is half that is a magic number which has come from basically experimentation pure experimentation like any other scientific experimentation okay.

So these are the actions we should say axioms of probability and these are just basically observed over time and people have understood that probably this should be the case there is no basis that this should be happening people have not solved the behind means underlying physics to try to come up with this they have seen they have tried testing with different number of coins in different location with different number of times with different kind of people who were tossing it.

So they have seen all these observation and everywhere they could see that it is close to this and that has given that axiom that okay that should be the probability from there something else comes into picture okay, so people started defining instead of this definition they started defining that whenever I do a experimentation how many equally likely events are possible in a particular experimentation like in this experiment I have head and tail which are equally likely okay. So whenever I start calculating the overall probability so I count those equally likely or equal possible events okay I count those things and then create a space okay. So like here I have a space of head and tail only two elements are there okay.

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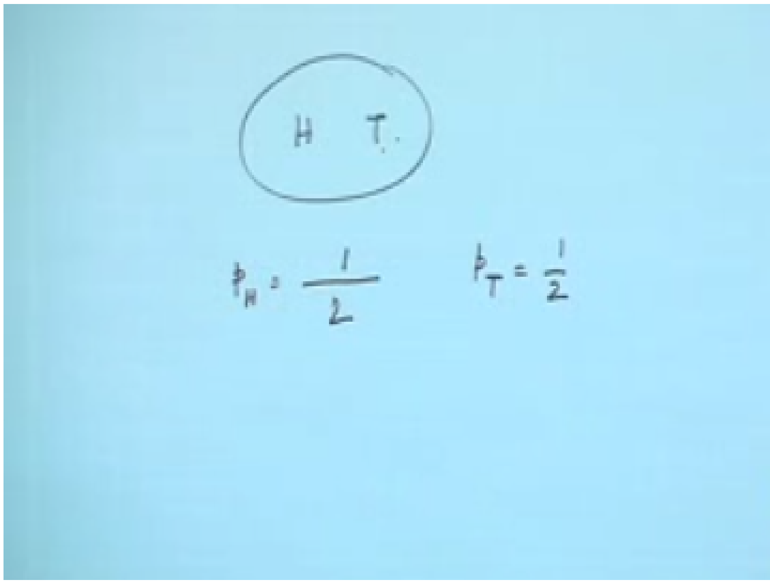

And we say the probability of head will be out of them for that particular event we are calling means this will be more clear or with a bigger example probably right now we just defining it, so for the event we are targeting how many equally likely elements are favorable to that event okay here for head there is only one event which is favorable tell is not favorable towards head because if it is tail then it is not head right.

So only one event are favorable towards this divided by as many events are there equally likely events so too I get the same answer same thing happens for PT that should be1 by 2 right there are two events out of them favorable to head is one okay. So let us just give one little bit bigger example and then probably you will understand a little bit more. So let us try to say suppose I have a six phase die and I roll it and we say that the die is fair that means any surface appearing at the top is equally likely okay.

So that means how many events equally likely events are there already by my definition I get six equally likely events right.

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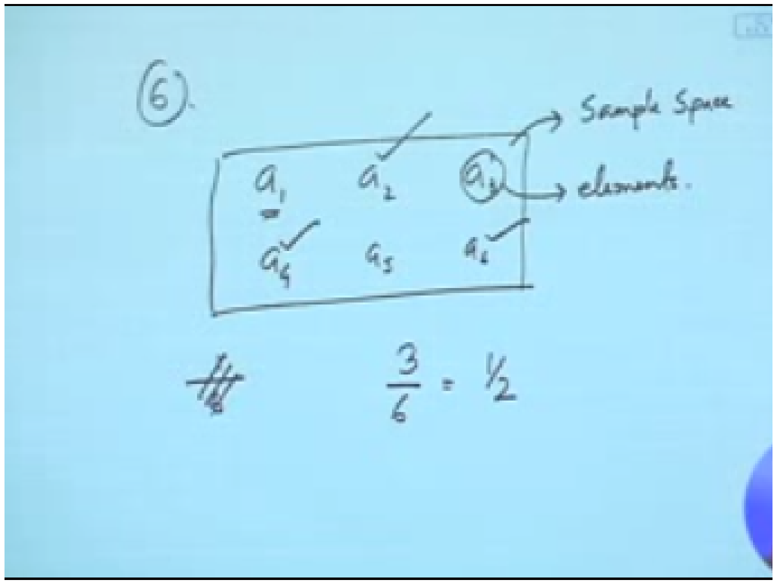

So I can define them as a $_1$ a $_2$ a $_3$ a $_4$ a $_5$ and a $_6$ that means in the top surface one comes top surface 2 comes top surface 3 comes and so on okay. So there are 6 such events and that define my entire space whenever I define this collection of events I call this a sample space that is a definition by definition I call this a sample space and these are the individual element of that sample space, so these are the elements it is just the abstraction we are trying to do okay all of them we have already told by definition are equally likely okay.

Now let us say that I define an event now we just diverting from this I could have defined an event which is that I will be means what is the probability that I will be receiving one in the top surface. So that is actually there are how many are favorable to this particular thing it is only one this one only will give me one all others will be giving me something else. So one divided by how many total events are there six so it should be 1by 6 similarly if I talk about 2 that should be also 1 by 6 and so on.

But if I just now change the definition or change the means the yes definition of event let us say my event is that whenever I roll a die what is the probability that I will be getting a even number okay that is the event definition now. So out of them now you can see the favorable elements, so one can appear to can appear three can appear for can appear five can appears ix can appear out of them this will give me a even number this will give me a even number and this will give me a even number so how many are favorable to my experimentation or my definition of event that is three divided by total number of elements that six so that probability of getting even number is half similarly probability of getting odd number is also half right. So that is how we define event over here okay, so let us say suppose the same thing we take it.

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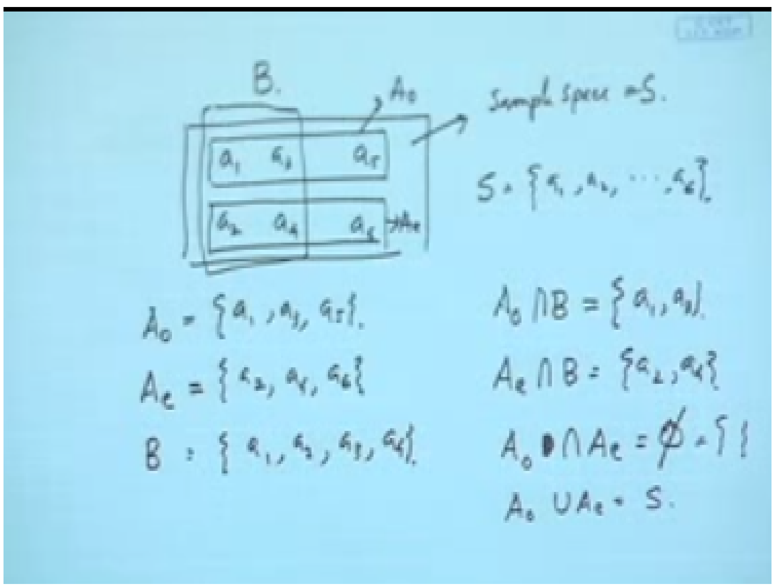

So let us say this is a_1 a_2 a_3 $a_4$$a_5$ and a_6 that is   my sample space I call it s okay, so the sample space now I will give us set theoretic definition of this particular thing so sample space actually is a means group of elements so I can call that as a set so this particular set is defined by these elements a_1 a_2 a_6 six elements are there okay. Out of that I define an event which is called a or I should say a o that means the event that has element which are actually showing me odd number in the top of the surface.

So a o which is this so I can write a o as within the set it is just a subset which has the element a_1 a_3 and a5 right similarly I can write a/e that is   the even number which happens to be this a_2 a_4 a

$_6$ right up to this it is all fine right now I start defining something else because now I will actually go inside the probability theory okay. So let us try to define another event which is telling me that whenever I roll a die my event is that I should observe a number which is less than equal to 4 okay so who should be the part of this particular event that should be these things so either 1 2 3or 4 those are all less than or equal to4 so I call that event be fine so I have defined now three events within the same sample space okay.

I have 6 elements and with that I started defining events so I have a definition of events and accordingly I have defined some three events so this is a $_1$ a $_2$ a $_3$ a $_4$ now in terms of set theory let us start calling the union of event and intersection of events because I have two events now let us try to see if I can define in set theory we know about Union that means taking the elements of means elements either that exist over here or there that is Union and intersection is it should be and that means it should be both existent on both the set or both the I should say collection or subset.

So if I just try to write a zero inter section B what do I get or a o intersection B so this is my ao this is my a so this has element this and this a_1 and a_3 right similarly I can define a e intersection B and this has element a_2 and a_4 okay. So far have still not gone into probability okay I am just trying to see the events and their intersection or Union if I try to write the union of this sorry intersection of this what do I get that is a null set because these two has no common event so I can write that as a null set or I can write it this way no events are included in that okay or no elements are included in that. And of course unions AE must be giving me as the overall set so these are some of the things we have observed now let us try to see if I wish to from there from that definition.

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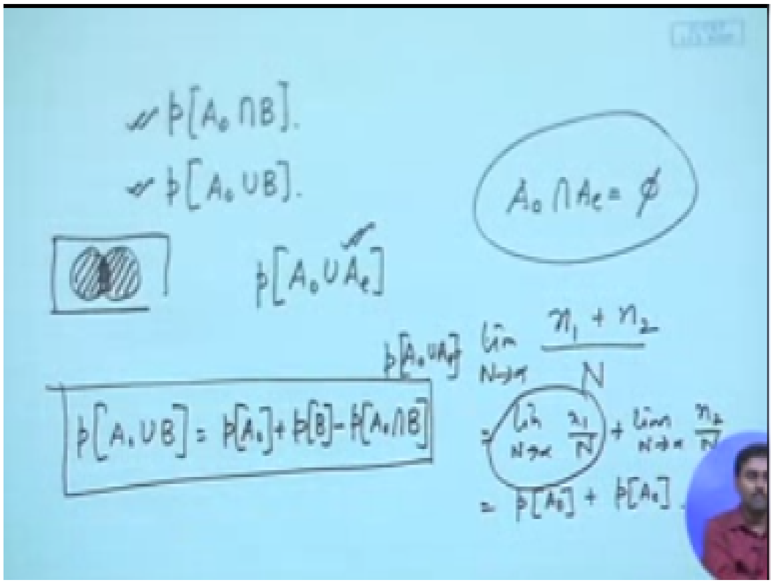

If I wish to calculate the probability that you know intersection B is happening or probability that a Oh union B is happening so this I should say joint event simultaneously a o and B happening so that means I would say that I roll a dice what is the probability that the number I will be seeing that is both even and less than 4 that is this event and the Associated probability and if I say that what is the probability that both are means the event that are happening those are both odd as well as less than 4 so that should be this and associated probability value that I wish to assign okay.

Probability means again that same definition that if I have all outcomes now it is becoming a little bit more complicated earlier it was much more simpler now it is becoming more complicated because I have joint event defined so both the things be satisfied what is the relative frequency of that how many times if I start doing this experiment means that means n number of times I roll the die and if n tends to infinity what should be the number of times that I will be seeing this particular joint event occurring okay so that is what we wish to evaluate okay.

So if I wish to evaluate that how do I evaluate this particular things okay, so first of all let us try to see if I wish to do it for this one let us say probability of this okay probability of a O Union a okay this is something I wish to do now a o and AE has no common element right what I can do I can basically individually calculate the probability axiom says that I can individually calculate the events and their associated probability and I can add them it is very simple it can be easily observable because if I just do the experiment for n number of times whichever number of times this will be happening this will not be happening okay.

So if I observe them as N 1 number of times and if I observe this one as n2 number of times they should be completely if this is happening this will never be happening that is not the case I cannot say that same thing for a o and B I cannot say these things okay. So whenever I will be calculating suppose a O Union B I won't be able to say this thing that a particular event that is favorable towards a oh that is not favorable towards B or it is probably that proper term is mutually exclusive that means if a O happens B will not be happening I cannot say that there are elements where both the things will be happening right and there are elements also where one will be happening the other one will not be happening.
So this kind of complicated definitions are already observed over here but in this case whenever I have this definition that means they are we call this as two sets are mutually exclusive they do not have any common elements if that is the case then whenever this will be happening suppose I keep counting this n1 is the count that you know has happened whenever a o has happened I am sure that the other one has not happened and whenever the other one has happened I am sure that this one has not happened.

So therefore if I just count this one and this one over all this union has happened can be just summation of them I am sure okay there will be no extra term counted if I just add these two wherever in this case there are some common commonality that means there might be one experimentation where I get suppose a o and B I am talking about so I get suppose it is odd number so I get one that is one valid experiment so that particular one I count for which one should I count it for this one or this one basically it will be counted for both of them but this is just one experiment I am counting two times if I just add those counting it will just take extra count.

And obviously that means derived probability value will be wrong of this joint event so that is the problem over here if they are mutually exclusive I can do this and then I can write limit n tends to infinity that is the probability of this event a O Union AE and then I can separate them limit n tends to infinity n 1 by n plus limit n tends to infinity n 2 by n this is exactly the probability of a o, so I can write this as probability of AO plus this is probability of a e so this is actually where probability gets distributed over the Union.

If the events are mutually exclusive okay so that is probably the first axiom of probability theory that we can get if we can define two events which are mutually exclusive then the probability gets added okay if they are not mutually exclusive then what happens so basically what we can see just see it from set theory thirty point of view so I have a set a I have a set B right I wish to do a union B so what will happen if I just take as many times a happens plus as many times B happens then this particular section will be counted twice.

So I have to take that out, so therefore this must be P (a0) + P (b) – p (a0) intersection B I can write this so that is probably the as long as they are not mutually exclusive that is probably the second axiom of probability theory that if two events are not mutually exclusive I have to do it always this way okay this particular case right. So now let us try to see that Union if I wish to calculate if I know individual event occurrence those are good but I also have that intersection, so I have to now try to evaluate the intersection of a occurrence.

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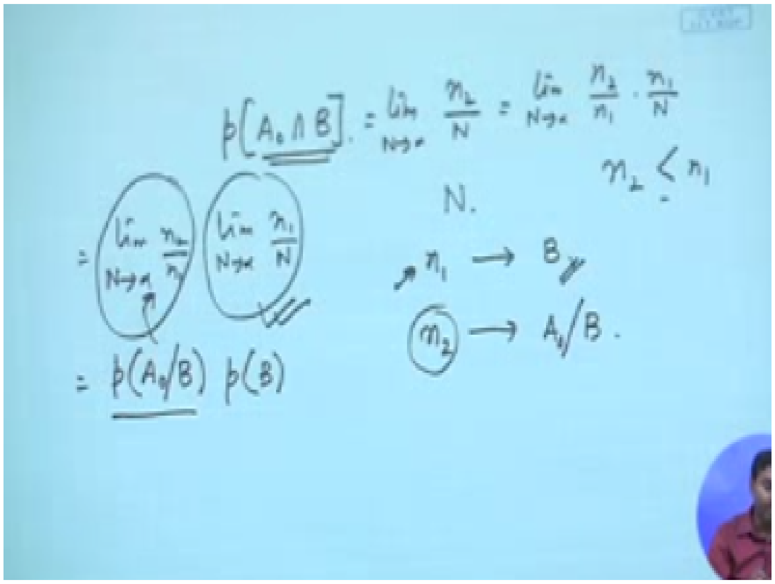

So let us say this I wish to calculate this right so this is something I want to calculate okay so how do I do that? This actually means that both the things has happened right, so let us say things event a zero has also occurred and event B also has occurred let us try to again do the frequency definition let us say I have roll the die for n number of times okay out of them I have seen that in one number of times be is occurring okay.

So basically I am just trying to see that n times I have rolled the dice I am just counting where B has occurred I am not bothered about a o now okay so I am trying to see where B has occurred because I wish to evaluate where a B are both occurring so if a B are both occurring B has to occur so I am just counting those numbers where B has occurred that is N 1 okay.

Out of them how many times a has occurred now there is a restriction given B has occurred because this n 1 times I am filtering out those are the event means those are the outcomes or those are the experiment experimental outcome where I have seen B event B has occurred that means I have seen some suppose B is that less than 4so that has occurred okay out of them I am now trying to evaluate how many times a has occurred let us say that is n_2 so n_2 is actually a number where it is basically this is counted from that set of N_1 those events only I have observed out of them wherever a also has been satisfied I am just counting out them.

So it should be I always know that N_2 must be less than equal to N_1 that should be the case because out of them only I am trying to see okay. So this is where a has occurred given B also has occurred okay so that is the definition or that is how it is being written that event a or I should say event a O has occurred given that B has also occurred okay.

So I can just now get this a intersection B what is the definition of that limit n tends to infinity finally a intersection B where both a and B has occurred that is actually n_2, so it should be n_2/ capital N this n_2 I can write so this I can write as n_2 / N_1 n 2 n_1 divided by n that is simple just both sides I have multiplied and divided by n_1 now let us divide that limit n tends to infinity n_2 / N_1 and limit n tends to infinity n_1 / n.

Now what is this and what is this? This is very clear this was the first experiment right were out of that n number N_1 number of time B has occurred so that must be the probability of B right what is this is actually probability that given B has occurred what is the frequency of a occurring so this must be the probability that a has occurred given B has occurred this particular part because it is n_2 by n_1 so that n_1 number is already telling that B has occurred out of them or out of those experiment where B has already occurred I am trying to see a has occurred so this must be the probability that a o has occurred given B has already occurred.

Because whenever I am trying to calculate this that means I have already ensured that B has occurred okay, so that is where the conditional probability comes into picture this is called the conditional probability this actually means that a particular event conditioned on that a particular event has already occurred what is the associated probability that another particular event will be occurring okay.

So we could now evaluate by this definition frequency definition that what is the intersection probability what will do in the next class we will try to see the implication of these things okay, so we will discuss these things in details in the next class, thank you.